Hash Functions and SHA-3

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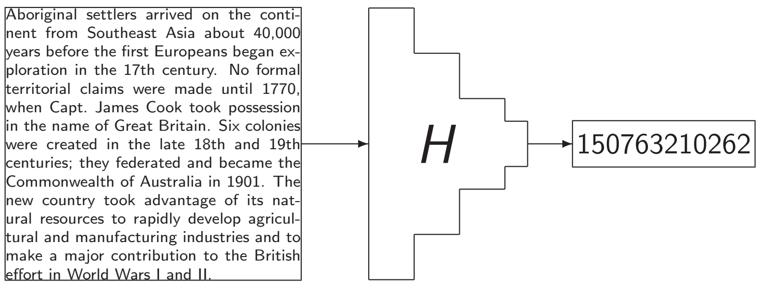
Definition - hash function



$H : \{0,1\}^* \longrightarrow \{0,1\}^n$, for fixed value of n

Generic attacks

For $H : \{0,1\}^* \rightarrow \{0,1\}^n$

attack	rough complexities
collisions	$\sqrt{2^n} = 2^{n/2}$
2nd preimages	2^n
preimage	2^n

Goal: generic attacks are best (known) attacks

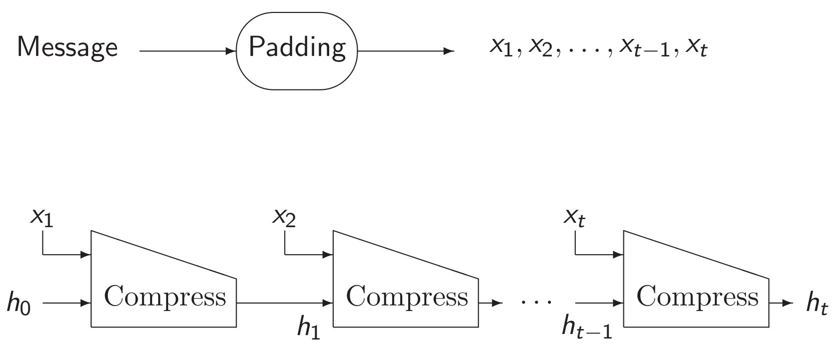
Further properties

- "Behave like" a random oracle
- Indifferentiable from random oracle
- Variants of (seond)-preimage resistance
  - aPre, ePre, aSec, and eSec
- Security against
  - Extension attack
  - Multi-collisions

Structure

- Classical Merkle-Damgård ?
- Sponge ?
- Two chains ?
  - RIPE-MD style
  - Checksums (MD2)
  - Double-pipe

Iterated hash functions - (Merkle-Damgård schemes)



Generic attacks - iterated hash functions

For $H : \{0,1\}^* \rightarrow \{0,1\}^n$

attack	rough complexities
collisions	$2^{n/2}$
2nd preimages	$k2^{n/2} + 2^{n-k}$ with 2^k blocks
preimage	2^n

Merkle (1989)

$h : \{0,1\}^m \rightarrow \{0,1\}^t$, assume $m > t$

- Split message, x, into blocks of $m - t$ bits.
- If last block incomplete, pad with zeros.
- Append extra block containing length of x (bits)
- Define
$$h_{i+1} = h(h_i, x_{i+1}),$$
$H(x) = h_s$.
- Collision for H means collision for h

Damgård (1989)

$h : \{0,1\}^m \rightarrow \{0,1\}^t$, assume $m > t + 1$

- Split message, x, into blocks of $m - t - 1$ bits.
- If last block incomplete, pad with d zeros.
- Append extra block containing bin. repr. of d (fixed length)
- Then define
$$\begin{aligned} h_1 &= h(\mathrm{iv} \mid 0 \mid x_1) \\ h_{i+1} &= h(h_i \mid 1 \mid x_{i+1}) \end{aligned}$$
$H(x) = h_s$.

Damgård (1989) (2)

Parallelizable hash: $h : \{0,1\}^{2t} \rightarrow \{0,1\}^t$

- Message x of j bits.
- Pad message with 0s until length is $2^j t$ for some j.
- Let h_0 be padded message of $2^j t$ bits
- Hash h_0 to h_1 of $2^{j-1} t$ bits using h
- Hash h_1 to h_2 of $2^{j-2} t$ bits using h
- Gives h_j of t bits
- $H(x) = h(h_j \mid \mathrm{length}(x))$

Merkle-Damgård Strengthening, Lai-Massey (1992)

Build $H : \{0,1\}^* \rightarrow \{0,1\}^n$ from $h : \{0,1\}^m \rightarrow \{0,1\}^n$, $m > n$

- Merkle's scheme
$$H : \{0,1\}^N \rightarrow \{0,1\}^n$$
- Damgård's scheme
$$H : \{0,1\}^* \rightarrow \{0,1\}^n$$
- Lai-Massey used Merkle's scheme and named the method *Merkle-Damgård Strengthening*
- collision for $H \Rightarrow$ collision for h

NB! Pad with '1', then zeros, then add message length (blocks) to message

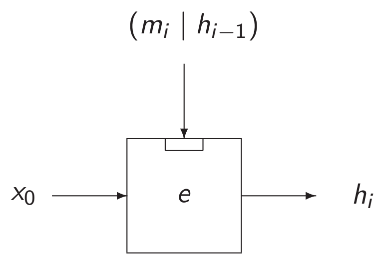
In the beginning there was ...

Diffie and Hellman, 1976. New directions in cryptography.

- Digital signatures .... for efficiency:
- "Let g be a one-way mapping from binary N-space to binary n-space...". "Take the N bit message m and operate on it with g to obtain the n bit vector m'."
- "It must be hard even given m to find a different inverse image of m'"
- "Finding such functions appears to offer little trouble"

Diffie-Hellman, $\kappa > n$

$e : \{0,1\}^\kappa \times \{0,1\}^n \rightarrow \{0,1\}^n$



- x_0 fixed block
- 2nd preimages hard if e secure against known-plaintext attack

Hash function using a block cipher

Why build on a block cipher?

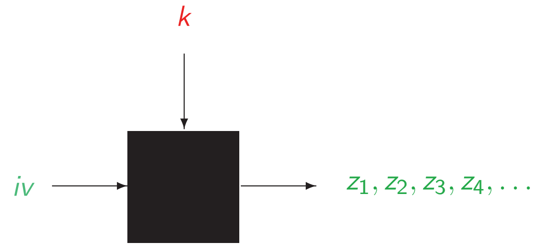
- it's natural !
- use existing technology
- transfer security (trust?!) to hash construction
- schemes "slow" (partly due to key-schedules)
- weaknesses of block cipher not relevant for encryption

Block cipher based hash functions

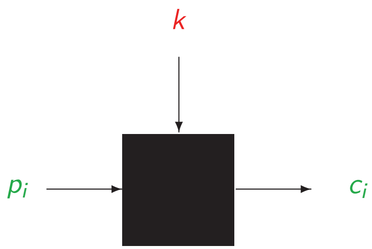
- "Diffusion is more important than confusion in hash functions"
- "Confusion is more important than diffusion in block ciphers"
- Why? Why not have S-boxes in hash functions ?
- How fast should/can a hash function be ?

Speed ..

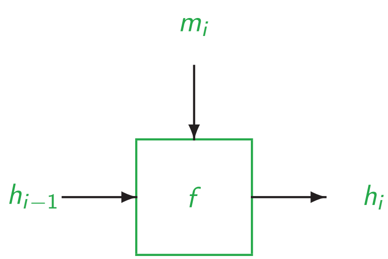
Additive stream cipher



Block cipher



Hash function



Speed ..

- Additive stream cipher, known/chosen plaintext attack
- Block cipher, chosen plaintext attack
- Hash function, known/chosen-key attack
-

Stream	4-8	cycles/byte
AES	20	cycles/byte
SHA-1	11	cycles/byte
SHA-512	18	cycles/byte

DES & AES

DES = Data Encryption Standard

AES = Advanced Encryption Standard

system	year	block size	key size
DES	1977	64	56
AES	2001	128	128, 192 or 256

Hash rate

- Given hash function built from block cipher

$$e : \{0,1\}^\kappa \times \{0,1\}^n \rightarrow \{0,1\}^n$$

- Rate usually is defined as

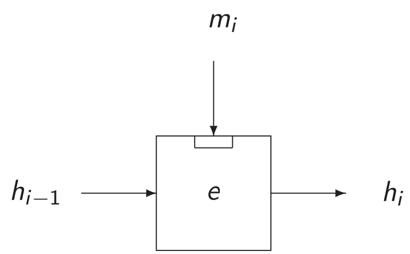
$$\frac{\#\ n\text{-bit blocks hashed}}{\#\ \text{invocations of } e}$$

- Ought perhaps be defined as

$$\frac{\#\ n\text{-bit blocks hashed}}{\#\ \text{invocations of } e + \#\ \text{key-schedules}}$$

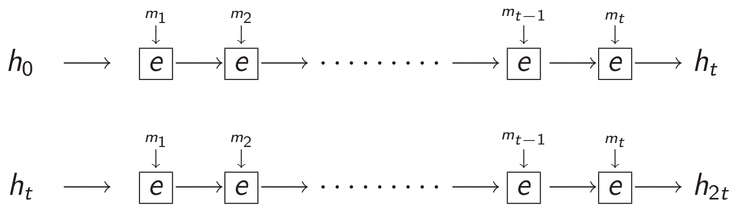
Rabin, 1978

$e : \{0,1\}^\kappa \times \{0,1\}^n \rightarrow \{0,1\}^n$



- rate $= (\kappa/n)/(1+1)$
- Yuval: collisions based on birthday paradox (79) (Merkle 79)
- Pre-images in approximately same time

Davies-Price variant of Rabin's scheme 1980



- Coppersmith 1985:
  - preimage attack on one-chain Rabin $\approx 2^{n/2}$
  - preimage attack on two-chains Rabin $\approx 2^{n/2+n/16}$ using multi-collisions

Single block hash

- $e : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$
- 12 secure ones (Preneel 93, Black et al 02), here three

$$h_i = e_{m_i}(h_{i-1}) \oplus h_{i-1} \quad \text{Davies-Meyer}$$
$$h_i = e_{h_{i-1}}(m_i) \oplus m_i \quad \text{Matyas-Meyer-Oseas}$$
$$h_i = e_{h_{i-1}}(m_i) \oplus m_i \oplus h_{i-1} \quad \text{Preneel-Miyaguchi}$$

- Hash rates. About 1/(1+1) (1/2 for DES and AES)
- Collisions (birthday attack) in $2^{n/2}$ operations

MD4-family

- MD4, Rivest 1990
- MD5, Rivest 1991
- SHA-0, 1993
- SHA-1, 1994
- all hash functions of Davies-Meyer form
- "block ciphers" with feed-forward
- hash rates for Davies-Meyer can be (arbitrarily) high

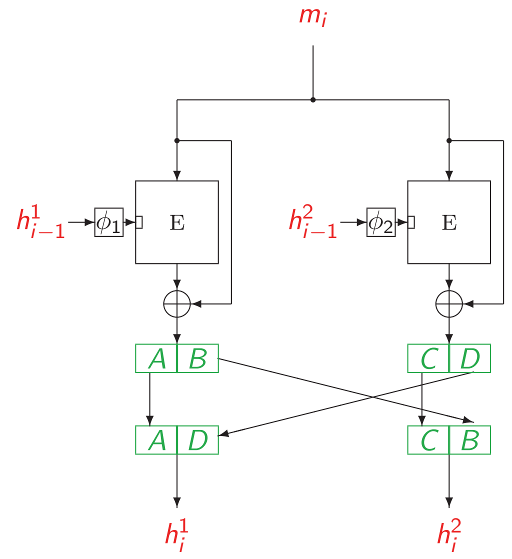
Double block hash - based on block ciphers

- Based on $e : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$
- Length of hash, $2n$ bits
- Aim: 2^n security level for collisions
  - Merkle, 1989
  - MDC-2, Brachtl, Coppersmith et al 1988/1990
  - PBGV, QG, LOKI-DBH, ..., 1990s
  - Hirose, Nandi, 2005

Merkle's double block schemes with DES (1989)

- "DES can be used to build a one-way hash function which is secure"
- if DES fails "it seems almost certain that some block cipher exist with the desirable properties"
- proof of security in ideal cipher model
- collisions $\approx 2^{55}$, inconvenient block sizes, low hash rates
- "recent proposal from IBM looks very hopeful", but no proof..

MDC-2



MDC-2

- designed for DES but can be used with any block cipher
- hash rate 1/(2+2) (1/4 for DES and AES)
- 1992: Coppersmith "defends" MDC-2

MCD-2 used with DES and AES

(Best known attacks)

	DES	AES
Preimage attack	2^{83}	2^{192}
2nd preimage attack	2^{83}	2^{192}
Collision attack	2^{55}	2^{128}
Hash rate	1/4	1/4

- For use with AES, "proof" that collision requries $> 2^{75}$ operations (Steinberger 2007)

Abreast-DM & Tandem-DM - Lai, Massey 1990

$e : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$, $\kappa > n$ $\qquad f(x,y) = e_x(y) \oplus y$

Abreast-DM scheme: $\begin{cases} h_i^1 = f(h_{i-1}^2 \parallel m_i\,,\ h_{i-1}^1) \\ h_i^2 = f(m_i \parallel h_{i-1}^1\,,\ \overline{h}_{i-1}^2) \end{cases}$

Tandem-DM scheme: $\begin{cases} h_i^1 = f(h_{i-1}^2 \parallel m_i\,,\ h_{i-1}^1) \\ h_i^2 = f(m_i \parallel (h_i^1 \oplus h_{i-1}^1)\,,\ h_{i-1}^2) \end{cases}$

AES-256, (128-bit block, 256-bit key), hash rate 1/4, conjectured security level for collisions 2^{128}

Knudsen-Preneel 1996 $\quad f_i(x,y) = e_x(y) \oplus y$

Compress: $(h_{i-1}^1, \ldots, h_{i-1}^5, m_i) \to (h_i^1, \ldots, h_i^5)$

$$\begin{aligned} h_i^1 &= f_1(h_{i-1}^1\,,\ h_{i-1}^2) \\ h_i^2 &= f_2(h_{i-1}^3\,,\ h_{i-1}^4) \\ h_i^3 &= f_3(h_{i-1}^5\,,\ m_i) \\ h_i^4 &= f_4(h_{i-1}^1 \oplus h_{i-1}^3 \oplus h_{i-1}^5\,,\ h_{i-1}^2 \oplus h_{i-1}^4 \oplus m_i) \\ h_i^5 &= f_5(h_{i-1}^1 \oplus h_{i-1}^3 \oplus h_{i-1}^4 \oplus m_i\,,\ h_{i-1}^2 \oplus h_{i-1}^3 \oplus h_{i-1}^5 \oplus m_i) \end{aligned}$$

Constructed from $[5,3,3]$ code over $GF(2^2)$: rate 1/(5+5)
Claimed security against collision attacks is 2^n

Knudsen-Preneel, more examples

Better rates using codes over larger fields

$GF(2^2)$		$GF(2^4)$		Collision
Code	Rate	Code	Rate	
$[5,3,3]$	$1/(5+5)$	$[6,4,3]$	$2/(6+6)$	$\simeq 2^n$
$[8,5,3]$	$2/(8+8)$	$[8,6,3]$	$4/(8+8)$	$\simeq 2^n$
$[12,9,3]$	$6/(12+12)$	$[12,10,3]$	$8/(12+12)$	$\simeq 2^n$

AES-128, rate 1/3, conjectured security level for collisions 2^{128}

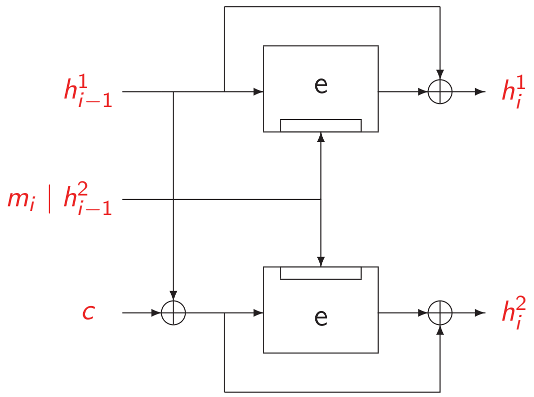
Hirose's double block mode 2006

Based on work by Nandi, 2005
$e : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$, $\kappa > n$, c nonzero constant

$$\begin{aligned} h_i^1 &= e_{h_{i-1}^2 \,|\, m_i}(h_{i-1}^1) \oplus h_{i-1}^1 \\ h_i^2 &= e_{h_{i-1}^2 \,|\, m_i}(h_{i-1}^1 \oplus c) \oplus h_{i-1}^1 \oplus c \end{aligned}$$

- Collision requires 2^n operations assuming $e(\cdot,\cdot)$ is ideal cipher
- AES-256, hash rate 1/3, security level 2^{128} for collisions

Hirose's double block mode, figure



Ideal cipher model ?

- proofs in model give protection against generic attacks
- no real-life cipher is an ideal cipher; "nearly ideal" cipher can be strong for encryption but very weak when used for hashing
- attacker in control of key can invest time in finding key(s) with certain properties

Known-key distinguishers - Knudsen, Rijmen 2007

- Block cipher cryptanalysis with applications to hash functions
- With a given (random) key, produce set of texts with "non-random" statistical behaviour
- Most short-cut attacks on block ciphers exploit statistical properties of plain- and ciphertexts in (reduced) cipher
- If such properties cannot be found given the key, it seems unlikely that they can be found when **not** given the key

Known-key distinguishers - examples

- Example 1. Generic 7-round Feistel cipher.
  - given a key, one can find (in time $\mathcal{O}(1)$) two texts such that

$$\Delta(\delta, \alpha) \rightarrow \Delta(\delta, \beta)$$

- Example 2. AES reduced to seven rounds
  - given a key, one can find 2^{56} texts balanced in all bytes of plain- and ciphertexts

Known-key distinguishers

- DES:
  - key-recovery attack, 2^{43} known texts
  - collision attack, 2^{32} operations (best known)
- SHACAL-1:
  - block cipher built from SHA-1
  - 160-bit blocks, 512-bit keys
  - best known attacks today:
    key-recovery attack on SHACAL-1 has complexity $\approx 2^{500}$
    collision attack on SHA-1 has complexity $\approx 2^{60}$

Known-key distinguishers

- SHACAL-1 has a weak key-schedule !
- Due to lack of S-boxes ?
- What makes a good key-schedule ? Very little research done

SMASH - Knudsen, 2005

- Idea: build collision-resistant hash function from one bijective mapping
- Why? we know how to make one, strong bijective mapping (Not a family of bijections !?)
- let f be a strong, bijective mapping of sufficient size

$$h(h_{i-1}, m_i) = f(m_1 + h_{i-1}) + m_1 + \theta h_{i-1}$$

- Compression function **not** collision-resistant
- 2nd preimages in $2^{n/2}$ operations
- Proposal broken by Rijmen, Rechberger, Pramstaller, 2005

Grindahl - Knudsen, Rechberger, Thomsen 2007

- Daemen-style hash construction, sponge
- Iterated hash function
- "Rijndael"-state, 4×13 byte-matrix
- MixColumns, SubBytes same as for AES
- Compression function invertible
- Meet-in-the-middle preimage attack with birthday attack complexity
- Short-cut attack, Peyrin 2007

Hash based on fixed functions

- Preneel, 1992
- Black et al, 2005: Provably secure (collision-resistant) iterated hash functions based on one bijective mapping do not exist (information-theoretic setting)
- Shrimpton-Stam, 2006:
  - let f_1, f_2, f_3 be three, distinct functions, then define:

$$h(h_{i-1}, m_i) = f_1(m_1) + f_3(f_1(m_1) + f_2(h_{i-1}))$$

  - collisions $\Theta(2^{n/2}/n)$, preimages suboptimal $(2^{2n/3})$
- Rogaway-Steinberger, 2008
  - at least three bijections needed
  - at least five bijections needed in double-block hash mode

SHA-3

SHA-3 - Call for candidates

- announcement: October 29, 2007
- must provide digests of 224, 256, 384, and 512 bits, not 160.
- available worldwide royalty-free, no IPR
- capable of protecting sensitive information for decades
- should be suitable for
  - digital signatures, FIPS 186-2
  - HMAC, FIPS 198
  - key establishment, SP 800-56A
  - random number generation, SP 800-90
- security strength at least that of the SHA-2s with **significantly** improved efficiency

SHA-3 - Desirable properties

- efficient integral options, e.g., randomized hashing, that "fundamentally improve security"
- parallelizable
- avoid "generic properties" of Damgård/Merkle constructions
- attack on SHA-2 should not lead to attack on SHA-3
- flexible for a wide variety of implementations
- a single family, except if good arguments for more families
- tunable security parameter, e.g., number of rounds, with recommendations

SHA-3 - Security

Message digest of n bits

- Collisions should take $2^{n/2}$
- Preimages should take 2^n
- 2nd preimages should take 2^{n-k} for messages shorter than 2^k bits

Higher levels of security against 2nd preimage will be viewed positively

- NIST open to other designs than Damgård/Merkle

SHA-3 - Timeline

- hard submission deadline: 31/10-2008
- submissions by 31/8-2008 checked by NIST for inconsistencies
- Round 1: 12 months. Workshop 1. Workshop 2.
  No modifications during Round 1.
- Round 2: $\approx$ 5 candidates selected. 12-15 months. Tweaks allowed. Workshop 3.
- AHS(s).
- documentation and testing like AES
- review is public

Outtro

- Hash functions are important for many things in cryptology and we are asking for very strong properties
- No apparent reason why such functions can/should be very fast... ?
- NIST do not really invite for block cipher based proposals
- NIST: "a successful collision attack on an algorithm in the SHA-2 family could have catastrophic effects for digital signatures"
- So better not make a hash of it...