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Public Key Perturbation of Randomized RSA Implementations

A. Berzati, C. Dumas & L. Goubin

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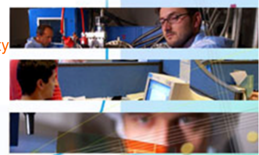

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Outline

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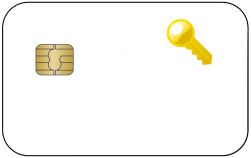
1. Introduction
   - Differential Fault Analysis
   - RSA Public Key Perturbations
2. Public Key Perturbation Against R2L Implementations
3. Application to Randomized RSA Implementations
4. Conclusion

Differential Fault Analysis (DFA) [BS97]

- Perturbation of an electronic device behavior
  - Supply voltage, clock or temperature variations
  - White light, ion or laser beams



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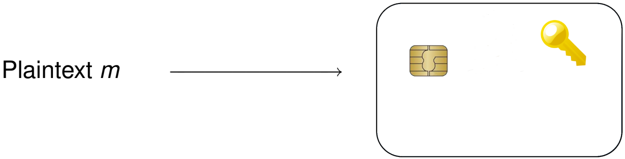

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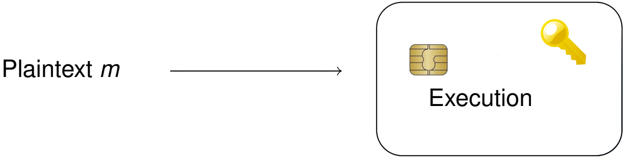

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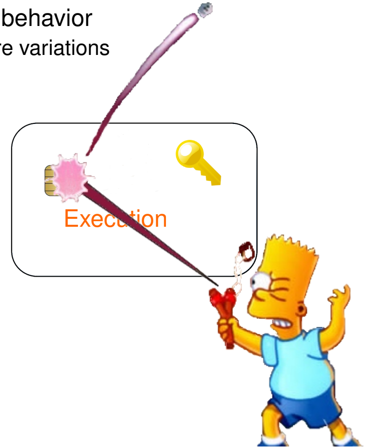
Plaintext m $\longrightarrow$



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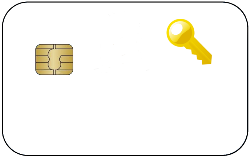
Plaintext m $\longrightarrow$

Ciphertext $\hat{C}$ $\longleftarrow$



Differential Fault Analysis (DFA) [BS97]

Analysis of the faulty output

- Identification of the perturbation
- Choice of a fault model
- Differentiation of correct and faulty outputs

$$\Delta_{\hat{C},C} = f(\varepsilon, k)$$

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Applications to implementations of cryptosystems

- Symmetric: DES [BS97], AES [DLV03], [HS04] ...
- Asymmetric: RSA [BDL97], RSA-CRT [BDL97], ...
- Stream ciphers: RC4 [HS04, BGN05], A5/1 [GKW05], Grain-128 [BCC+09], ...

Outline

RSA Signature scheme

Key Generation

- Pick large primes $\mathbf{p}$ and $\mathbf{q}$ and compute $\mathbf{N} = \mathbf{p} \cdot \mathbf{q}$
- Pick a random $\mathbf{e}$ such that $\gcd(\mathbf{e}, \varphi(N)) = 1$
- Compute $\mathbf{d} \equiv \mathbf{e}^{-1} \bmod \varphi(N)$
- The public key is $(\mathbf{e}, \mathbf{N})$
- The private key is $\mathbf{d}$

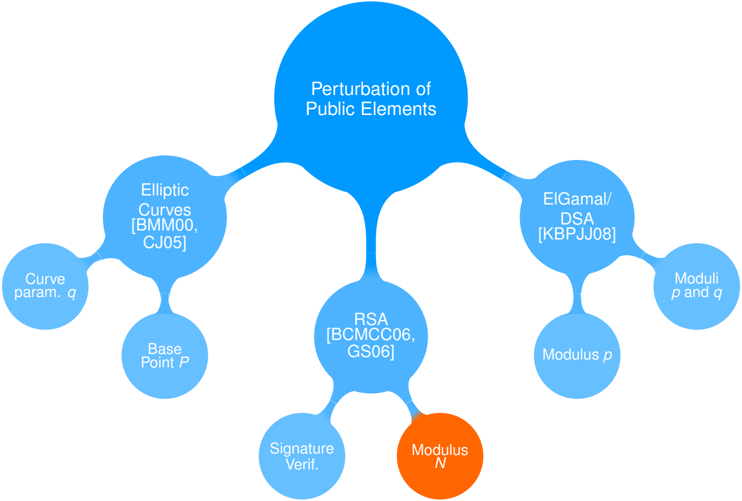
Signature

- Compute $\mathbf{S} \equiv h(m)^{\mathbf{d}} \bmod \mathbf{N}$
- Return $(\mathbf{S}, m)$

Verification

- Check that $\mathbf{S}^{\mathbf{e}} \equiv h(m) \bmod \mathbf{N}$

Previous Work



Previous Work for RSA

- *"Why one should also secure RSA Public Key Elements"*
  E. Brier *et al.*, CHES'06 [BCMCC06]
  - Provide a full private key extraction
  - The modulus is modified before the modular exponentiation:

$$\hat{S} = h(m)^{d} \bmod \hat{N}$$

  - Countermeasure: Exponent randomization

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Our contributions

- CHES'08 [BCG08]: Exploit faults on the modulus that occur during a *"Right-To-Left"* modular exponentiations
- CT-RSA'09 [BCDG09]: Generalization to *"Left-To-Right"* modular exponentiations

Outline

Fault Model

Fault model

- Transient random byte modification of **N**
- Perturbation of a modular square t bits before the end of the exponentiation
- Time location of the fault known by the attacker

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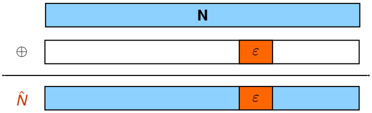
- Illustration of a faulty modulus **N**



where ε is a random byte value

Example of Faulty Execution

(N, e) and **d**

(N, e)

"*Right-to-Left*" Algorithm

Input: m, d, N
Output: $A = m^d \bmod N$

```
A:=1;
B:=m;
for i from 0 upto (n − 1)
    if (d_i == 1)
        A := (A · B) mod N;
    end if
    B := B^2 mod N;
end for
return A;
```

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Faulty RSA Signature

- Fault injection:
  $A_i = A_{i-1} \cdot B_{i-1}{}^{d_i - 1} \bmod \mathbf{N}$ et
  $B_i = B_{i-1}^2 \bmod \hat{N}$

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- Subsequent iteration:
  $A_{i+1} = A_i \cdot B_i{}^{d_i} \bmod \hat{N}$ et
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Example of Faulty Execution

$(\mathbf{N}, \mathbf{e})$ and $\mathbf{d}$

$\left(m, \hat{S}\right) \longrightarrow$

$(\mathbf{N}, \mathbf{e})$

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- Returned faulty signature:

$$\hat{S} = (A_i \bmod N) \cdot m^{d_{[t]}} \bmod \hat{N}$$

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2 Public Key Perturbation Against R2L Implementations

- Attack Model
- Fault Analysis

3 Application to Randomized RSA Implementations

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Private Key Recovery from Faulty RSA Signatures

Faulty RSA signature analysis

- A part of the private key $d_{[t]}$ is isolated
- Recovery of $d_{[t]}$ and $\hat{N}$ from the pair $\left(S, \hat{S}\right)$ and the fault model
  $\Rightarrow$ *Guess-and-determine* approach
- The right pair is found with high probability

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1. Gather sufficiently many signatures faulted at different steps
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Key extraction on a PC for a 1024-bit RSA

- 250 faulty signatures
- A few dozen minutes for the analysis

Outline

Exponent Randomization

- Proposed by P. Kocher in 1996 [Koc96], formalized by J.S. Coron at CHES'99 [Cor99] to defeat side channel attacks
- Based on Fermat's theorem:

$$m^{\varphi(N)} \equiv 1 \bmod N$$

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RSA Exponent Randomization Algorithm

Input: $\dot{m}, N, \varphi(N), d$ and the length l
Output: $S = \dot{m}^d \bmod N$

1: *//Randomize the private exponent*
2: Pick a random $\lambda \in [\![0; 2^l - 1]\!]$;
3: $\bar{d} = d + \lambda\varphi(N)$;
4: *//Perform the exponentiation*
5: $S = \texttt{PowMod}(\dot{m}, \bar{d}, N)$;
6: **return** S;

- Typically for a 1024-bit RSA : $l = 20$ or 32 bits

Efficiency Against Public Key Perturbations

Difficulty due to Exponent Randomization

- The fault injection isolates a part of $\bar{d}$ instead of d
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Solution

- Randomization is not homogeneous
- Such a perturbation isolates a part of $\bar{d}$

$\Rightarrow$ $\bar{d}$ may leak information on d

Outline

Non-Homogeneity of the Masking Operation

Details of the Randomization

$$\bar{d} = d + \lambda\varphi(N)$$

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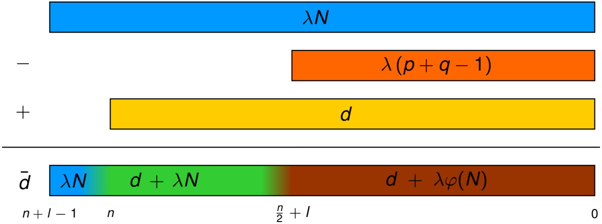

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Principle of the Attack: MSB Case (1/2)

Approximation on MSB

$$\bar{d} \approx d + \lambda N$$

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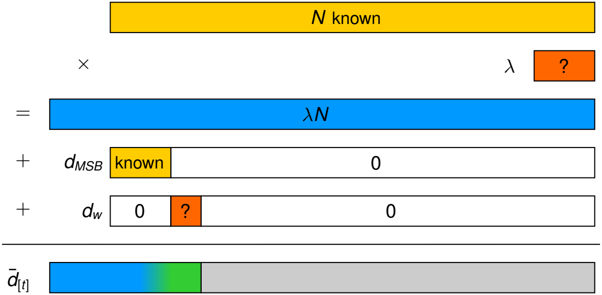

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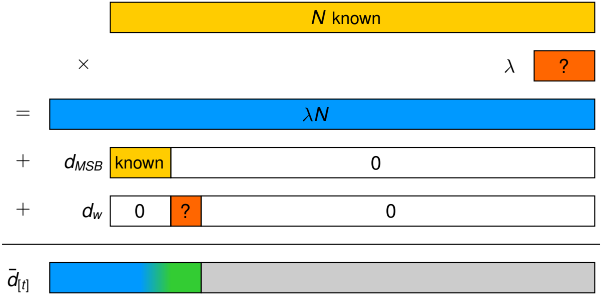

$\Rightarrow$ Guessing (λ, d_w) enables to compute good candidate values for $\bar{d}_{[t]}$

Principle of the Attack: MSB Case (2/2)

Theorem

Let $\hat{S}_t$ be a faulty signature performed under an exponent randomized by λ, and S the corresponding correct signature. For all candidate pairs $(d'_w, \lambda') \in [\![0; 2^w]\!] \times [\![0; 2^l]\!]$, if $\lambda' > \lambda$, then (8) can not be satisfied.

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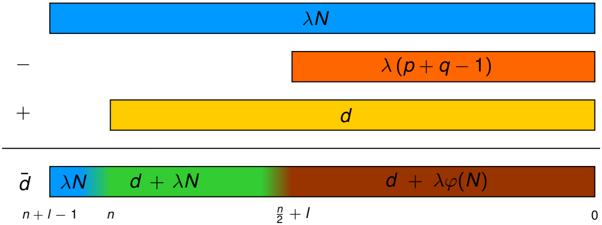
$\Rightarrow$ If $t > l$, λ can be exactly determined by building good values for $\bar{d}_{[t]}$

Faulty randomized RSA signatures analysis

1. Inject a fault on **N** during a signature
   $\Rightarrow$ A part of the blinded key $\bar{d}_{[t]}$ is isolated
2. Determine $\bar{d}_{[t]}$ and $\hat{N}$ from the pair $\left(S, \hat{S}\right)$
   $\Rightarrow$ Good candidates for $\bar{d}_{[t]}$ are built from a known part of the private key d_{MSB}, and candidate pairs for d_w and λ
   $\Rightarrow$ Candidates for $\hat{N}$ are built according to the fault model
3. Update the known part of the key d_{MSB} and repeat the analysis on signatures faulted earlier until the most significant part of **d** is determined

Principle of the Attack: LSB Case

- Previous approximation not valid for LSB



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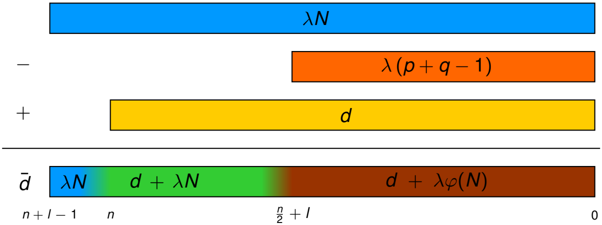

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Solution

- Get one more faulty signature to analyze (2 in practice)
- Make a variable change to boil down to the MSB case
- Solve a system to extract bits of **d**

Results

Complexity evaluation

- Estimated fault number

$$\mathcal{F} = \mathcal{O}\left(\frac{n}{w}\right) \text{ signatures}$$

- Computational complexity

$$\mathcal{C} = \mathcal{O}\left(\frac{2^{(l+w)} \cdot n^2}{w}\right) \text{ exponentiations}$$

$\Rightarrow$ Possible improvement: combine it with Coppersmith Attacks

Key extraction on a PC for a 1024-bit Randomized RSA

- $l = 20$ bits, $w = 2$ (bits of d recovered by pairs)
- 1000 faulty signatures
- Roughly 2^{40} exponentiations

Conclusion

- Physical robustness of the countermeasure
  - Randomized Exponentiation
    $\Rightarrow$ *"Doubling Attack"* [FV03]
    $\Rightarrow$ Small Public Exponent [FKJM^{+}06]
  - Blinding Operation
    $\Rightarrow$ *"Carry Analysis"* [FRVD08]

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First fault attack against randomized RSA

- Answer an open problem raised by E. Brier *et al.* at CHES 2006 [BCMCC06]
- Realistic fault model
- Reasonable performances

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- Realistic fault model
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Perspectives

- Exponent blinding does not provide a strong hardware security
- What about homogeneous blinding operation ?

Thank you !


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