

Algebraic Side-Channel Attacks on the AES: Why Time also Matters in DPA

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CHES'09 – September 2009



Outline

Introduction

Algebraic Side-Channel Attack

Comparison with DPA

Advanced scenarios

Conclusion



Outline

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Algebraic Side-Channel Attack

Comparison with DPA

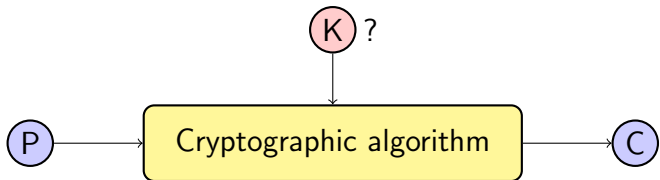
Advanced scenarios

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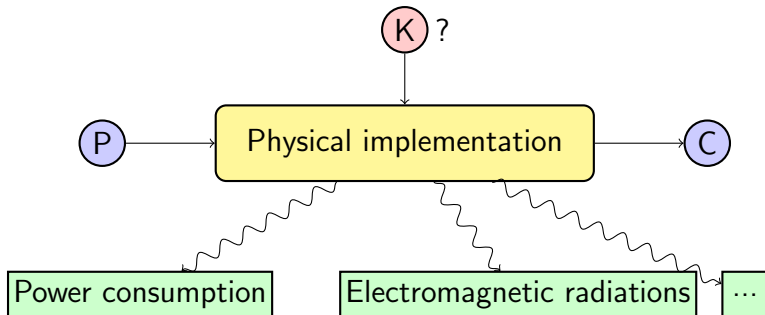
Introduction

Classical cryptanalysis



Introduction

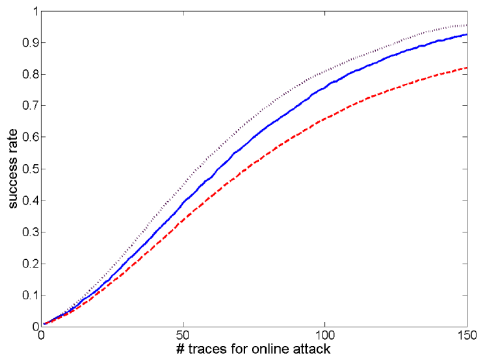
Side-Channel cryptanalysis



Introduction

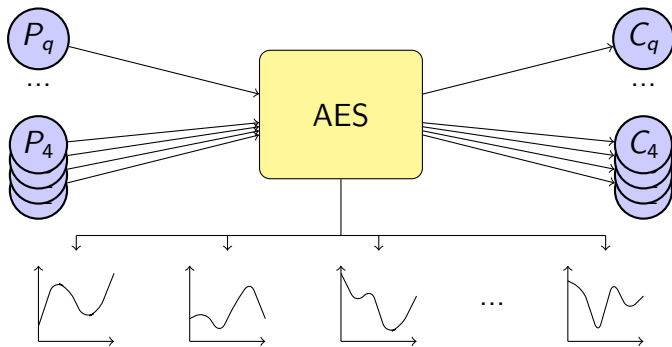
Open questions

What is the smallest possible data complexity?



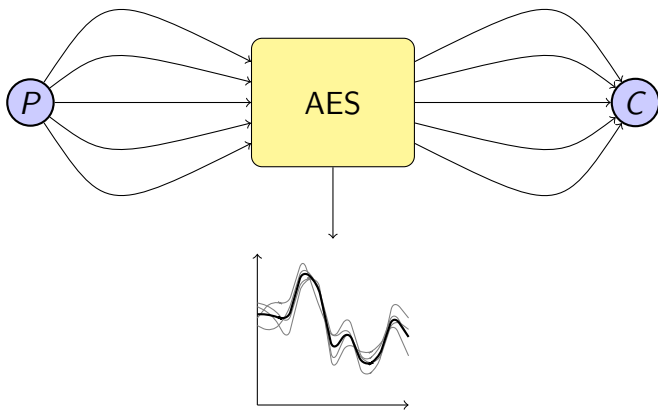
Introduction

Data complexity q

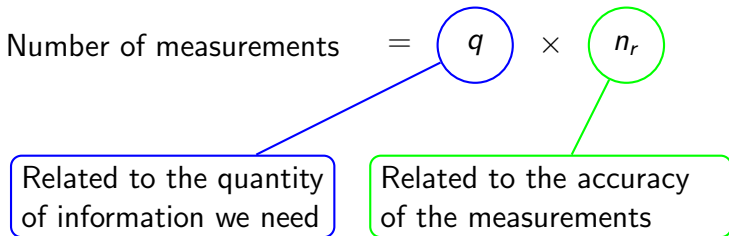


Introduction

Repetition number n_r



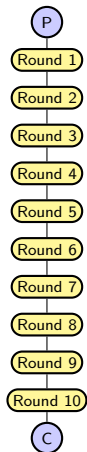
Introduction



What is the minimum value for q ?



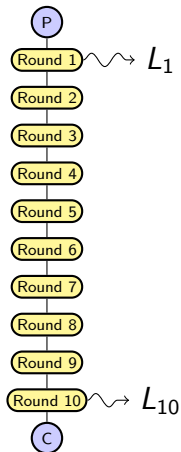
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Which information can be used?



Introduction

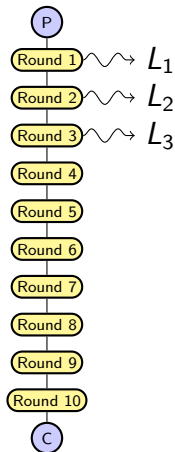


DPA uses information from the first or last round (low diffusion)

Which information can be used?



Introduction

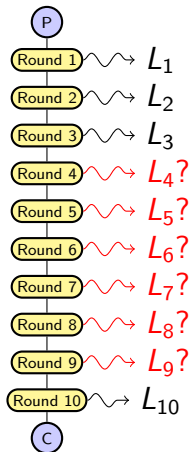


Collision attacks can use leakages up to the third round

Which information can be used?



Introduction



What about the middle rounds?

Which information can be used?



Introduction

Most of the side-channel attacks aim to directly recover the key bits



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Is it possible to use side-channel to recover less informative (easier) targets ...

... and combine this with a classic cryptanalysis phase to recover the key?



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Is it possible to use side-channel to recover less informative (easier) targets ...

... and combine this with a classic cryptanalysis phase to recover the key?

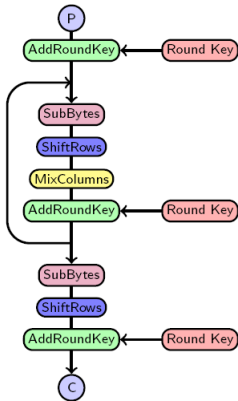
We decided to use *algebraic cryptanalysis*.



Introduction

A block cipher ...

becomes a big set of low degree boolean equations

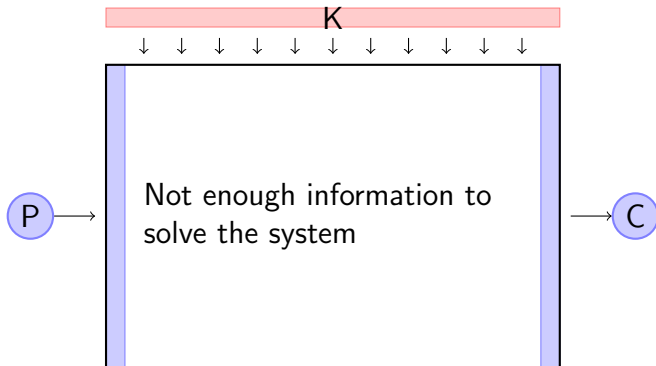


$$\begin{aligned} 0 &= x_2 + x_{11} + x_{14} + x_2 x_5 + x_4 x_{10} \\ 0 &= x_5 + x_9 + x_1 x_6 + x_3 x_{10} x_{13} + x_5 x_7 x_{16} \\ 1 &= x_7 + x_{15} + x_{16} + x_3 x_7 + x_9 x_{11} \\ 0 &= x_7 + x_1 x_5 + x_6 x_{11} + x_4 x_5 x_{12} \\ 1 &= x_8 + x_{10} + x_{11} + x_{15} + x_2 x_6 + x_6 x_{10} \\ 0 &= x_{10} + x_{15} + x_{16} + x_6 x_{12} + x_9 x_{16} + x_2 x_9 x_{14} \\ 0 &= x_{13} + x_{14} + x_5 x_{13} + x_8 x_{12} + x_4 x_7 x_{12} + x_8 x_9 x_{11} \\ &\dots \end{aligned}$$



Introduction

Classic algebraic attack

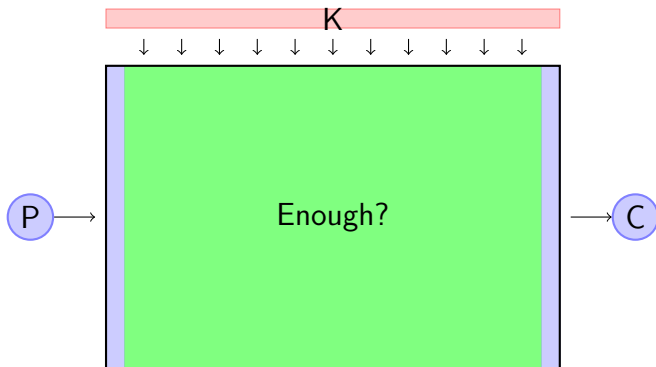


256 known bits



Introduction

Algebraic attack + side-channel information



256 known bits + side-channel information



Introduction

Related work

Square attacks + side-channel

- ▶ V. Carlier, H. Chabanne, E. Dottax, H. Pelletier, 2005.

Differential cryptanalysis + side-channel

- ▶ H. Handschuh, B. Preneel, 2006.

Collision attacks

- ▶ A. Biryukov, D. Khovratovich, 2007.
- ▶ A. Bogdanov, 2007.
- ▶ A. Bogdanov, I. Kizhvatov, A. Pyshkin, 2008.



Outline

Introduction

Algebraic Side-Channel Attack

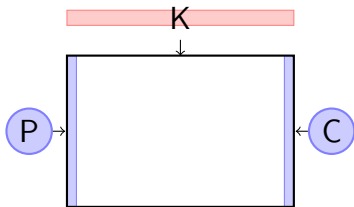
Comparison with DPA

Advanced scenarios

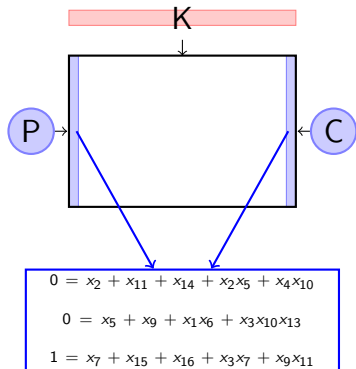
Conclusion



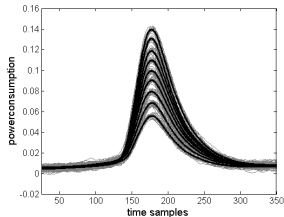
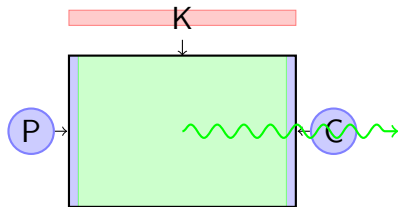
Algebraic Side-Channel Attack



Algebraic Side-Channel Attack



Algebraic Side-Channel Attack



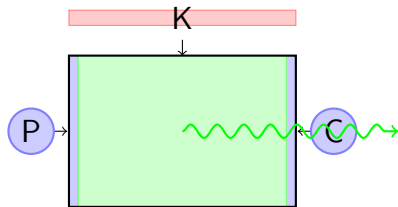
$$0 = x_2 + x_{11} + x_{14} + x_2 x_5 + x_4 x_{10}$$

$$0 = x_5 + x_9 + x_1 x_6 + x_3 x_{10} x_{13}$$

$$1 = x_7 + x_{15} + x_{16} + x_3 x_7 + x_9 x_{11}$$



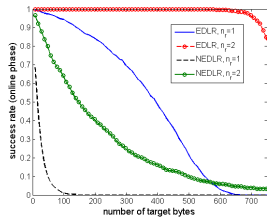
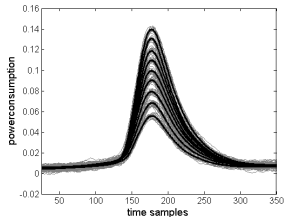
Algebraic Side-Channel Attack



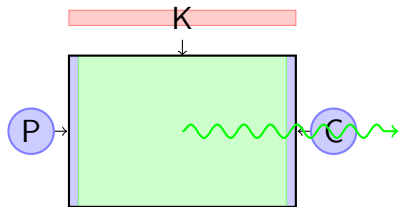
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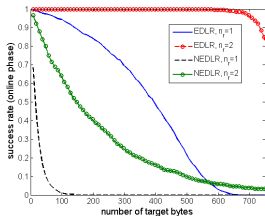
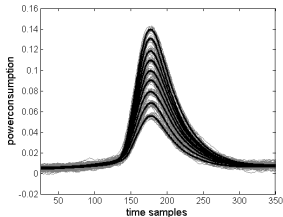


Algebraic Side-Channel Attack

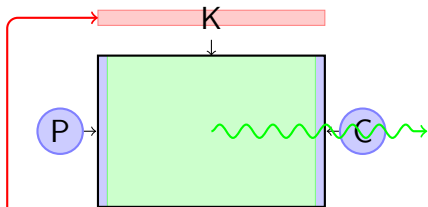


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 \end{aligned}$$

$$\begin{aligned}
 0 &= x_7 + x_1 x_5 + x_6 x_{11} + x_4 x_5 x_{12} \\
 1 &= x_8 + x_{10} + x_{11} + x_{15} + x_2 x_6 + x_6 x_{10} \\
 0 &= x_{10} + x_{15} + x_{16} + x_6 x_{12} + x_9 x_{16}
 \end{aligned}$$



Algebraic Side-Channel Attack



$$0 = x_2 + x_{11} + x_{14} + x_2 x_5 + x_4 x_{10}$$

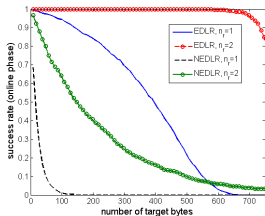
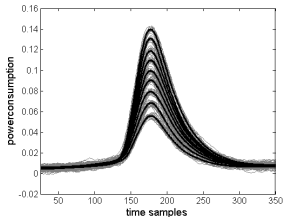
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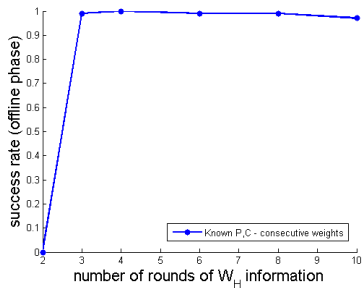
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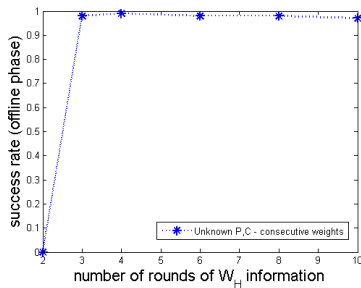
$$0 = x_{10} + x_{15} + x_{16} + x_6 x_{12} + x_9 x_{16}$$



Algebraic Side-Channel Attack



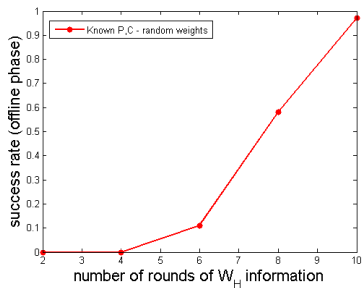
Known P, C, consecutive weights



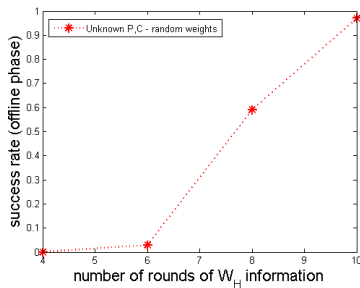
Unknown P, C, consecutive weights



Algebraic Side-Channel Attacks



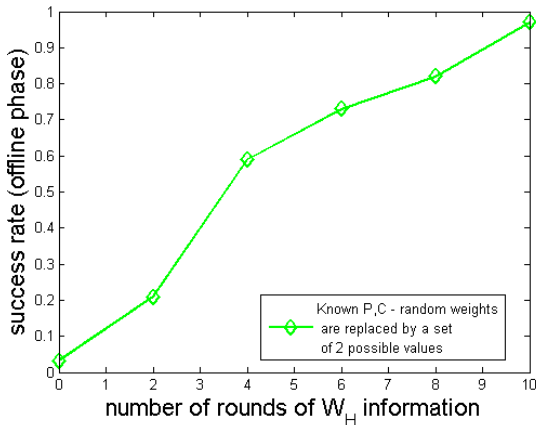
Known P, C, random weights



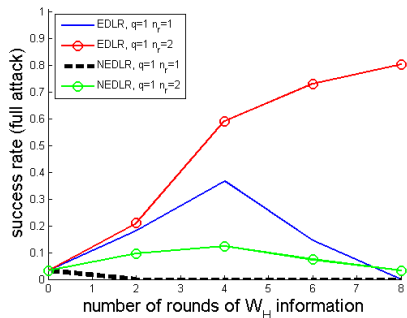
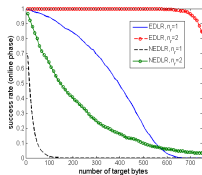
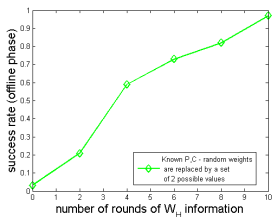
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Algebraic Side-Channel Attacks



Algebraic Side-Channel Attacks



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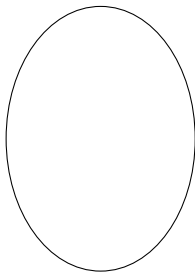
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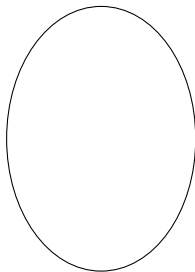
Comparison with DPA

Classical power analysis attack (DPA)

Target 1

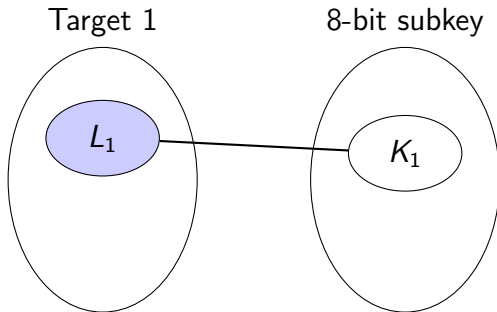


8-bit subkey



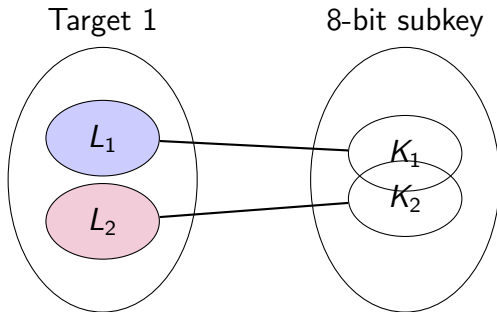
Comparison with DPA

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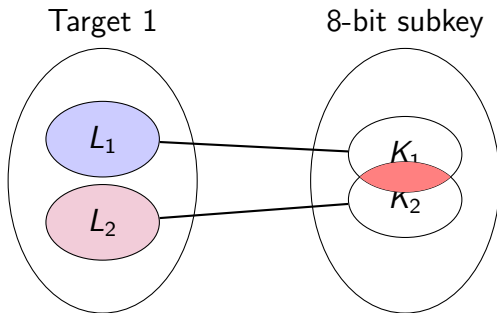
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Classical power analysis attack (DPA)



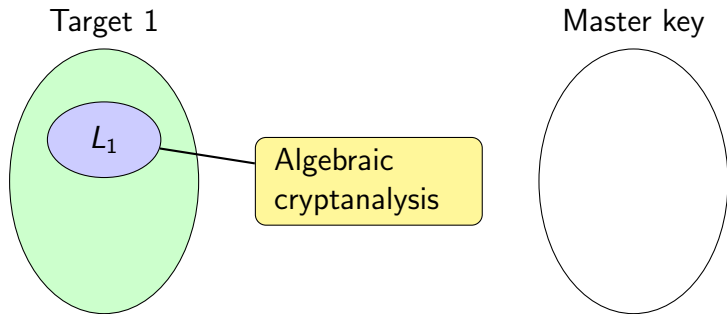
Comparison with DPA

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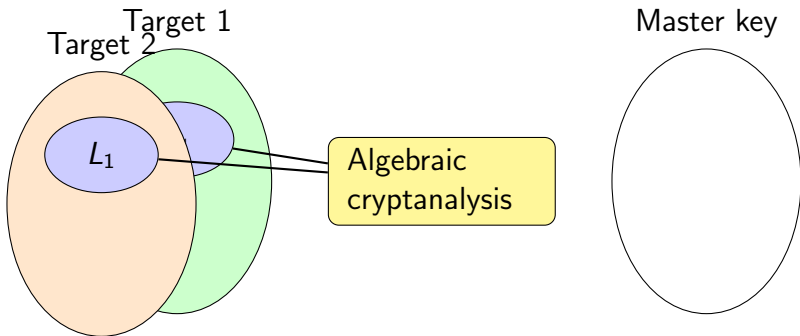
Comparison with DPA

Algebraic side-channel attack



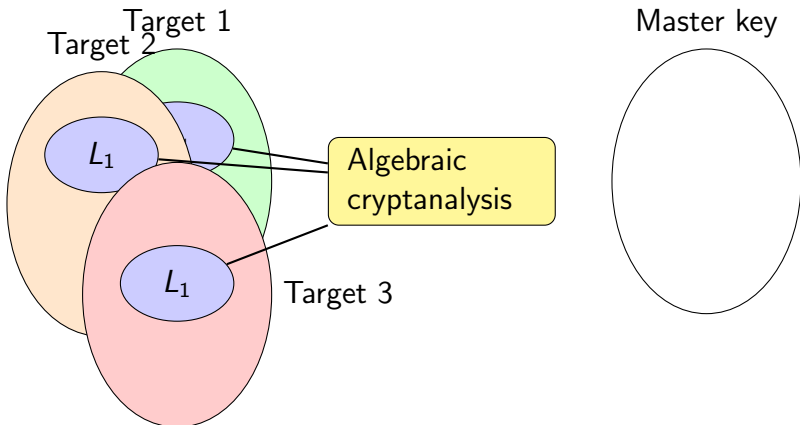
Comparison with DPA

Algebraic side-channel attack



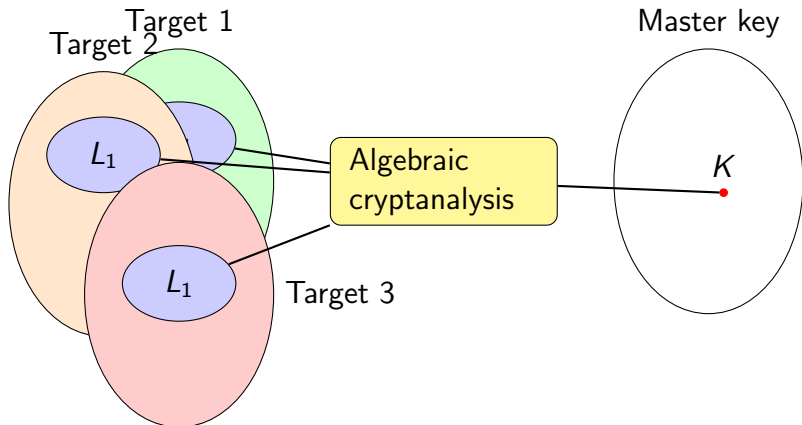
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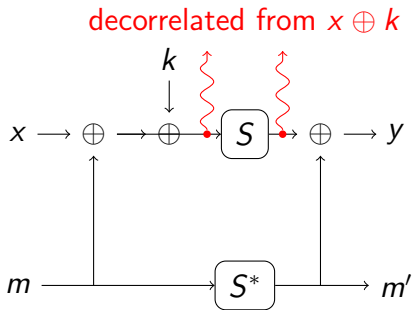
Advanced scenarios

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Advanced scenarios

Masked implementation

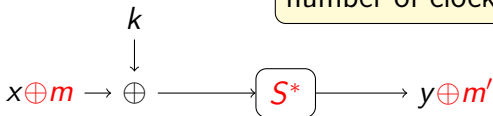


Advanced scenarios

Masking with S-box pre-computation (Herbst et al., 2006).

Mask size = 48 bits

Little impact on the number of clock cycles



New table look-up S^* ; S^* is pre-computed on the fly for m and m' .

Some additional targets, but increased algebraic complexity
 \Rightarrow **harder** to attack.

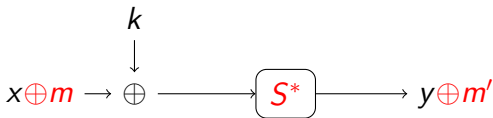


Advanced scenarios

Masking in $GF(2^4)^2$ (Oswald and Schramm, 2005).

Each S-box can have a different mask

Bigger impact on the number of clock cycles



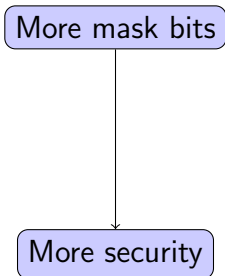
Masked inversion in $GF(2^4)^2$ via 14 table look-ups and 15 XOR operations.

A lot more targets \Rightarrow **easier** to attack.



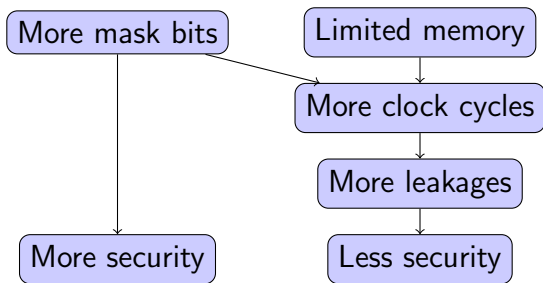
Advanced scenarios

Classical assumption



Advanced scenarios

Algebraic side-channel attack



Time also matters.



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Conclusion

Advantages

- ▶ data complexity $q = 1$
- ▶ side-channel information from everywhere in the cryptosystem
- ▶ efficient in an unknown plaintext/ciphertext context
- ▶ efficient against some masked implementations

Disadvantages

- ▶ require a strong profiling phase
- ▶ not error-tolerant



Conclusion

Between theoretical and practical

require a strong
profiling phase

data complexity = 1

What about the rekeying strategies?



Thank you for your attention.

