First-Order Side-Channel Attacks on the Permutation Tables Countermeasure

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Side Channel Attacks and Countermeasures

Attacks

- Algorithm Processing leaks information about the manipulated data
- Results in information leakage about secret keys
- Side Channel Analyses (SCA) exploit this leakage:
  (HO-)CPA [BrierClavierOlivier04],
  MIA [GierlichsBatinaTuylsPreneel08],
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Software Countermeasures

- Shuffling [HerbstOswaldMangard06]: each signal containing information about a sensitive variable is spread over random signals leaking at different times.
- Masking [ChariJultaRaoRohatgi99,GoubinPatarin99]: every sensitive data is modified by a random transformation.

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  - flawed (zero is never masked) [FumaroliMayerDubois07].
- Permutation [Coron08]: randomly generate a permutation P

$$\tilde{\mathbf{U}} = P(\mathbf{U}) \ ,$$

  + data/masked-data relation is more complex.
  - less efficient than the two others and flawed [This paper]

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Protect a Block Cipher: every intermediate data **U** is presented under the form $P(\mathbf{U})$.

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Obvious Answer: at each execution, generate the look-up table of the function $\mathtt{XT}_8(x, y) = P(P^{-1}(x) \oplus P^{-1}(y))$.

Then, to compute $P(\mathbf{U} \oplus \mathbf{V})$ from $P(\mathbf{U})$ and $P(\mathbf{V})$ process

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- allocation of table of size 2^{16} in RAM

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More Tricky Answer: generate two random permutations P_1 and P_2 operating on 4-bit data and define P such that $P = P_2 || P_1$

Then ...

The Permutation-Table Countermeasure - 2

- Design two 8-bit to 4-bit look-up tables s.t.:

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It equals $P_1(U \oplus V)$.

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     It equals $P_2(U' \oplus V')$.
  3. Concatenate
     $$P_2(U' \oplus V')||P_1(U \oplus V).$$
     We get $P(\mathbf{U} \oplus \mathbf{V})$.

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For instance: if U equals V then $P_1(U)$ equals $P_1(V)$.

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- Let $\hat{\phi}$ be a consumption model.
- For every key hypothesis compute a prediction $\hat{U}||\hat{V}$ on $U||V$ and estimate:

$$\rho(L, \hat{\phi}(\hat{U}||\hat{V})) \quad .$$

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Why? Because in this case the mean of $\phi(P_1(U)||P_1(V))$ does not depend on $U||V$.
What to do? Focus on higher order statistical central moments, *e.g.* the central moments of order 2:

$$\rho((L-\mathrm{E}\,[L])^2, f(\hat{U}||\hat{V})) \ ,$$

where f is a well-chosen function.

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Proposal: use the function $f(\hat{U}||\hat{V})) = \delta_{\hat{U}}(\hat{V})$ defined by $\delta_{\hat{U}}(\hat{V})$ equals 1 if $\hat{U} = \hat{V}$ and equals 0 otherwise.

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Proved to be an optimal choice in the Gaussian Model with $\phi = \mathrm{HW}$. It corresponds to an estimation of the function

$$\hat{u}, \hat{v} \mapsto \mathrm{E}\left[(L - \mathrm{E}[L])^2 | \hat{U} = \hat{u}, \hat{V} = \hat{v}\right] \quad ,$$

(proved to be the optimal choice in [ProuffRivain09]).

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Alternative (more complex) functions are proposed in more general models.

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2. During the first MixColumn operation:

$$U = S_l[X \oplus K] \text{ and } V = S_l[X' \oplus K'] \ ,$$

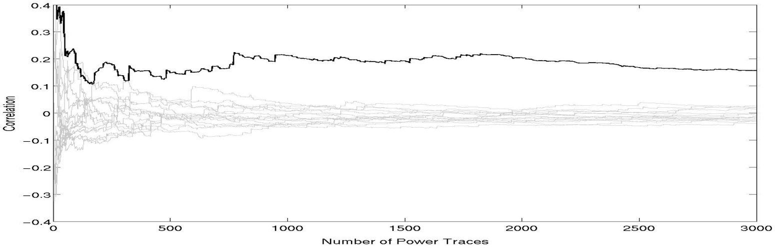
where (X, X') is a pair of plaintext bytes, (K, K') is a pair of key bytes and S_l corresponds to the 4 lowest bits of the AES Sbox.
Goal: retrieve (K, K').

First Scenario

Simulations

Noise std	0	0.5	5	7	10
Nb of measurements	100	1,000	60,000	230,000	900,000

Experiments



Second Scenario

Simulations

Noise standard deviation	0	0.5	1	2	5	7	10
Nb of measurements [MIA with Kernel]	2,500	20,000	60,000	290,000	$> 10^6$	$> 10^6$	$> 10^6$
Nb of measurements [Parametric MIA]	na	3,000	4,000	25,000	250,000	500,000	800,000
Nb of measurements [CPA with f_{opt}]	1,000	1,000	1,500	6,500	120,000	550,000	$> 10^6$

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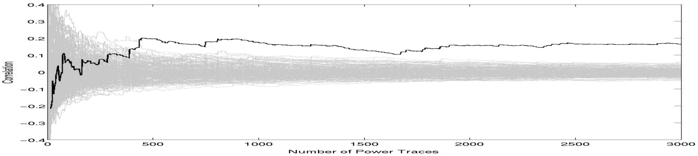

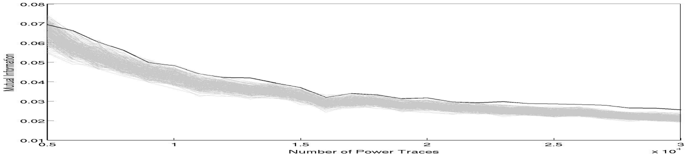

Conclusion

- A first-order flaw exists in the permutation tables countermeasure proposed in [C08].
- To exploit this leakage, 2 attacks have been developed in different scenarii: CPA and MIA.
- Attacks have been verified in both simulation and practice.
- A patch for the permutation tables countermeasure is proposed in the extended version of this paper.
- Even if the permutation tables countermeasure is flawed, exploiting this flaw requires more traces than an attack on a flawed masking scheme: when patched this masking must therefore be a good alternative against HO-SCA.

Thank you!
Questions and/or Comments?