Boomerang attacks on BLAKE-32

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About BLAKE

- BLAKE is now one of the five finalists in SHA-3 competition anounced by NIST.
- One of the two (Addition-Rotation-Xor)ARX designs in the final round
- It is one of the fastest functions on various platforms in software

Hash function BLAKE-32

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- *Initialization*

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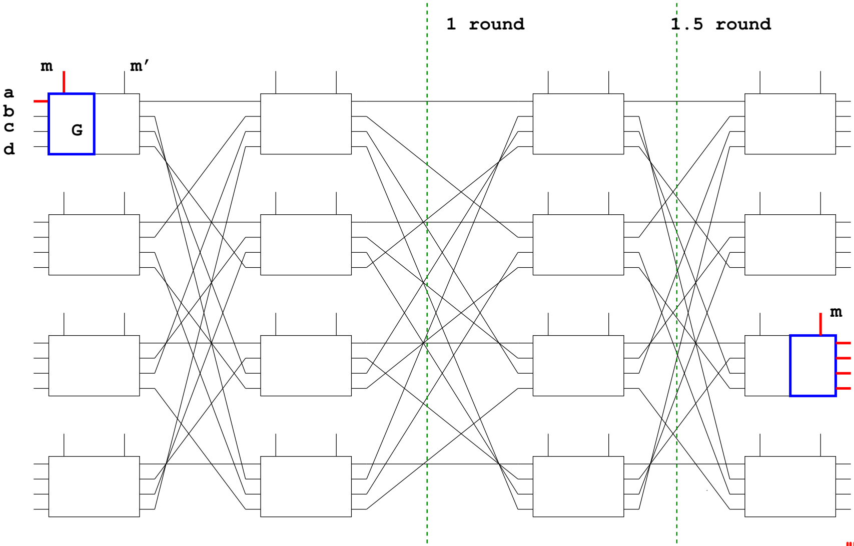
- *Finalization* procedure is linear

High probability differential trail



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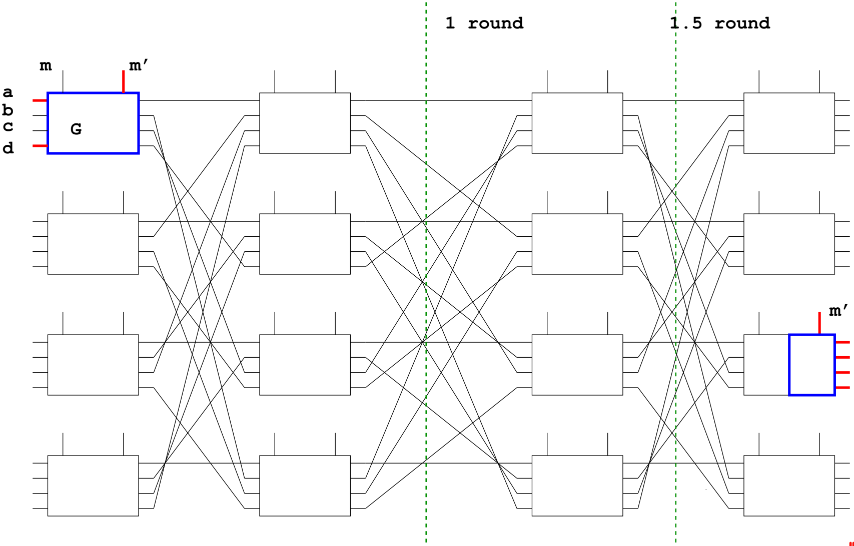

High probability differential trails

- We obtain a 2-round differential trail with probability 2^{-1} with active MSB
- 3-round differential trail with probability 2^{-s} where $s = 6, 7, 8$
- 3.5-round differential trail with probability $\geq 2^{-32}$

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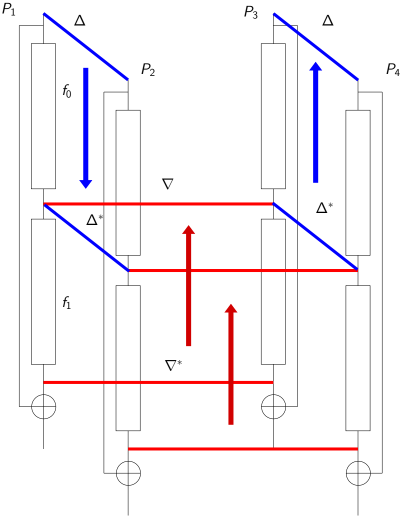
- 2-round differential trail with ith and $(i+16)$th bit active with probability 2^{-9}(when ith bit is MSB) otherwise probability is $\geq 2^{-14}$

Boomerang attack on Compression Function



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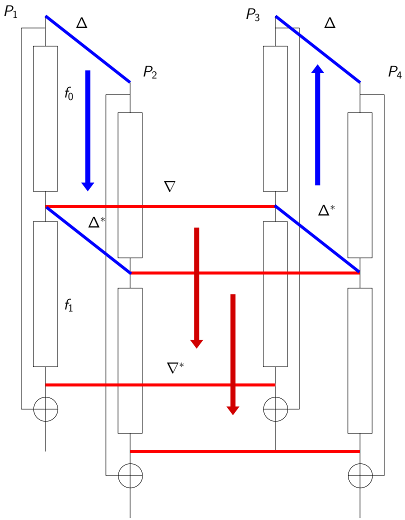

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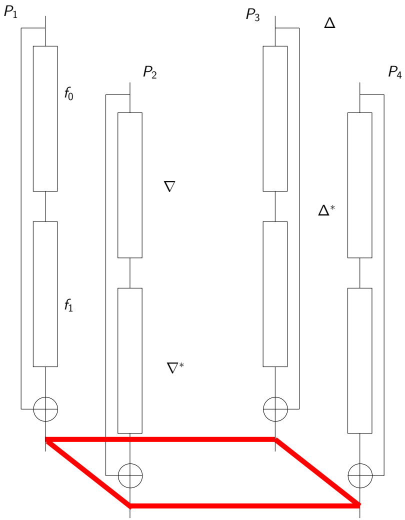
- For a random n-bit compression function finding such quartet will have complexity 2^n(with a fixed difference)
- To get a boomerang distinguisher for compression function F we need $p^2q^2 > 2^{-n}$

Zero-sum distinguisher



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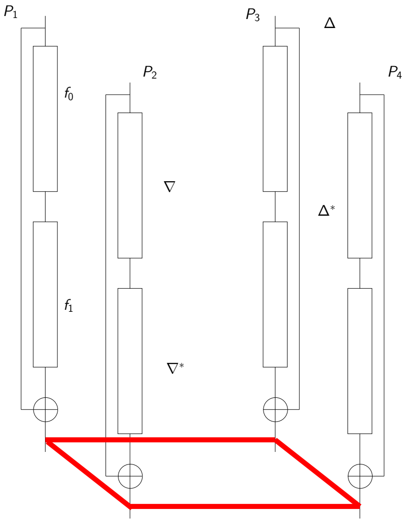

- From the last equations we get: $F(P_1) \oplus F(P_2) \oplus F(P_3) \oplus F(P_4) = 0$
- For a random permutation complexity is $2^{n/4}$. But with fixed difference the complexity rises to $2^{n/2}$

Boomerang attack on BLAKE-32

- The real probability of the Boomerang is $\hat{p}^2\hat{q}^2$, where $\hat{p}, \hat{q}$ are the amplified probability defined as:
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- For the attack on Hash function, the returned pairs are consistent if $v_{12} \oplus v_{13}$ and $v_{14} \oplus v_{15}$ are fixed. This increases the complexity of the attack by a factor of 2^{64}

Summary of our attack

CF/KP[1]	Rounds	CF/KP calls
CF	4	2^{67}
CF	5	$2^{71.2}$
CF	6	2^{102}
CF	6.5	2^{184}
CF	7	2^{232}
KP	4	2^{3}
KP	5	$2^{7.2}$
KP	6	$2^{11.75}$
KP	7	2^{122}
KP	8	2^{242}

[1] CF = Compression Function, KP = Keyed Permutation

Conclusion

- Application of the concept of boomerang distinguisher to compression function
- Shown such distinguisher for CF of BLAKE-32
- Classical boomerang distinguisher for KP of BLAKE-32
- Attack works for 2/3 of the total number of rounds of the CF and 4/5 of the total number of rounds of the KP
- The attack can be equally applied to other versions of BLAKE
- BLAKE-32 has been tweaked to 15 rounds in the final round

THANK YOU!