Random Oracle Reducibility

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Introduction

Two Cryptographic Schemes...

$\boxed{A}$ Secure under assumptions $\mathbb{A}$

$\boxed{B}$ Secure under assumptions $\mathbb{B}$

- Possible comparison criteria
  - which scheme is more efficient?
  - how do $\mathbb{A}$ and $\mathbb{B}$ relate?
  - purpose-specific properties (e.g. ciphertext size)?
- rather easy to compare in the standard model

Two Cryptographic Schemes #2

$\boxed{A}$ $\boxed{B}$

Secure under $\mathbb{A}$ Secure under $\mathbb{B}$

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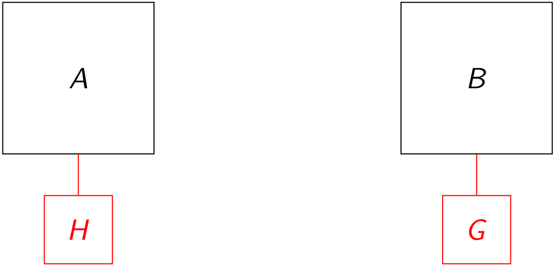

Secure under $\mathbb{A}$
in the ROM

Secure under $\mathbb{B}$
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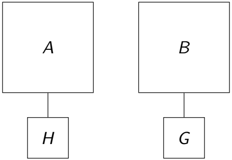
- Comparison “biased” by random oracle dependency

Comparing The Schemes

- Comparison "biased" by random oracle dependency
- e.g. $\mathbb{A} \subsetneq \mathbb{B}$, but H more demanding than G
  - RO G: provide randomness
  - RO H: POWHF, CR, ...
- perhaps H even uninstantiable!



Secure under $\mathbb{A}$ in the ROM

Secure under $\mathbb{B}$ in the ROM

The Reduction Approach

- Formalizing exact requirements is tedious
- instead, use the cryptographer's approach: reduction
  - A^H secure $\Rightarrow$ B^{T^H} secure
  - any hash function which makes A secure also makes B secure
  - uninstantiability of B implies uninstantiability of A

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  - any hash function which makes A secure also makes B secure
  - uninstantiability of B implies uninstantiability of A
- may require a non-trivial transformation T (stateless, deterministic, efficient)
  - guarantee "structural compatibilty"
- i.e., relative security amongst two schemes

Random Oracle Reducibility

Semi-formal Definition

Scheme A $\{$strictly,strongly,weakly$\}$ reduces to scheme B if for every H there exists a transformation T such that

- strictly:
  A is G_A^H-secure under $\mathbb{A} \Rightarrow B$ is $G_B^{T^H}$-secure under $\mathbb{B}$

where $G_S^{\mathcal{O}}$ defines a security game (think IND-CCA for example) for scheme S

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- weakly:

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- strongly:

  $$A \text{ is } G_A^H\text{-secure under } \mathbb{A} \Rightarrow \left\{ \begin{array}{ll} B \text{ is } G_B^{T^H}\text{-secure under} & \mathbb{A} \cup \mathbb{B} \text{ and} \\ B \text{ is } G_B^{T^{H'}}\text{-secure under} & \mathbb{B} \text{ for some } H' \\ & \text{relying on } \mathbb{H}' \end{array} \right.$$

- weakly:

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Example

Example: Hashed ElGamal

- Twin hashed ElGamal (THEG) encryption scheme [CKS09]
- extends hashed ElGamal (HEG) encryption scheme, but milder assumption
  - DH assumption as opposed to strong DH assumption
  - IND-CCA secure given an IND-CCA symmetric scheme
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- hence superior at first glance
- our result: THEG* is strongly reducible to HEG

Proof of Reducibility

- THEG* is strongly reducible to HEG
- Proof strategy
  1. show weak reducibility from THEG* to HEG
  2. prove THEG* secure on its own (in the ROM)
- strong reducibility then follows

Scheme Details

HEG (scheme A)

$\mathbf{Enc}_A(m)$:

$y \leftarrow \mathbb{Z}_q$
$k \leftarrow H(g^y, X^y)$
$c \leftarrow \mathbf{E}_k(m)$
return (g^y, c)

THEG* (scheme B)

$\mathbf{Enc}_B(m)$:

$y \leftarrow \mathbb{Z}_q$
$k_0||k_1 \leftarrow G(g^y, X_0^y, X_1^y)$
$c \leftarrow \mathbf{E}_{k_0}(m)$
return (g^y, c, k_1)

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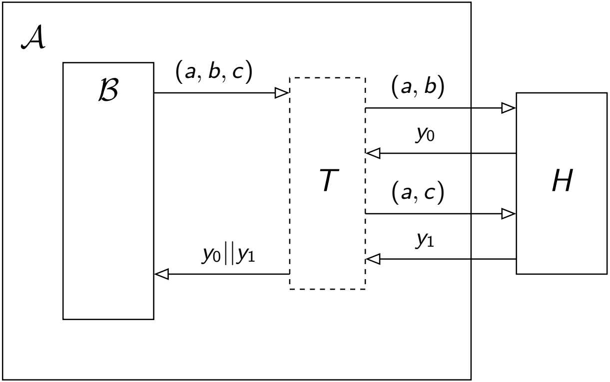
return (g^y, c, k_1)

- Oracles H and G: need transformation function
- $T^H(a, b, c) = H(a, b)||H(a, c)$

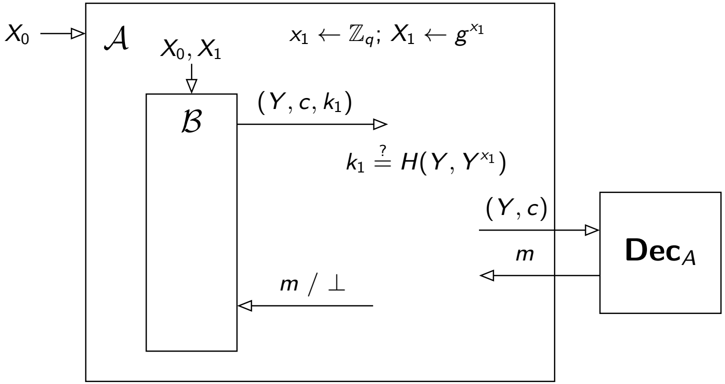
Proof Details

- Handling hash oracle queries
- alleged adversary $\mathcal{B}$ against THEG*
- algorithm $\mathcal{A}$ performs $T^H(a, b, c) = H(a, b)||H(a, c)$



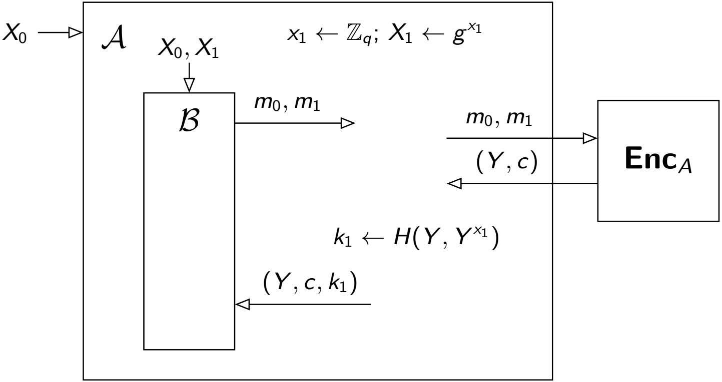
Proof Details

- Handling decryption queries
- algorithm $\mathcal{A}$ simulates second key half



Proof Details

- Handling the encryption challenge query
- algorithm $\mathcal{A}$ simulates second key half



Proof Details

- Algorithm $\mathcal{A}$ outputs whatever $\mathcal{B}$ outputs
- all queries are simulated perfectly
- thus, $\mathcal{A}$ is successful whenever $\mathcal{B}$ is

- THEG* is secure in the ROM (rather technical, see paper)
- hence strongly reducible

Further Results/Applications

Results on Signature Schemes

More examples of (strict) random oracle reductions

- probabilistic RSA FDH signatures reducible to Guillou-Quisquarter signatures
- probabilistic RSA FDH signatures reducible to PSS signatures
- Schnorr signatures reducible to BLS signatures

recall: reducibility allows to argue about instantiability

The End

Thank you!

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References

David Cash, Eike Kiltz, and Victor Shoup.
The twin DiffieHellman problem and applications.
Journal of Cryptology, 22(4):470–504, October 2009.