Multiple Results on Multiple Encryption

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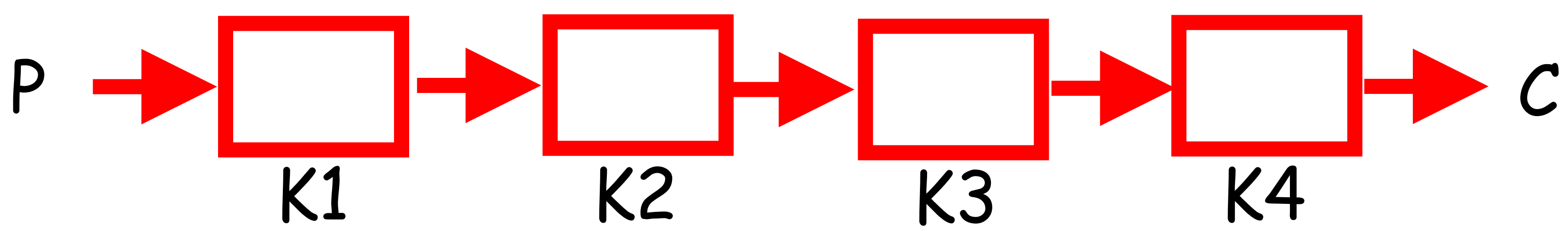
The Security of Multiple Encryption:

- Given a block cipher with n-bit plaintexts and n-bit keys, we would like to enhance its security via sequential composition
- Assuming that
  - the basic block cipher has no weaknesses
  - the k keys are independently chosen

how secure is the resultant composition?



Double and Triple Encryptions:

- Double DES and triple DES were widely used by banks, so their security was thoroughly analyzed

- By using a Meet in the Middle (MITM) attack, Diffie and Hellman showed in 1981 that double encryption can be broken in $T=2^n$ time and $S=2^n$ space. Note that $TS=2^{2n}$

- Given the same amount of space $S=2^n$, we can break triple encryption in time $T=2^{2n}$, so again $TS=2^{3n}$

How Secure is k-encryption for k>3?

- The fun really starts at quadruple encryption (k=4), which was not well studied so far, since we can show that breaking 4-encryption is not harder than breaking 3-encryption when we use 2^n space!
- Our new attacks:
  - use the smallest possible amount of data (k known plaintext/ciphertext pairs which are required to uniquely define the k keys)
  - Never err (if there is a solution, it will always be found)

The time complexity of our new attacks (expressed by the coefficient c in the time formula T=2^{cn})

k =										
c =										

The time complexity of our new attacks (expressed by the coefficient c in the time formula $T=2^{cn}$)

k =	2									
c =	1									

The time complexity of our new attacks (expressed by the coefficient c in the time formula $T=2^{cn}$)

k =	2	3								
c =	1	2								

The time complexity of our new attacks (expressed by the coefficient c in the time formula $T=2^{cn}$)

k =	2	3	4							
c =	1	2	2							

The time complexity of our new attacks (expressed by the coefficient c in the time formula T=2^{cn})

k =	2	3	4	5						
c =	1	2	2	3						

The time complexity of our new attacks (expressed by the coefficient c in the time formula $T=2^{cn}$)

k =	2	3	4	5	6					
c =	1	2	2	3	4					

The time complexity of our new attacks (expressed by the coefficient c in the time formula $T=2^{cn}$)

k =	2	3	4	5	6	7				
c =	1	2	2	3	4	4				

The time complexity of our new attacks (expressed by the coefficient c in the time formula T=2^{cn})

k =	2	3	4	5	6	7	8			
c =	1	2	2	3	4	4	5			

The time complexity of our new attacks (expressed by the coefficient c in the time formula $T=2^{cn}$)

k =	2	3	4	5	6	7	8	9		
c =	1	2	2	3	4	4	5	6		

The time complexity of our new attacks (expressed by the coefficient c in the time formula T=2^{cn})

k =	2	3	4	5	6	7	8	9	10	
c =	1	2	2	3	4	4	5	6	7	

The time complexity of our new attacks (expressed by the coefficient c in the time formula $T=2^{cn}$)

k =	2	3	4	5	6	7	8	9	10	11
c =	1	2	2	3	4	4	5	6	7	7

The "Magic Numbers" of rounds:

- We gain some time at each magic number, and the savings accumulate as k increases
- There is an infinite number of magic numbers, starting with k=4, 7, 11, 16, 22, 29, 37, 46, 56,… which grow quadratically
- We can prove the optimality of our new attacks within a broad class of possible algorithms which we call Dissection Attacks

Using the New Techniques to Solve Non-cryptographic Combinatorial Search Problems

- Consider for example the knapsack problem of finding a 0/1 solution for
  $x1*a1+x2*a2+x3*a3+\ldots xn*an = v$

- We can represent the knapsack problem as a k-encryption problem for any desired k

Using the New Techniques to Solve Non-cryptographic Combinatorial Search Problems

- Example: Given the 6 generators a1,…,a6, we describe the knapsack problem of representing the number v as a triple encryption with the three independent 2-bit keys (x1,x2),(x3,x4),(x5,x6)

- Starting with plaintext P=0, we first add to it x1*a1+x2*a2 to get the first ciphertext. We then encrypt it a second time by adding to it x3*a3+x4*a4, and finally encrypt it a third time by adding to it x5*a5+x6*a6 to get the final ciphertext C=v

Using the New Techniques to Solve Non-cryptographic Combinatorial Search Problems

- The knapsack problem can thus be described as the problem of finding the k keys of n/k bits each that map the initial plaintext 0 to the final ciphertext v

- By using our new 7-encryption attack, we can solve hard knapsack problems in time $T=2^{4n/7}$ and space $S=2^{n/7}$

- This is a faster attack than the best previously published knapsack solving algorithm (by Becker, Coron, Joux) for such a small memory complexity

Concluding Remarks:

- Breaking multiple encryption is much easier than previously believed
- Many combinatorial search problems can be described as k-encryption problems, and then solved more efficiently by our new generic techniques