Secure Database Commitments and Universal Arguments of Quasi Knowledge

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Size hiding protocols

- Traditional secure computation:
  - Parties learn nothing more than f(x,y)
  - *And the size of the inputs*

All existing *definitions* and *constructions* reveal input size

- Sometimes the size of the inputs may be private/confidential
  - No fly list, phishing lists, company databases

- Can we hide the size of the players inputs and still achieve strong security?

Yes! We will show a construction for secure commitments which hides the input size

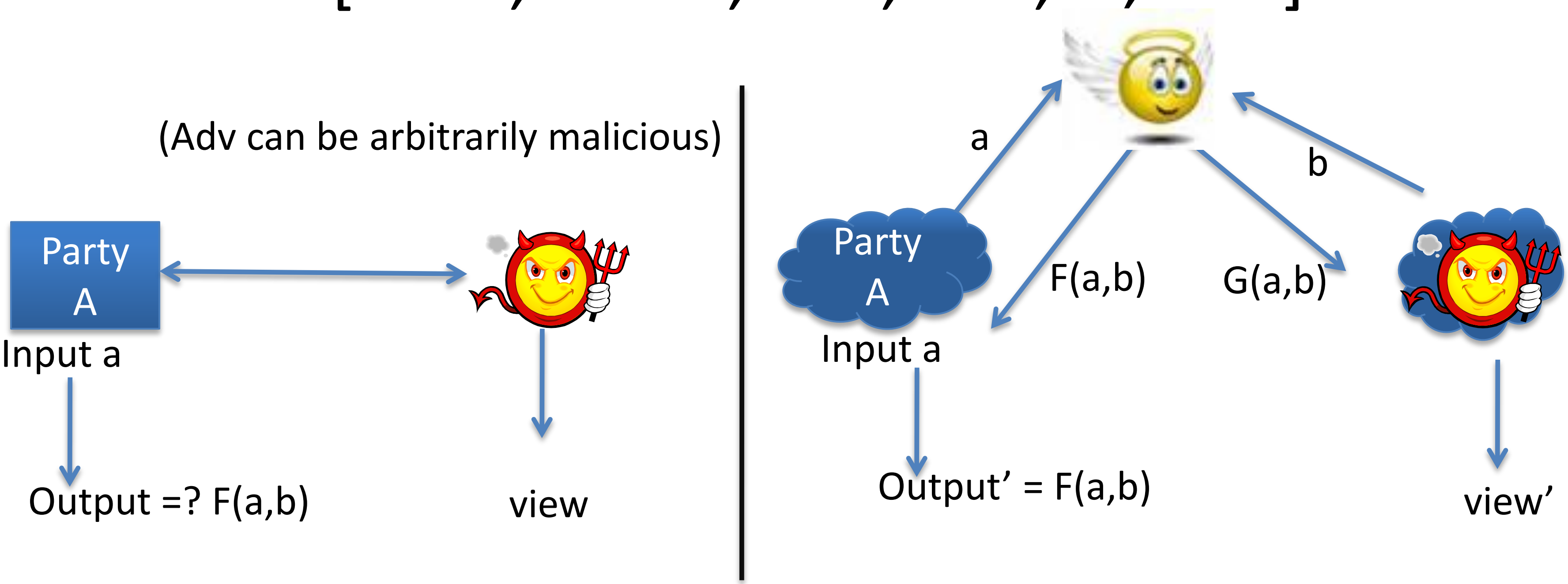
Size hiding protocols

- Input size must be poly(k), but can be any polynomial
  - No polynomial upper bound
- Prior work
  - Zero Knowledge Sets (ZKS) [MRK03, …]
  - Size-hiding Set Intersection [ADT11]
  - Branching Programs [IP07]

Semi honest security and/or ad hoc definitions

- How do we usually define secure computation?
  - Real/Ideal model

Real/Ideal model
[GL90, MR91, B91, C95, …, G04]



- Idea: Any attack in the real world could also occur in the ideal world

Traditionally: All parties know the size of the inputs (part of the description of F)

Our work

- Goal: realize size-hiding secure computation
  - Real/ideal model with malicious adversaries
- We focus on a very basic functionality: *Commitments*
- We give
  - Real/ideal model definition for size hiding (database) commitments
  - Constant-round construction based on CRHFs
  - Key building block: Universal Argument of Quasi Knowledge

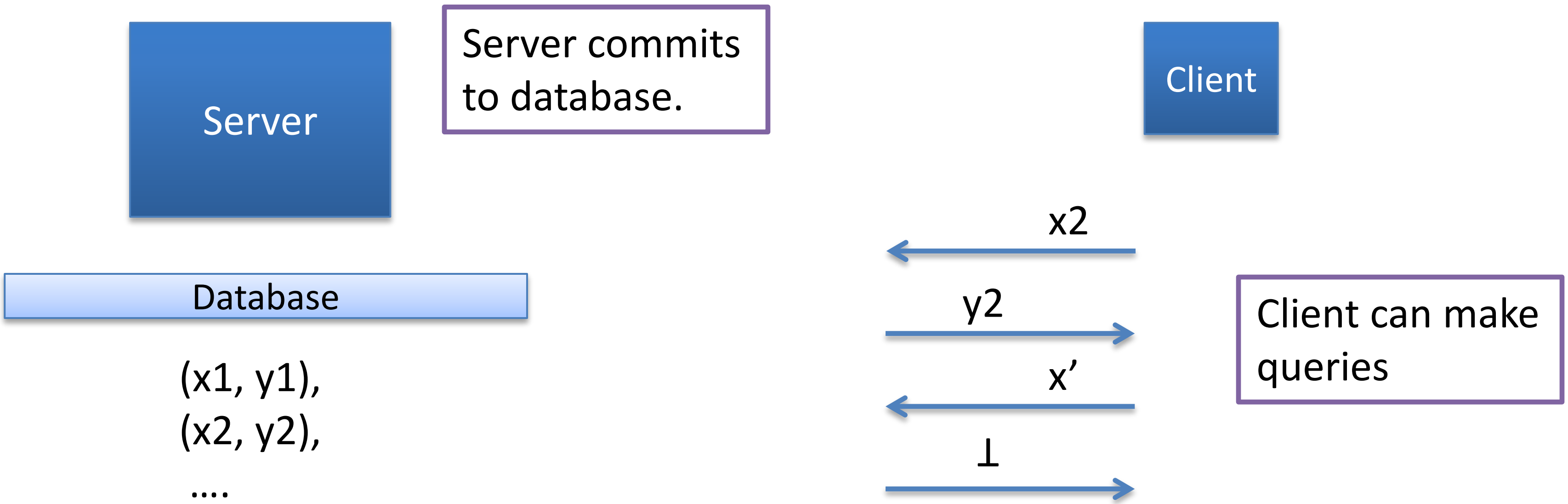
First size-hiding protocols in the real/ideal model

Roadmap

- Defining database commitments in the real/ideal model
- Universal Arguments of Knowledge [BG02]
  - and why they don't directly apply
- A new tool: Universal Arguments of Quasi Knowledge
- Constructing secure database commitments

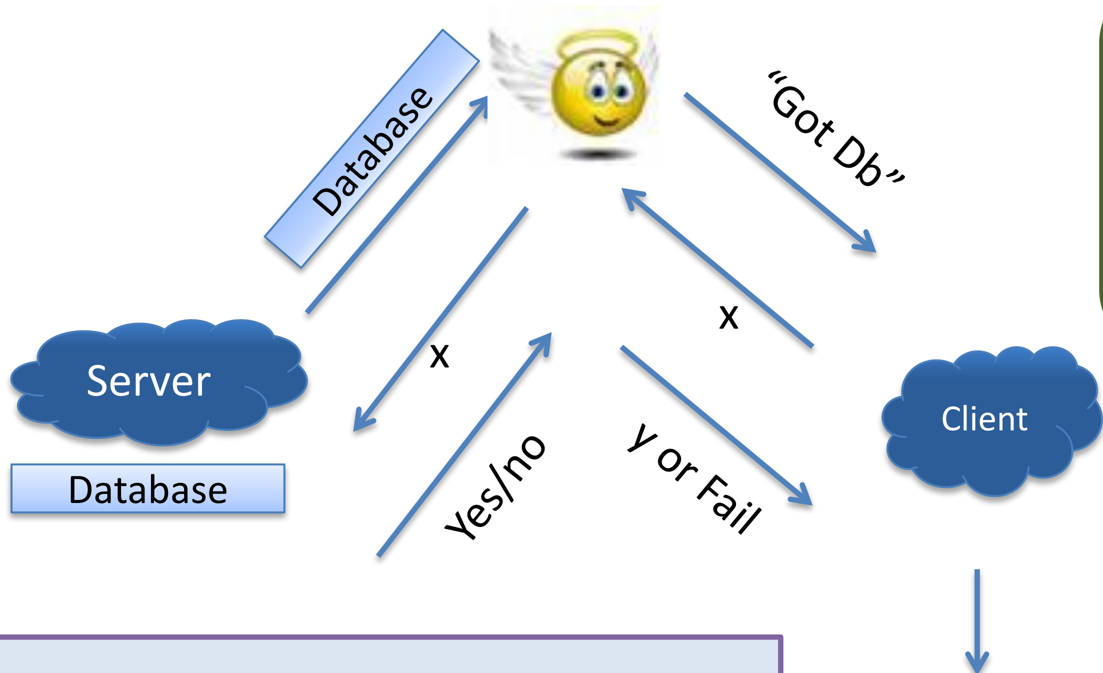
Secure Database Commitments

- High level idea: server can
  - Commit to a large input
  - Open it incrementally
- Elementary databases as in MRK03:



- Server can't change his mind later - must answer consistently with original database
- Client only learns answers to his queries (Does not learn size of database!)

Secure Database Commitments: Ideal model (first attempt)



y, …

Could be viewed as ideal/real world version of Zero Knowledge Sets [MRK03]

We put no limits on the size of the database

y can be
$\perp$ = "not in Db"

- Server must "know" what he's committing to from the beginning
- Client only learns answers to queries
- Query responses must be consistent with original database

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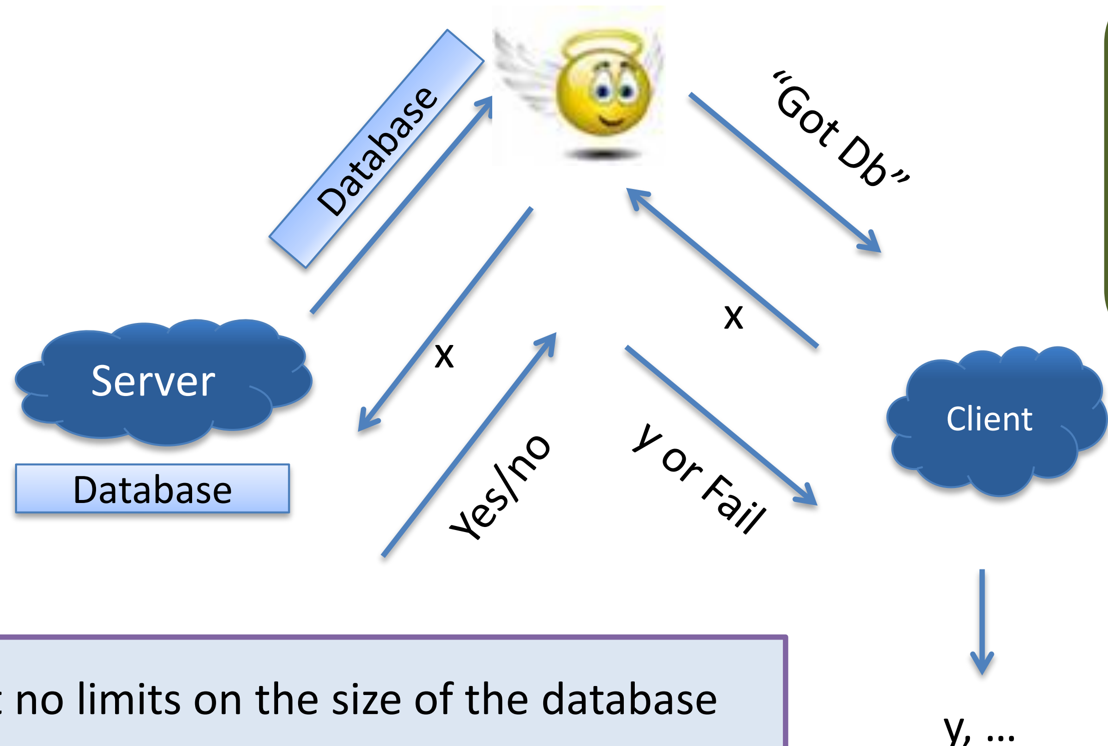

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- **Server must "know" what he's committing to from the beginning**
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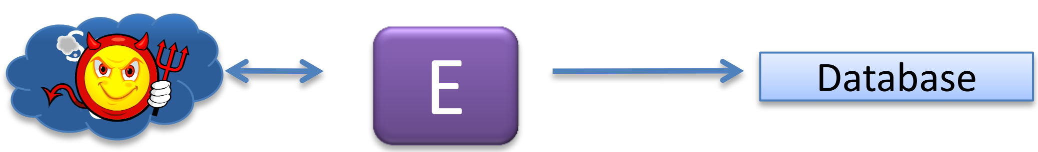
Implications of the definition

Server must "know" what he's committing to from the beginning

What happens when we want to realize this part of the definition?

Standard approach: there exists an extractor



Traditionally: commit + proof of knowledge, encryption, etc

But, communication needs to be independent of input size!

Recall: can't assume a fixed poly(k) upperbound

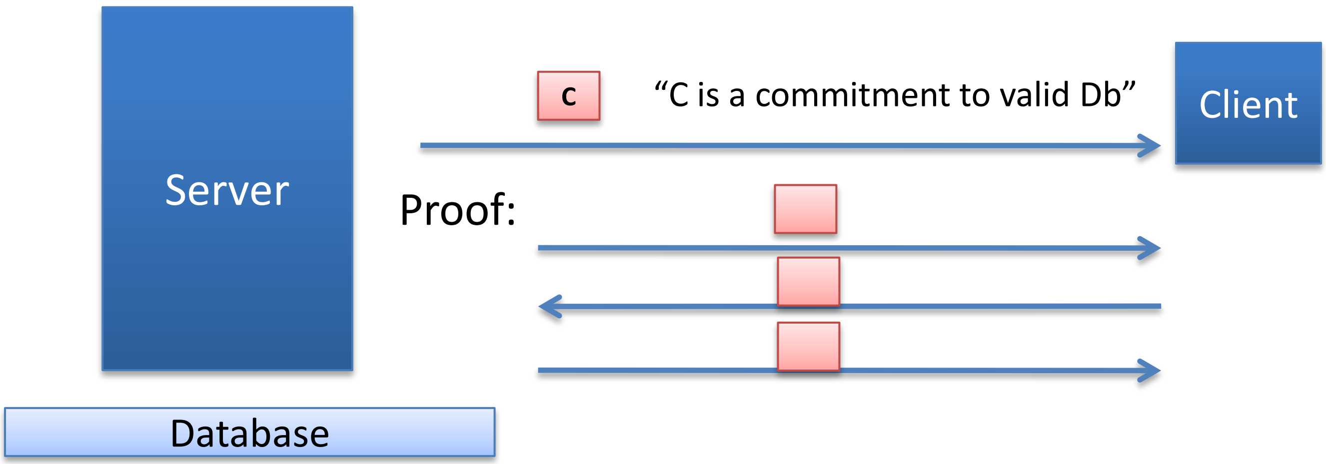
Implications of the definition

- We need a proof system where
  - 1) communication is much shorter than the witness
  - 2) must be a proof of knowledge so we can extract the witness
- Then perhaps we can apply commit and prove methodology
- Is there such a proof system?
  - What about Universal Arguments of Knowledge [BG02]?

Universal Arguments [BG02]

Original Application: Concurrent ZK

- Short proofs (even when witness is long)



- Witness Indistinguishability
  - Can't tell which database was used for proof

Database1 OR Database2

- Proofs of Knowledge weak

UAoK: Weak proof of knowledge

Why is it weak?

- E produces a circuit describing the witness

where w_i is the i-th bit of the witness

Address with modification to functionality

- If A produces a good proof with probability 1/p, E produces a good circuit with probability 1/p'
- We can't tell when C is a good circuit
  - (extracting t bits may take too long)
- E needs to be given a lower bound on the success probability of A
  - (running time is polynomial in this lower bound)

Compile a UA with weak PoK into ***new UA*** with stronger property

Note: we might get around these issues using superpolynomial simulation and/or non-standard assumptions, but we want to avoid those routes

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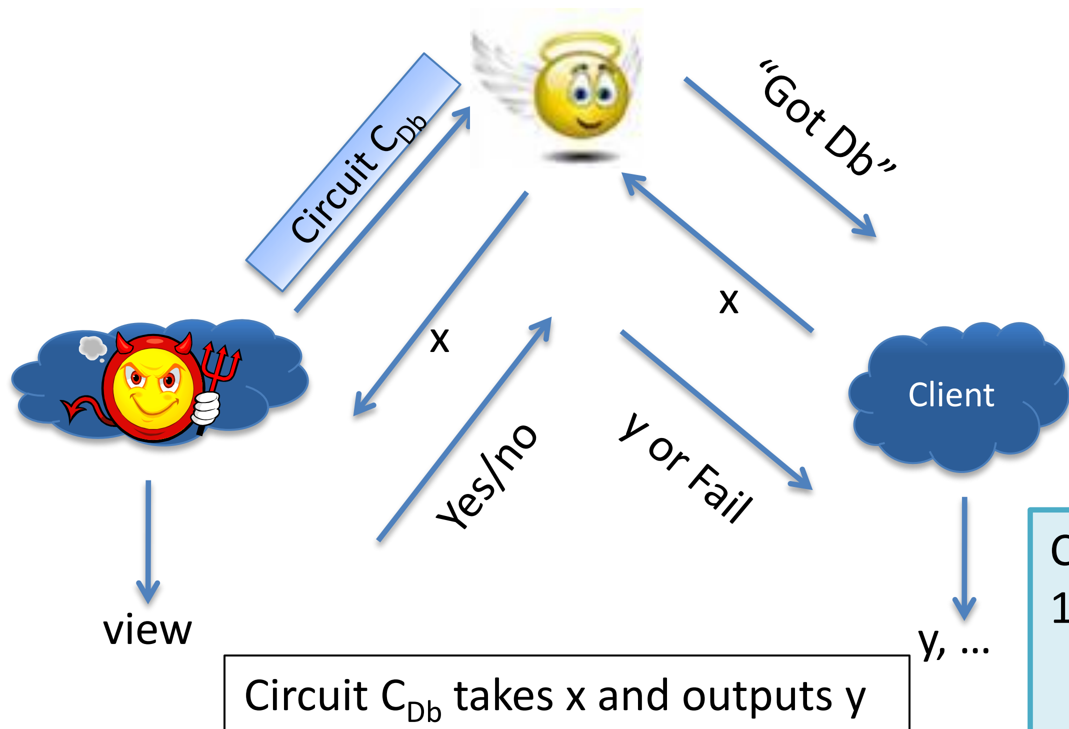
Secure Database Commitments: Ideal model (final version)



Caveats:

1) We will require honest server to know explicit set
2) We will allow ideal parties to run in expected polytime

- Note that this is does not reduce functionality
  - Adversary is still committed to a set
  - Adversary is still required to reply consistently
  - Any polynomial sized set can be converted into a polynomial sized circuit

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A new tool: Universal Argument of Quasi Knowledge

- There exists extractor E

- Suppose Adv convinces verifier with probability 1/p
  1) E runs in time p * poly(k)
  2) With all but negligible probability, C is *good enough*
- Good enough: there exists valid witness $w=w_1, \dots w_t$
  – In any application, C will always* produce bits of w
  – Negligible probability that any poly-time process can find i such that

where $\omega_i \neq w_i$

Compiler for achieving quasi-knowledge

- Build UAQK from any universal argument with (slightly stronger) weak proof of knowledge property
- Gives constant round, WI UAQK based on CRHFs

This stronger property is satisfied by BG02 UAQK construction

Note: To get UAQK that succeeds with probability p, just run Adv first, and then continue with extraction iff Adv produces an accepting proof

Using UAQKs

- The idea: commit using size-hiding commitment, give a UAQK proof of knowledge of the opening
- Issues
  - UAQK extract circuit that produces bits of witness
    - But ideal input C_{Db} takes x and outputs y
  - Need contradiction if responses are not consistent with extracted database

Solution based on
- Careful formatting of witnesses
- Property-based size-hiding commitments with special structure

  - Also need a couple other pieces: statistically hiding ZKAoK, trapdoor commitments, CRHFs

Summary

Size hiding is possible in the real/ideal model.

Specifically, we can achieve secure size hiding commitments

We give:

- Definition for size hiding database commitment
- Construction which is
  - Constant round
  - Based on CRHFs
  - Non-interactive responses
- New tool: Universal Argument of Quasi Knowledge

Questions

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