# SECURE IDENTITY-BASED ENCRYPTION IN THE QUANTUM RANDOM ORACLE MODEL

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# Random Oracle Model (ROM)

- Sometimes, we can’t prove a scheme secure in the standard model.
- Instead, model a hash function as a random oracle, and prove security in this model [BR 1993]

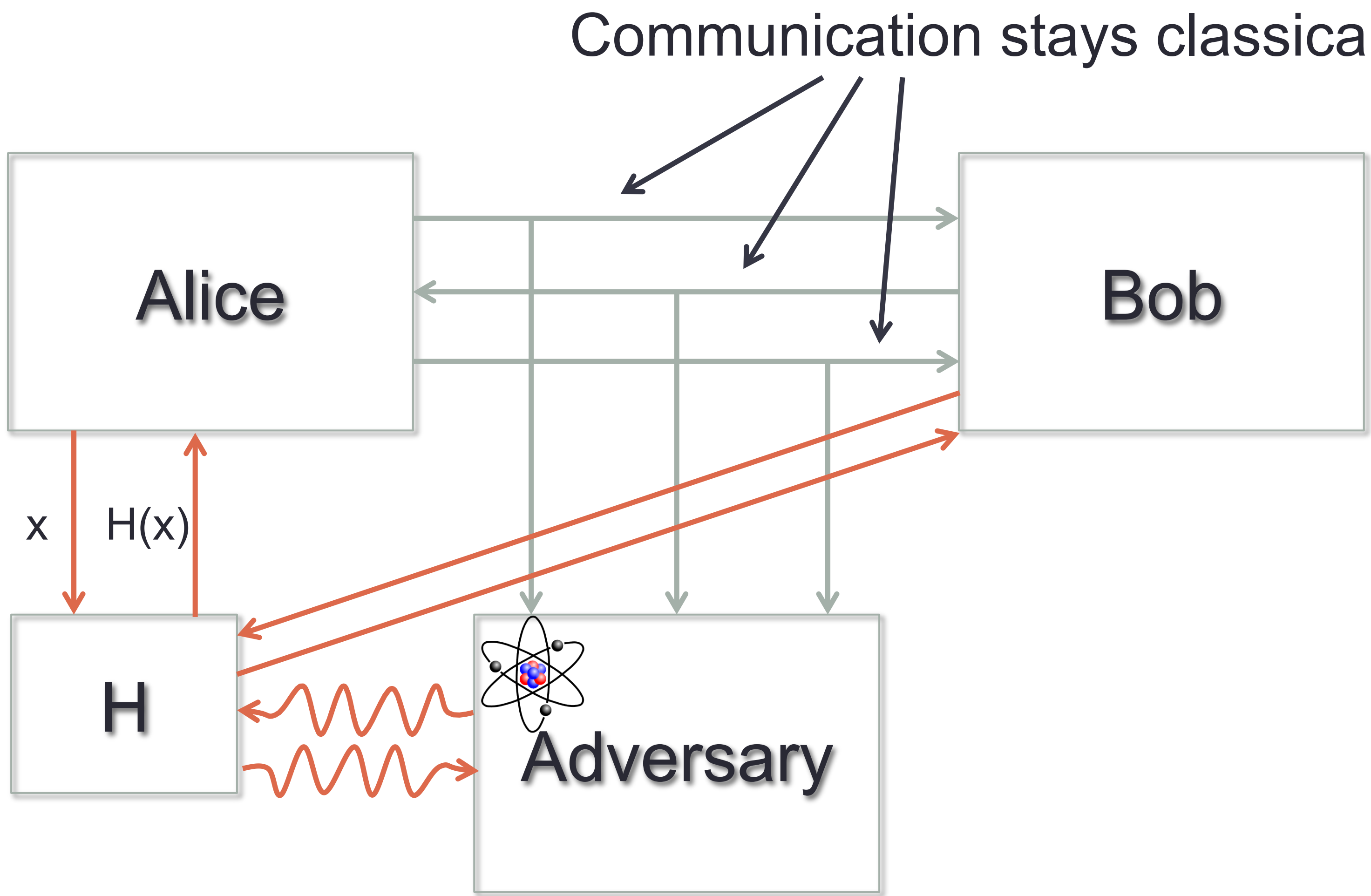
# Why Use the Random Oracle Model?

- Most efficient schemes are often only proved secure in the random oracle model
- True even in post-quantum world
  - RO-based GPV signatures more efficient that non-RO CHKP and ABB signatures [GPV 2009, CHKP 2010, ABB 2010]
  - RO-based Hierarchical IBE more efficient than non-RO versions
- Unfortunately, these schemes are only proved secure in the classical ROM
  - Only consider classical queries to the random oracle

# The Quantum Random Oracle Model

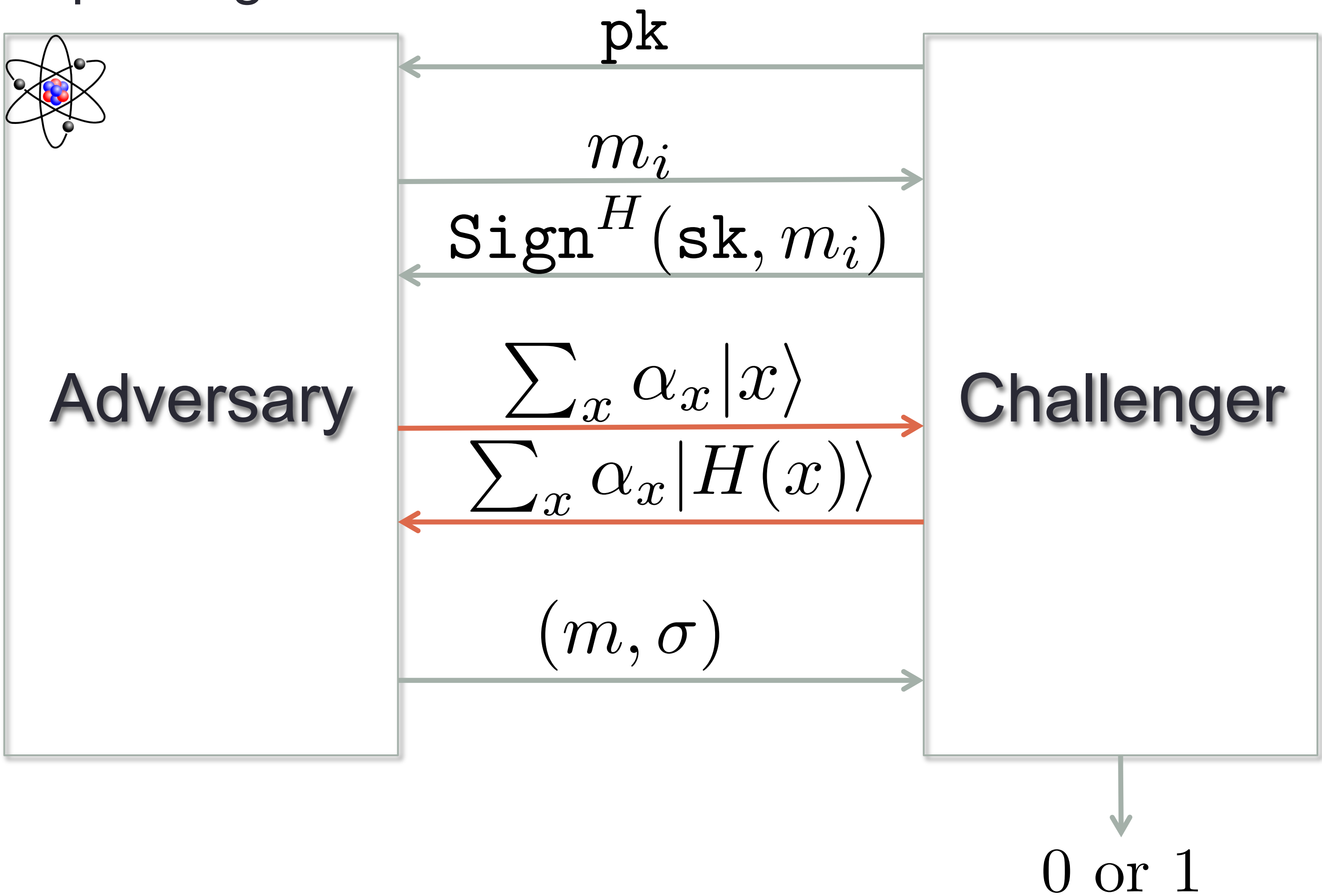
- Interaction with primitives is still classical
- Allow quantum queries to random oracle
  - When instantiated, random oracle replaced with hash function
  - Code for hash function is part of specification
  - Adversary can evaluate hash function on quantum superposition

# The Quantum Random Oracle Model (QROM)



# Security in the QROM

Example: Signatures



# Security Proofs in the QROM

- Classical random oracle model security proofs do not carry over to the quantum setting
- Difficulties:
  - Simulating the random oracle
  - Peaking into the adversary
  - Programming the random oracle

# Previous Results [Boneh et al. 2011]

- **Separation**: there exist schemes secure in the classical ROM against quantum adversaries, but that are insecure in the quantum ROM
- Some classical proofs can be adapted to the quantum setting:
  - Answer RO queries randomly, same across all queries
  - Use pseudorandom function to generate randomness
  - Examples: GPV Signatures [GPV 2009]
    Full Domain Hash with specific trapdoor permutations [Coron 2000]
    Katz-Wang Signatures [KW 2003]
    Hybrid encryption scheme

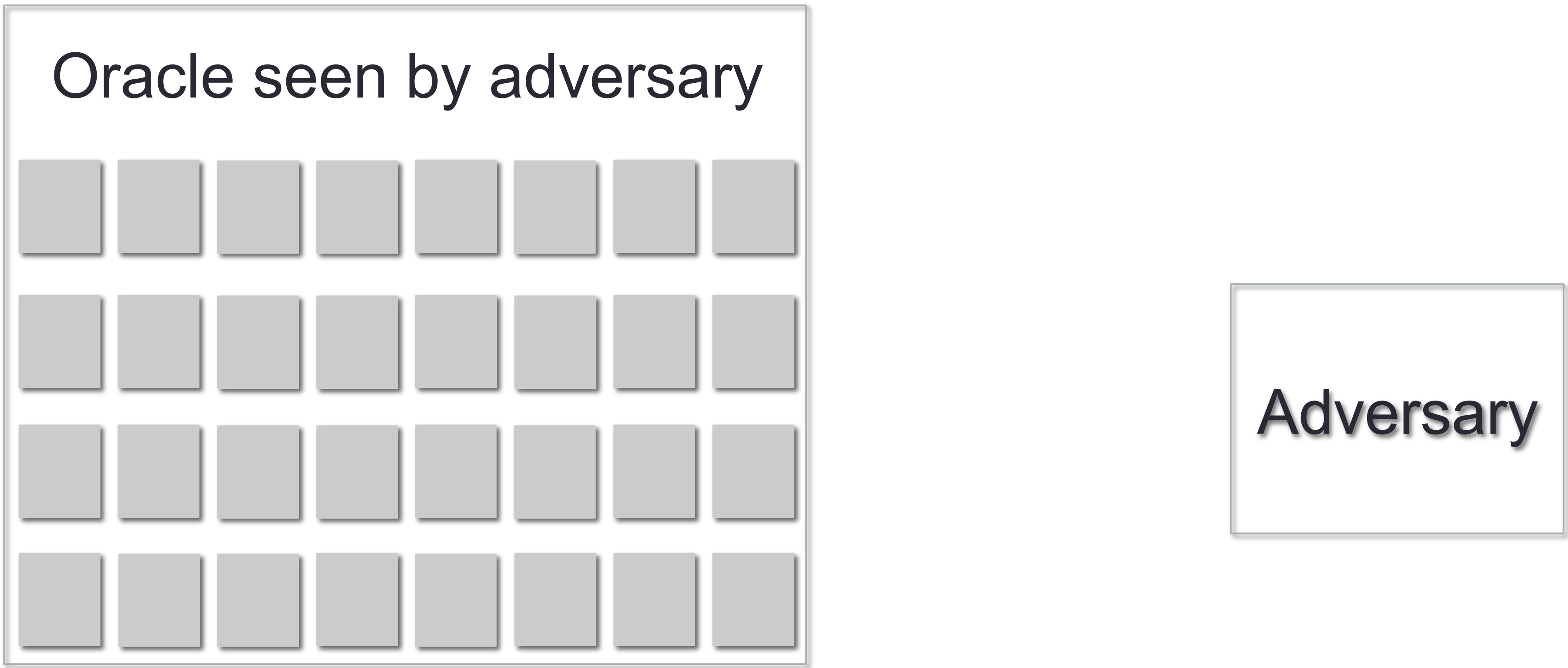
# Our Results

- Simulating the random oracle without additional assumptions
- New security proofs in the quantum random oracle model
  - Identity-Based Encryption
  - Hierarchical Identity-Based Encryption
  - Generic Full-Domain Hash
- New tools for arguing the indistinguishability of oracle distributions by quantum adversaries.

# Common Proof Technique in Classical ROM

- Start with an adversary A that makes q queries to random oracle H
- Construct B that solves some problem:
  - Pick a random query i
  - For all other queries, answer in way that looks random
  - For query i, plug in some challenge c
  - If A happens to use query i, then we can solve our problem
  - A uses query i with probability 1/q, so happens with non-negligible probability

# Common Proof Technique in Classical ROM



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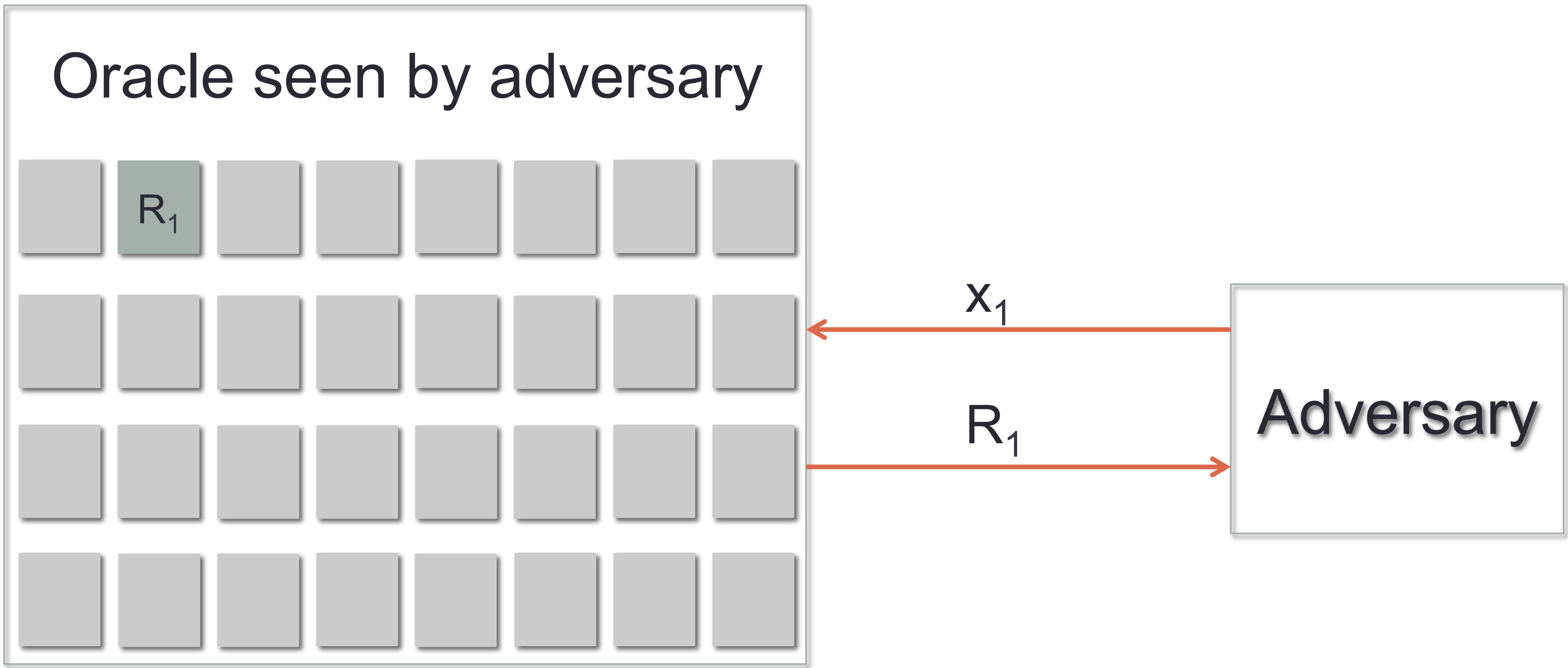

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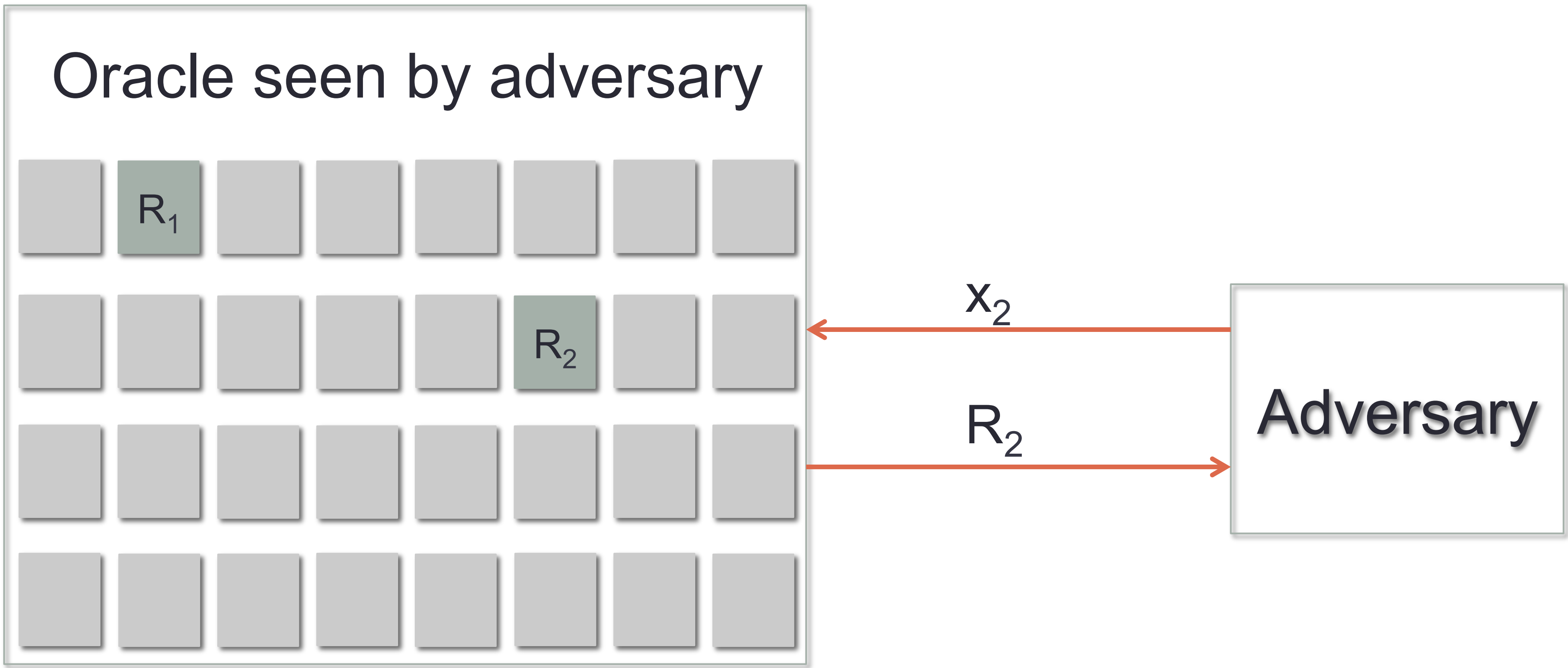

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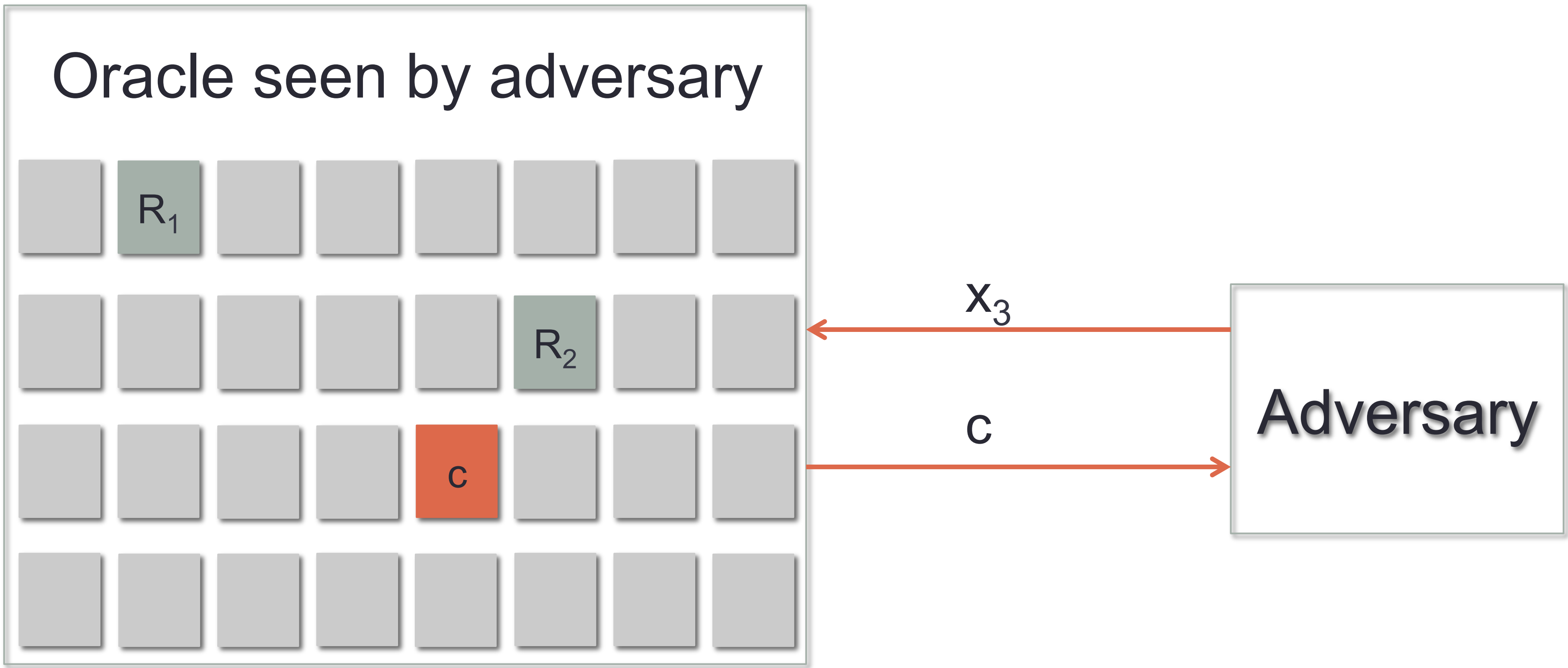

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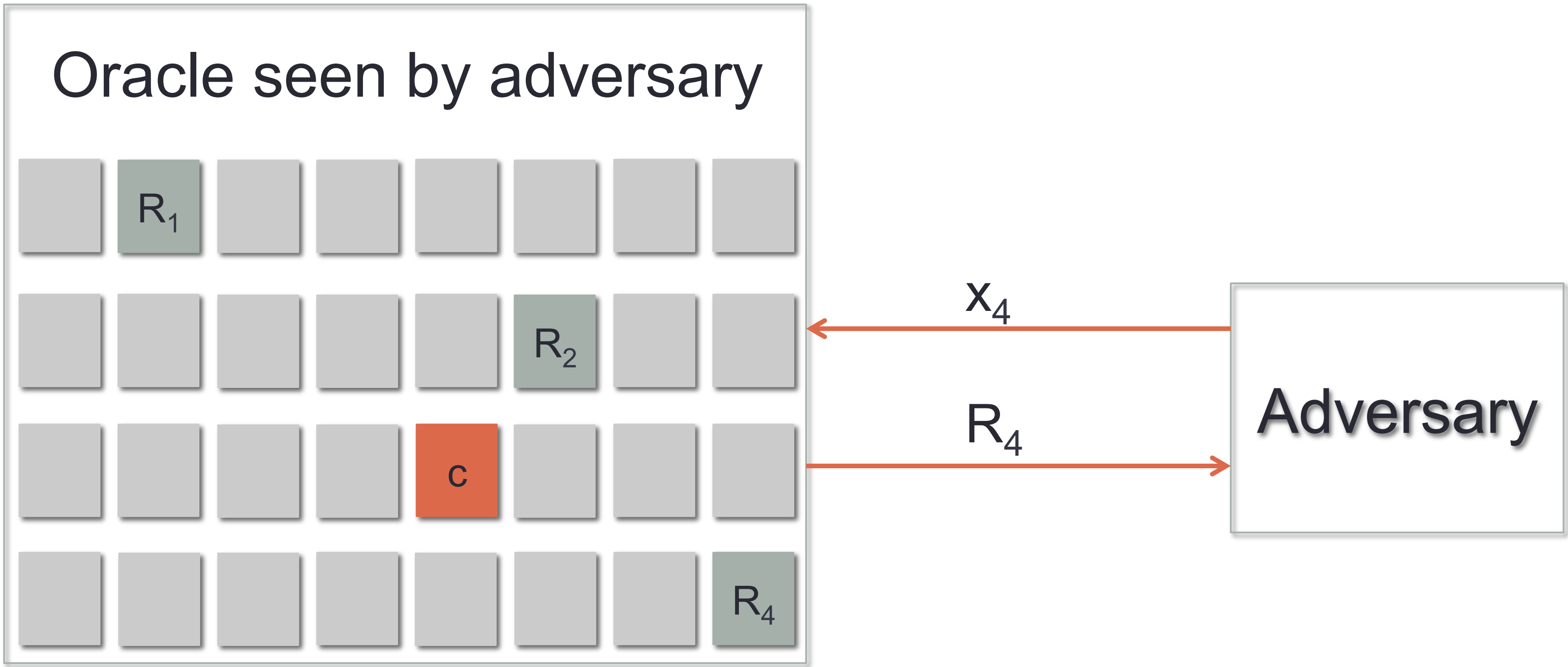
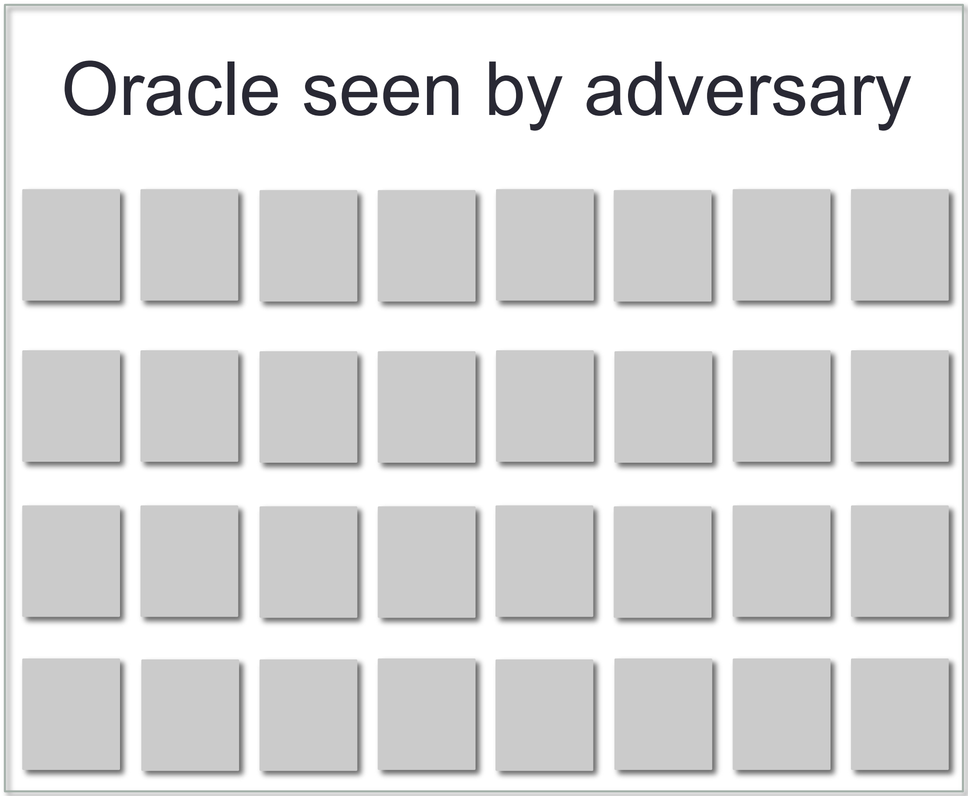


# Quantum Attempt 1

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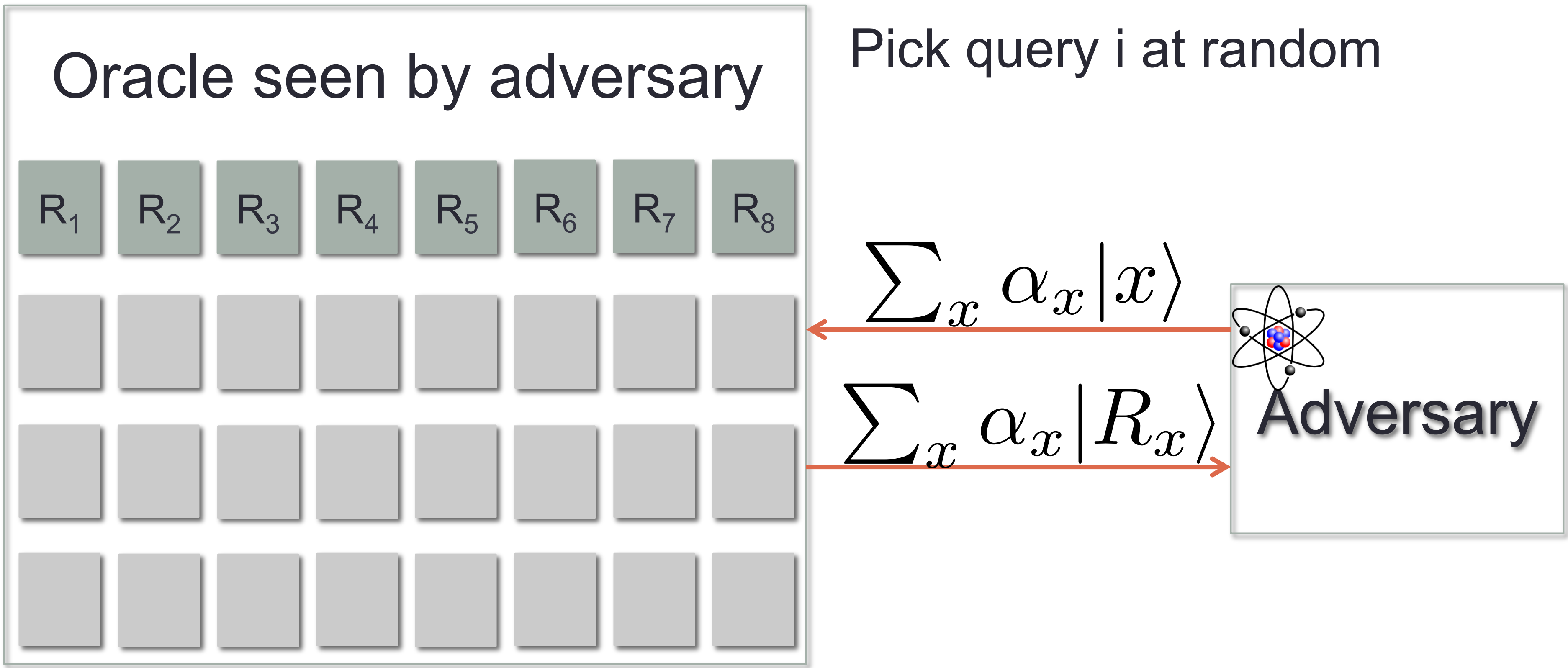

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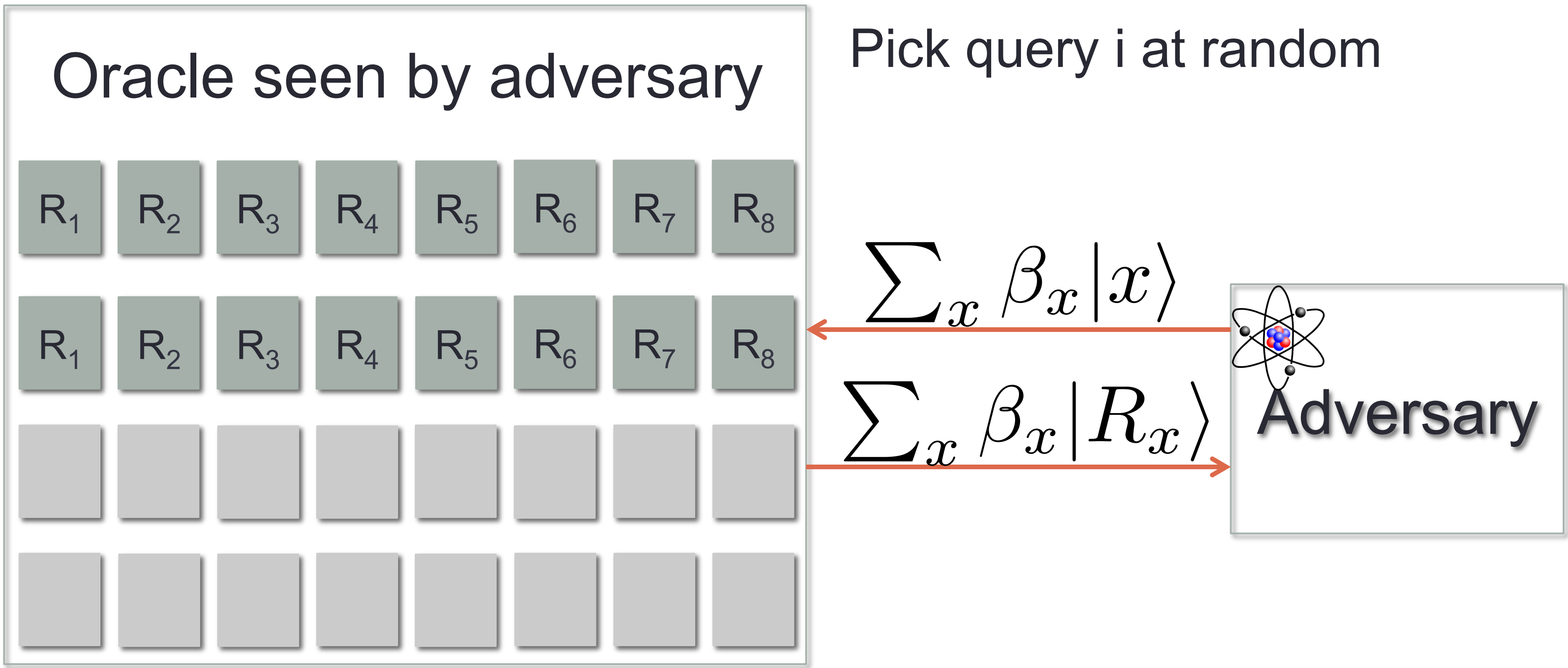

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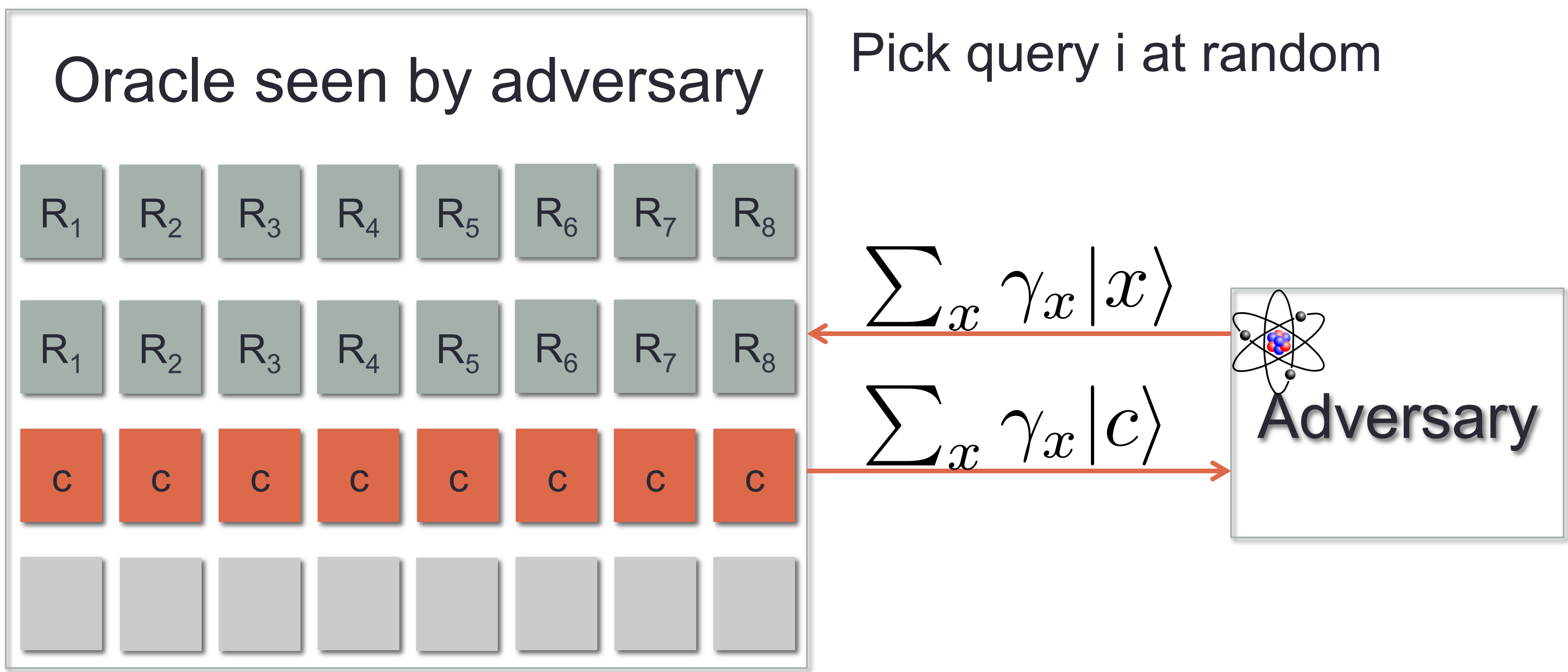

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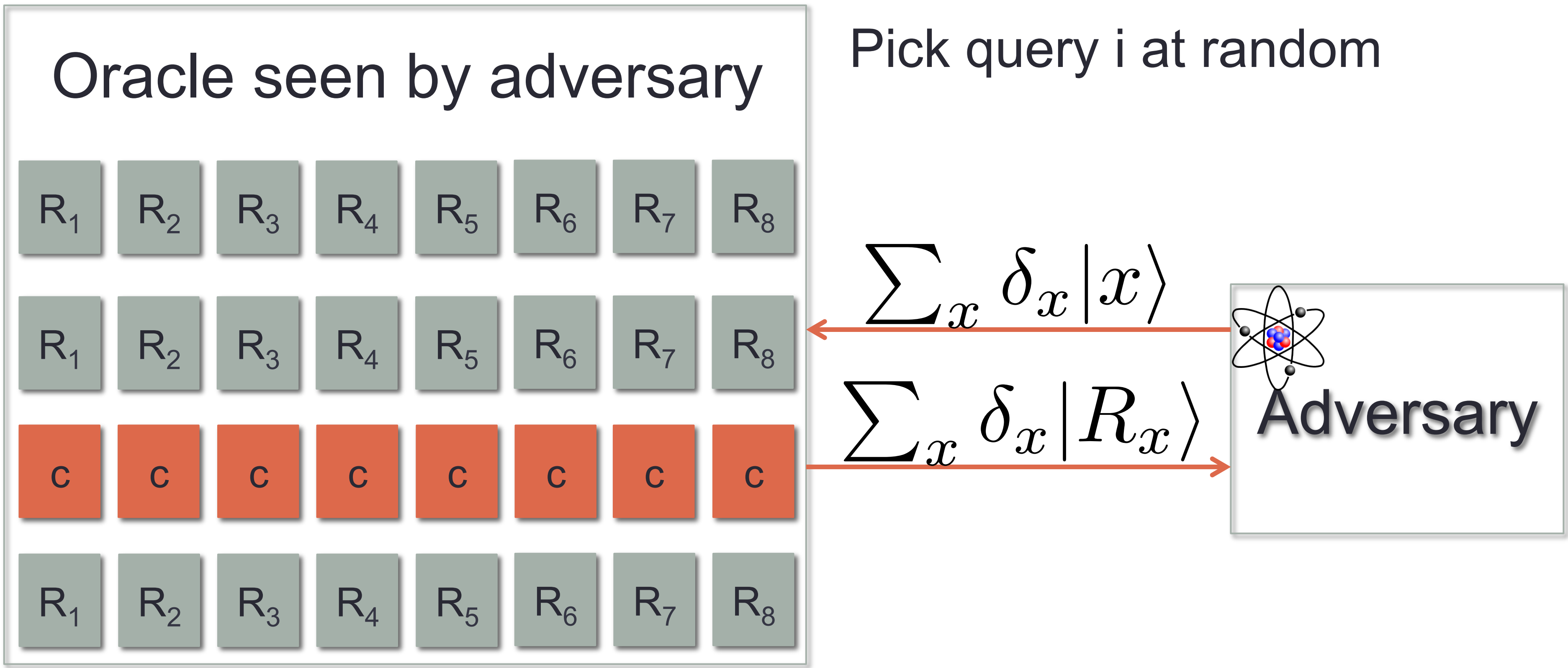

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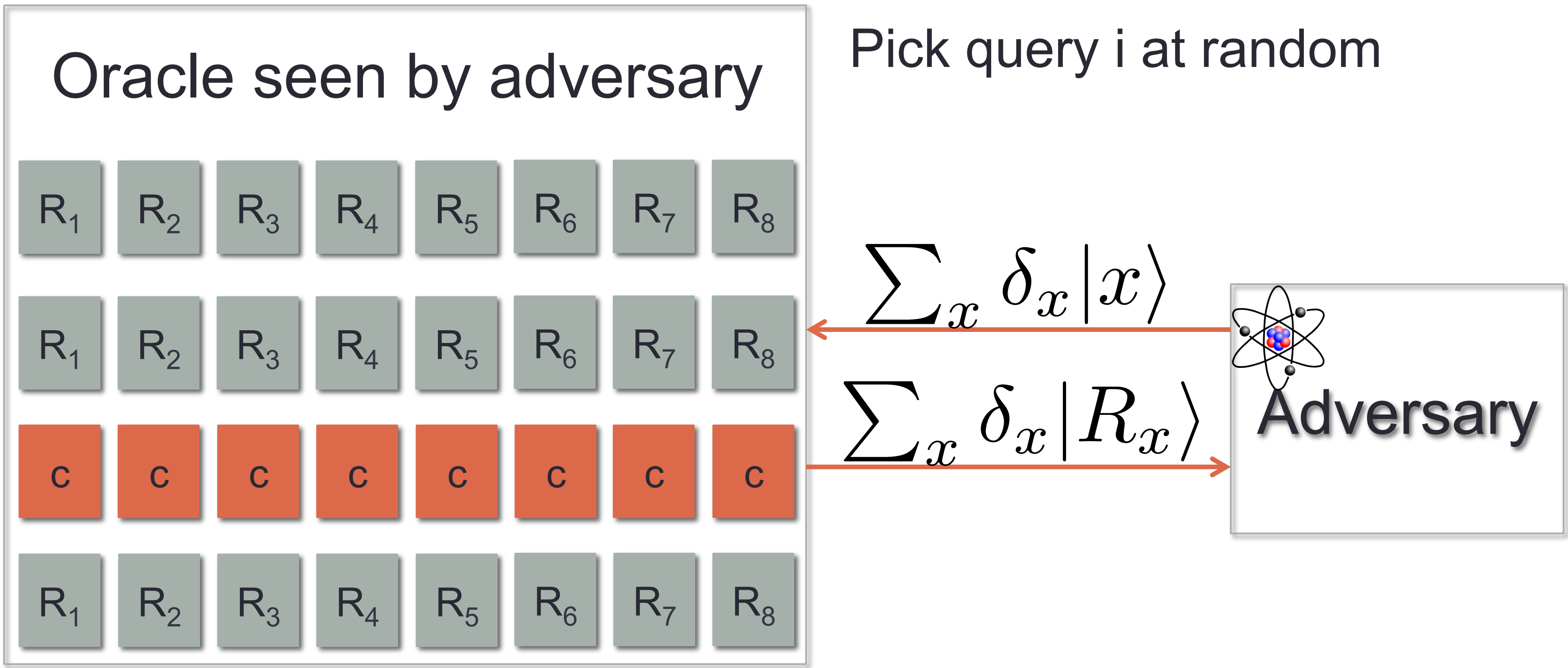
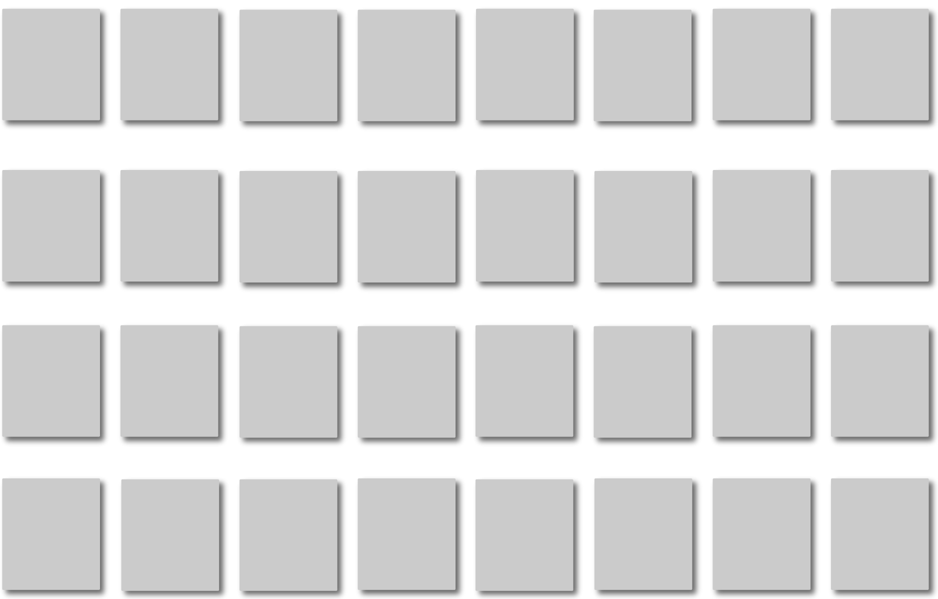


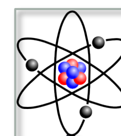

Query i is inconsistent and
does not look random

# Quantum Attempt 2

Oracle seen by adversary



Pick x* at random



Adversary

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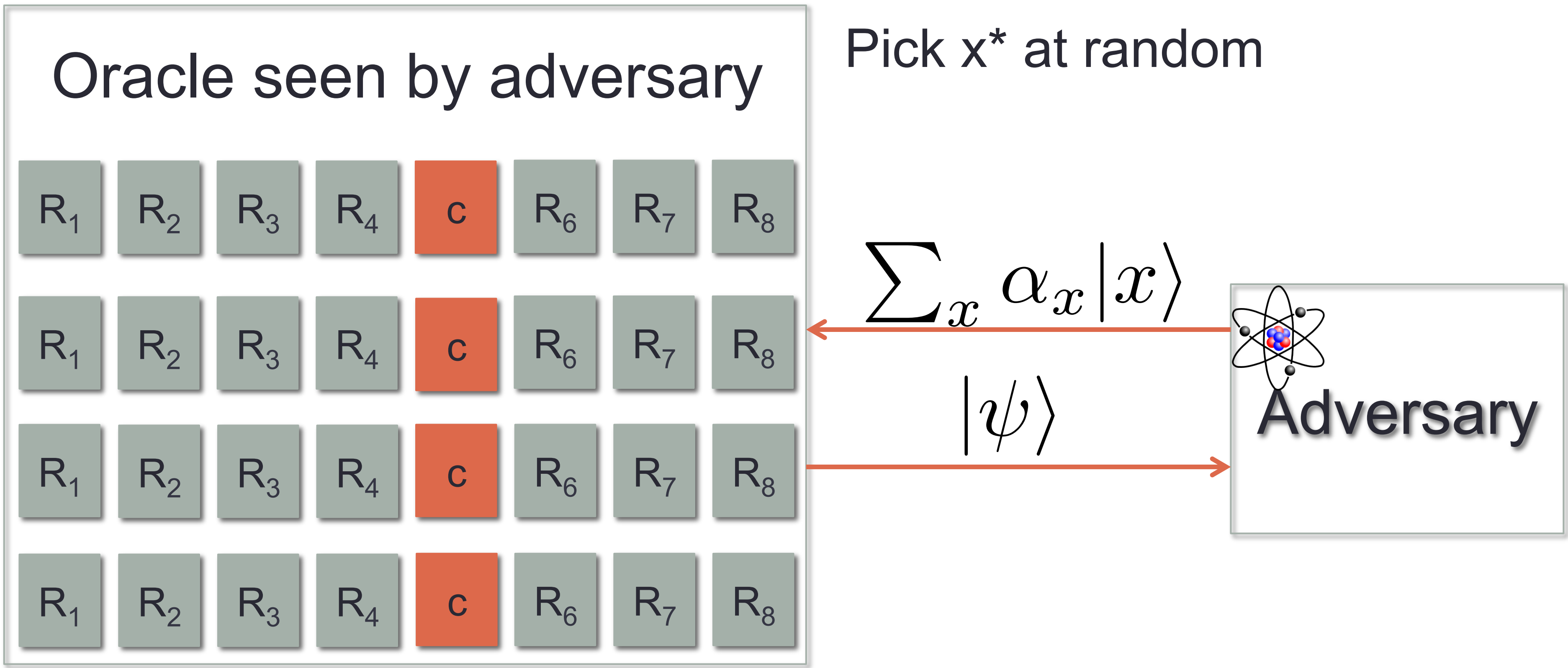


$$|\psi\rangle = \sum_{x \neq x*} \alpha_x |R_x\rangle + \alpha_{x*} |c\rangle$$

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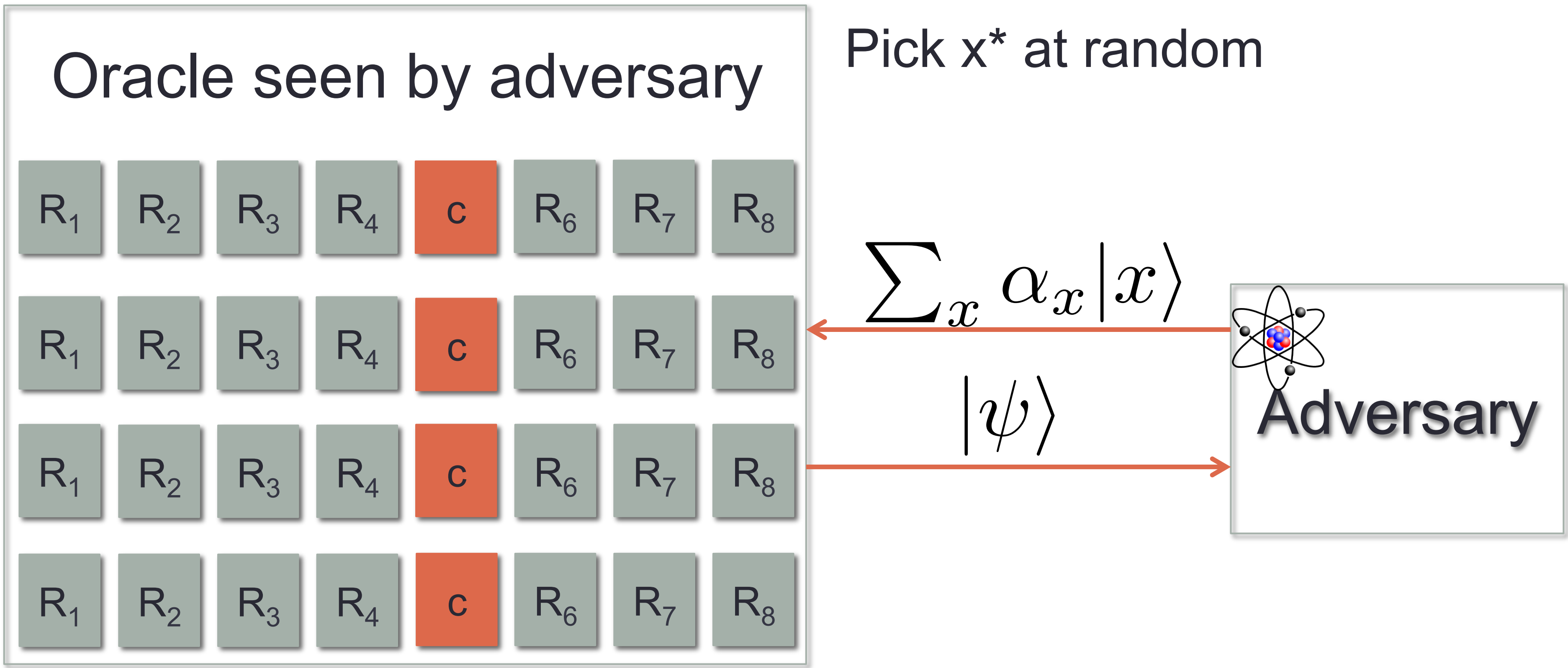
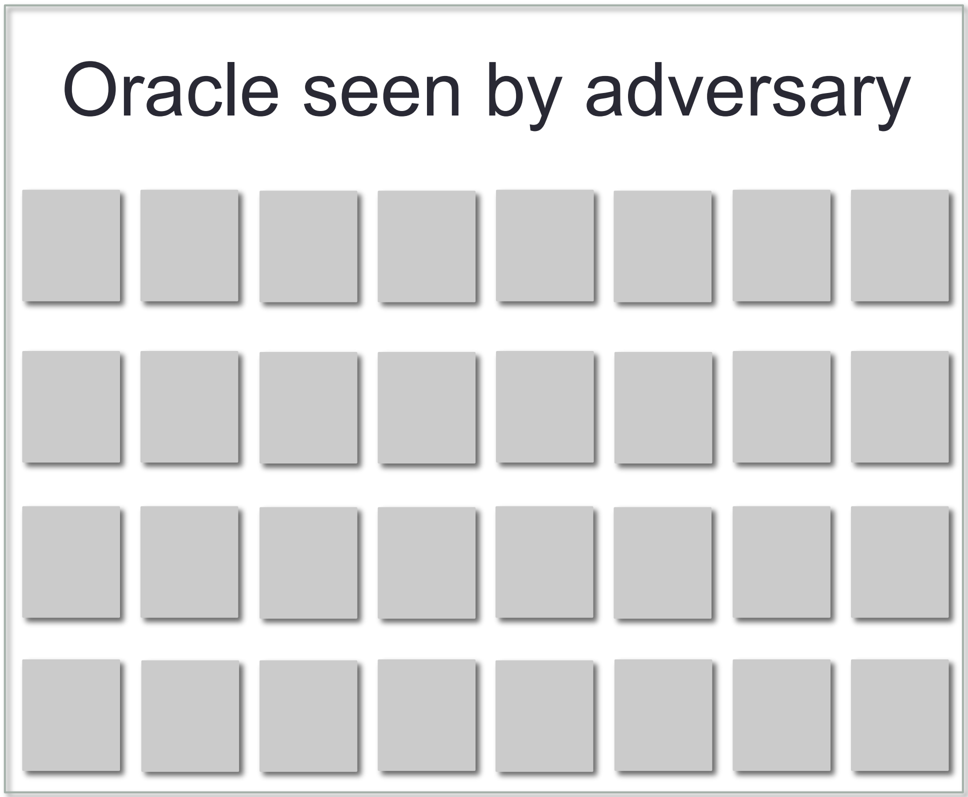



Adversary uses c with exponentially small probability

# Our Solution

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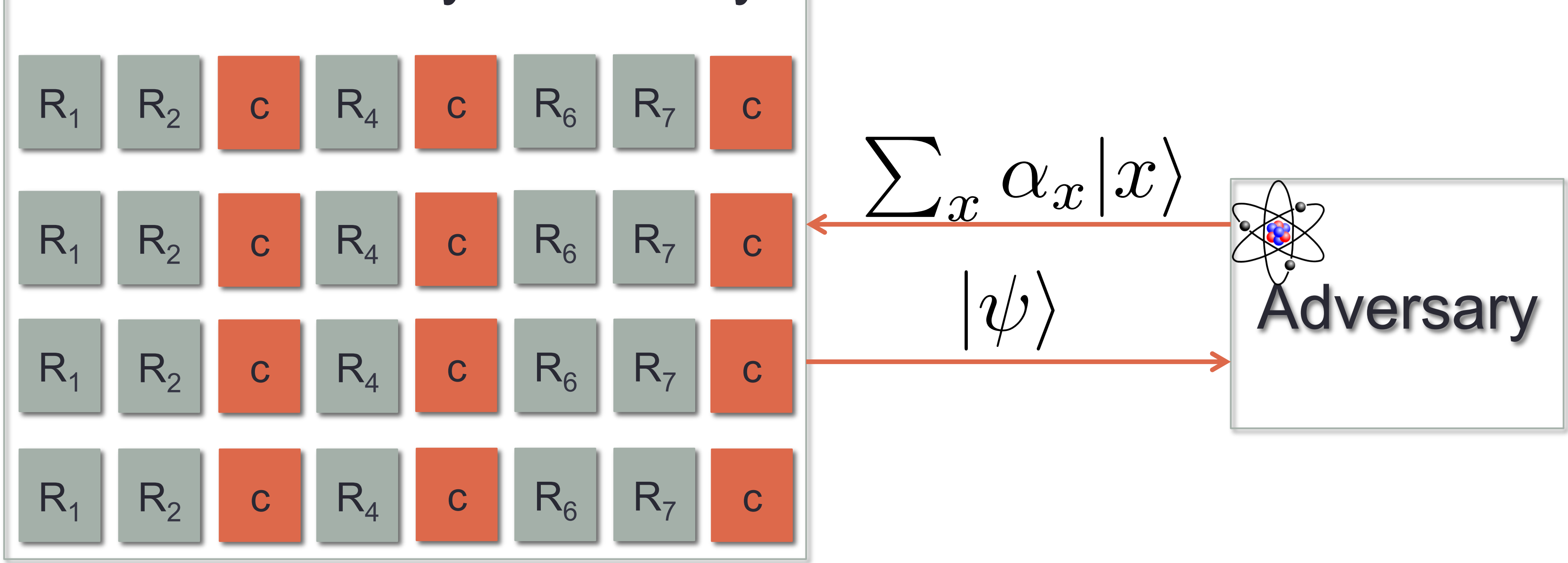


$$|\psi\rangle = \sum_{x \notin S} \alpha_x |R_x\rangle + \sum_{x \in S} \alpha_x |c\rangle$$

# Semi-Constant Distributions

- Parameterized by $\lambda$
- Pick a set S as follows: each x in the domain is in S with probability $\lambda$
- Pick a random c
- For all x in S, set H(x) = c
- For all other x, chose H(x) randomly and independently

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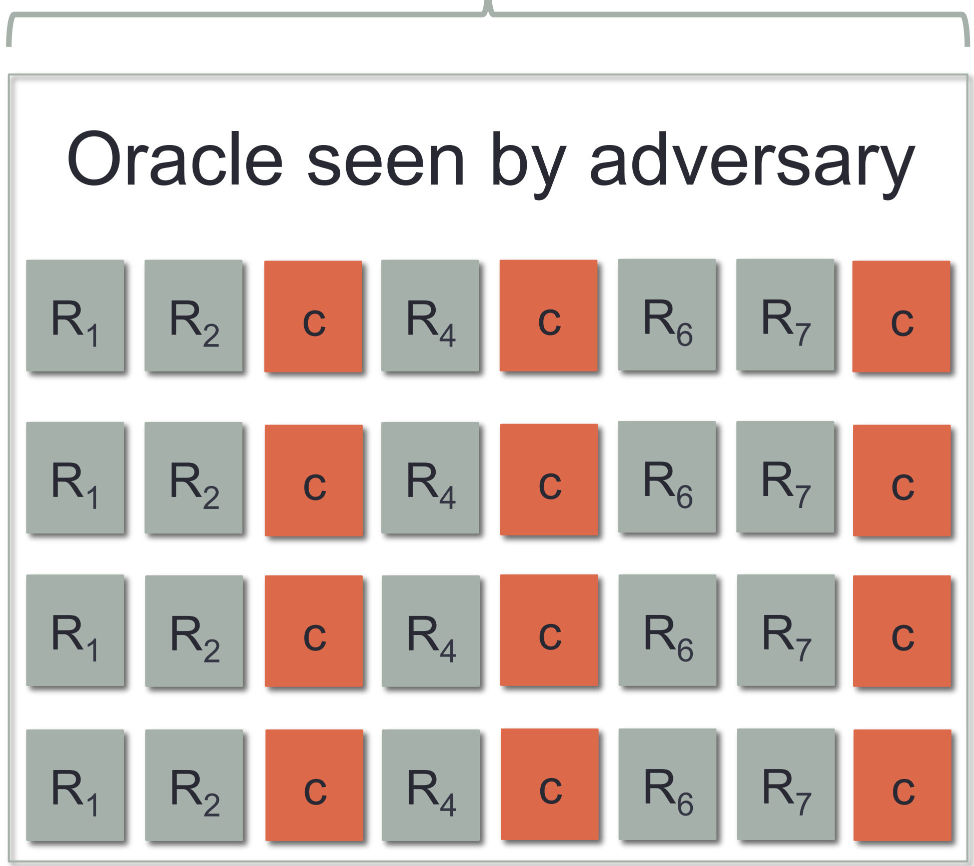
Theorem: Any quantum adversary making q queries to a semi-constant function can only tell it's not random with probability $O(q^4\lambda^2)$

# Quantum Security Proof

- Suppose adversary wins with probability $\varepsilon$
- Pick the set S, still let oracle be random
- Probability adversary uses one of the points in S: $\lambda$
- Probability wins and uses a point in S: $\lambda\varepsilon$
- Set $H(x) = c$ for all x in S
- Probability we succeed: $\lambda\varepsilon - O(q^4\lambda^2)$
- Choose $\lambda$ to maximize
- Succeed with probability $O(\varepsilon^2/q^4)$

# Generating the Random Values

Need to generate random values
for exponentially many positions



# Generating the Random Values

- $BDF^+$ 2011:
  - Assume existence of quantum-secure PRF
  - Pick a random key k before any queries
  - Let $R_x$ = PRF(k,x)
- Our solution:
  - Adversary makes some polynomial q of queries
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We can remove the quantum-secure PRF assumption from prior results as well

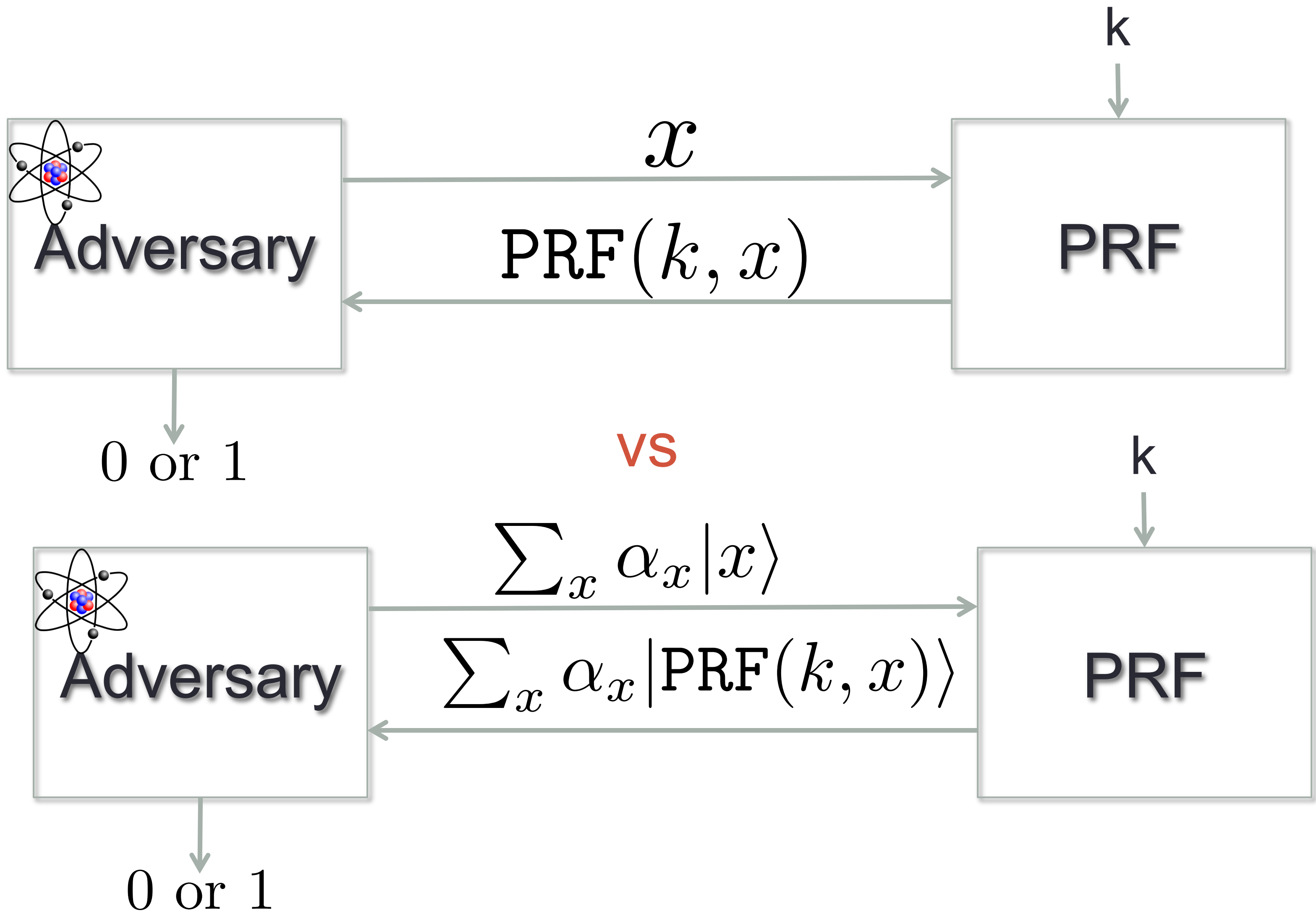
# Applications of this method

- IBE scheme [GPV 2009]
- Generic Full Domain Hash
  - Previous results only showed for specific trapdoor permutations
- Apply iteratively for Hierarchical IBE [CHPK 2010, ABB 2010]
  - Security degrades doubly exponentially in depth of identity tree
  - Classically, only singly exponential

# Quantum-Secure PRFs [Zhandry, FOCS 2012]

- So far, only considered case where interaction with primitive remains classical
- What if we allow quantum queries to primitive?
  - Example: pseudorandom functions

# Standard Security vs Quantum Security



# Quantum-Secure PRFs

- Results [Zhandry, FOCS 2012]
  - In general, PRF secure against classical queries not secure against quantum queries
  - However, several classical constructions remain secure, even against quantum queries
    - From pseudorandom generators [GGM 1984]
    - From pseudorandom synthesizers [NR 1995]
    - Direct constructions based on lattices [BPR 2011]
- Also have MACs secure when adversary can get tags on a superposition

# Open Questions

- Proving the quantum security of constructions based on Fiat-Shamir [FS 1987]
  - Signatures
  - Group Signatures
  - CS Proofs
- Other constructions
  - CCA security from weaker notions [FO 1999]

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Thank You!