Privacy-enhancing auctions using rational cryptography

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Rational cryptography

- goal: merge methodologies of cryptography & game theory

  → design & analysis of multi-agent protocols

- significant body of work

  (1) **honest, adversarial ≠ selfish & rational**

  - game-theoretic extensions of cryptographic protocols [HT04, GK06, ADGH06, LT06, KN08a, KN08b, MSR08, OPRV09, MS09, FKLN09…]

  (2) **mechanism design ≈ secure multiparty computation**

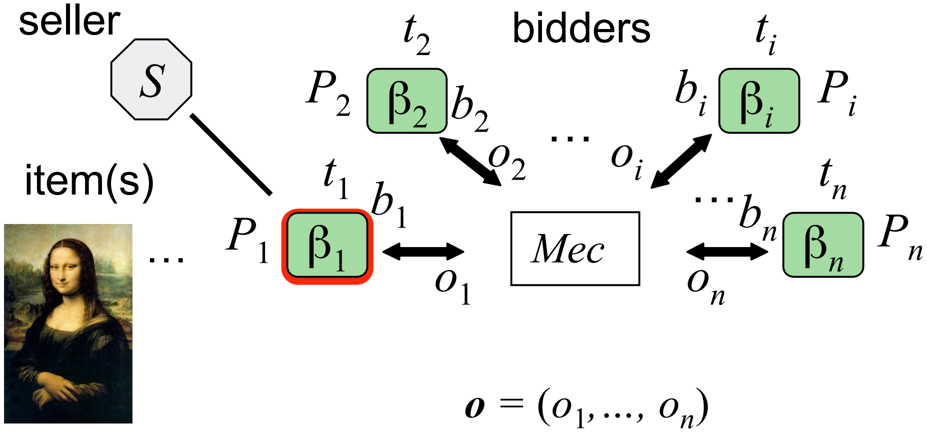
  - crypto-based realization of games without trusted mediator [DHR00, LMPS04, LMS05, ILM05,…]

- this work considers a concrete problem

  running a privacy-aware auction over the Internet

Classical auctions

- games for mapping items/prices to buyers
  (e.g., 2nd price auction)
  - distribution over private valuations or types ***t***
  - strategy β for submitting bids ***b***
  - allocation mechanism for specifying output ***o***



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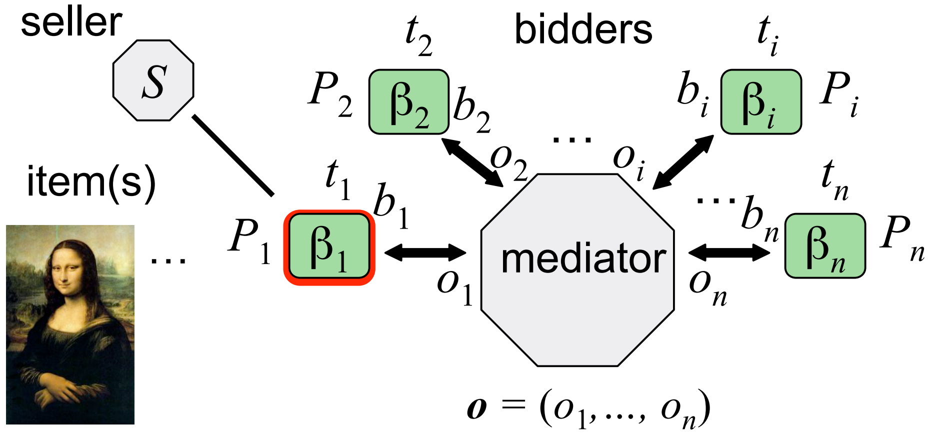


mediated

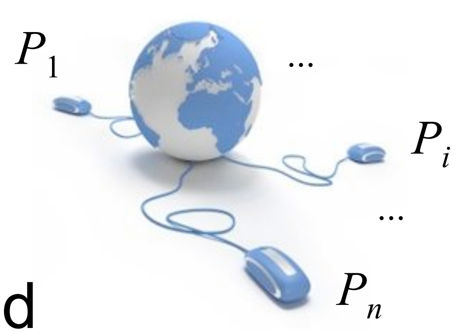
- defined by an abstract functionality
- realized through a concrete implementation

privacy-oblivious

- monetary utilities
- private bids/types may be revealed to participants

The problem

- goal: design an auction protocol for the Internet which considers privacy and which is rational to follow
- we wish the auction game to be



+

rich prior work, e.g., [NPS99, PRST06…]

Internet-based

- use realistic communication
  - secure & authenticated **point-to-point channels**
- towards practical implementation

privacy-aware

- model privacy concerns
  - bidders wish to prevent revealing information related to their valuations but would appreciate learning others’ valuations
- protect bidders’ valuations

Challenges

- goal: design an **auction** protocol for the **Internet** which considers **privacy** and which is **rational** to follow

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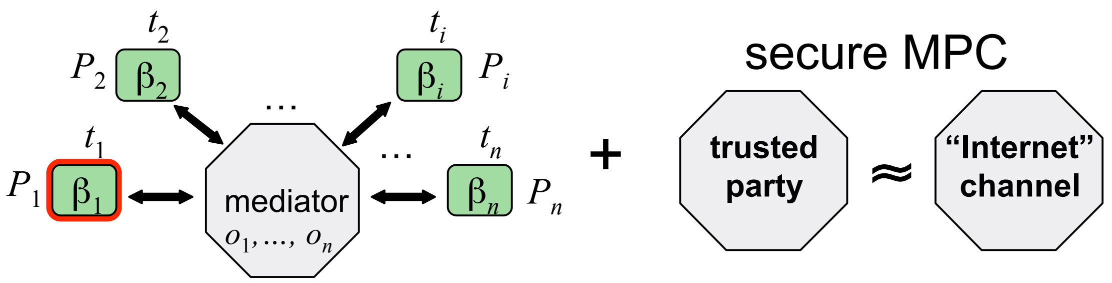


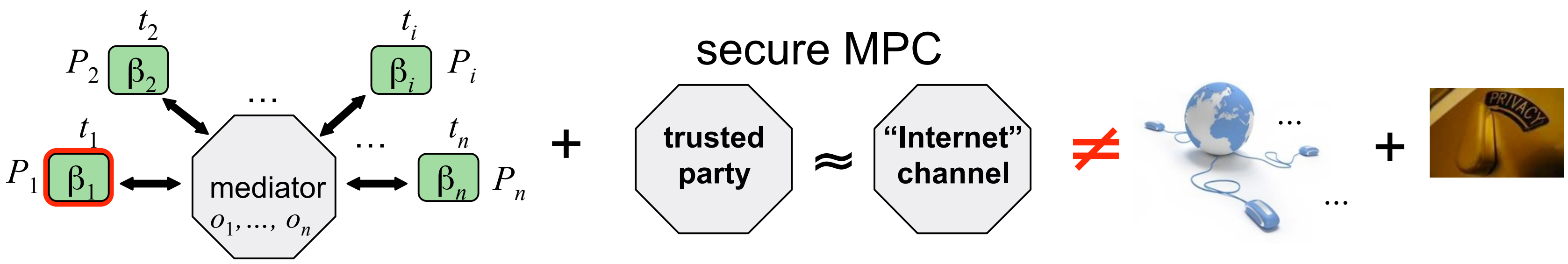
"use secure MPC to implement your favorite auction"

such an approach is insufficient

- **rational execution**
  - equilibrium analysis for secure privacy-aware auction in computational setting
- **information leakage**
  - consistent model for any information leakage that occurs in mediated auction
- **transaction completion**
  - definition of "winning" state given that winner is never forced to buy

Challenges

- goal: design an **auction** protocol for the **Internet** which considers **privacy** and which is **rational** to follow



at least, not yet…

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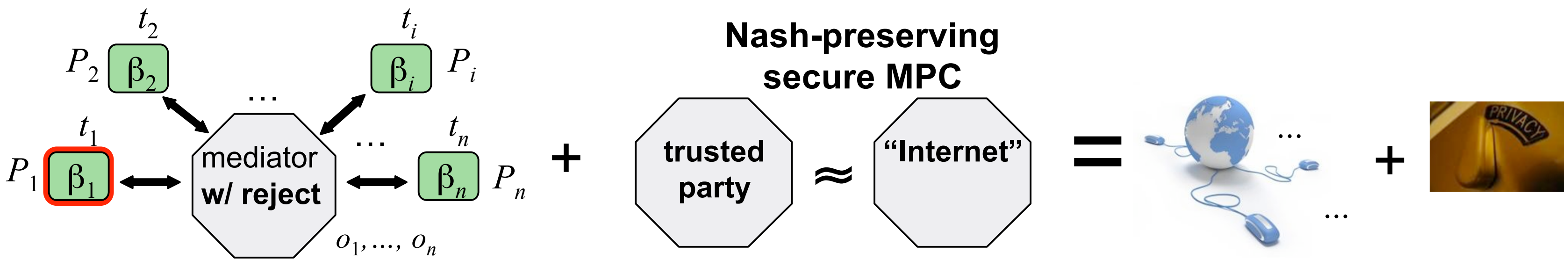
Our approach

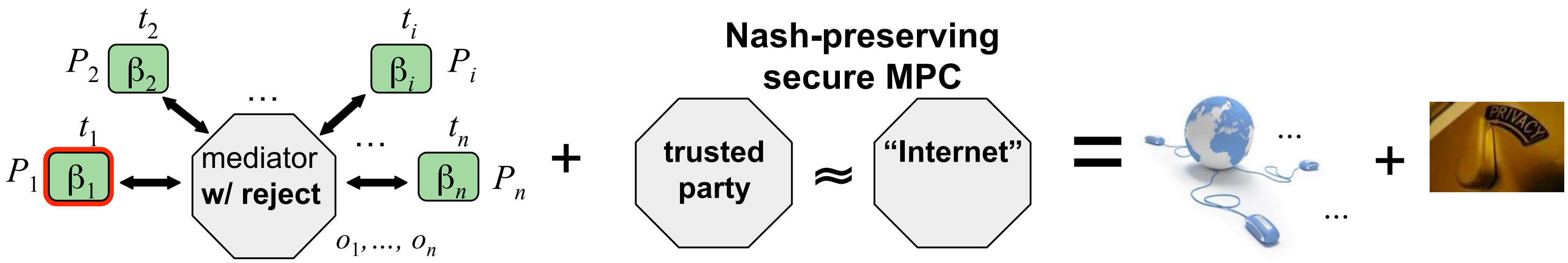
- rectify previous unsuccessful attempt



- define appropriate game-theoretic framework
  - protocol games over the Internet w/ hybrid utility model → **rational execution**
  - privacy-enhanced approximate Nash (PE ε-Nash) → **information leakage**
  - contracts and mediation w/ reject → **transaction completion**
- design Internet-based Nash-preserving sec-MPC protocol
  - unknown contract revelation point → **participation**
  - decoupling of winning information and winning contracts → **multi-winners**

Our result

- general design principle and possibility result



consider any PE ε-Nash auction mech. in the mediated w/ reject setting

generic method for Internet-based Nash implementation

PE ε-Nash auction mech. in the Internet setting with utility profile **negligibly close** to the original one

do such mechanisms exist?
yes, all "**predictable**" ones
(e.g., 1^{st} price auctions)

condition: players' "expected fiscal utility" sufficiently larger than "privacy weight", i.e., "greedy-then-paranoid"

≠

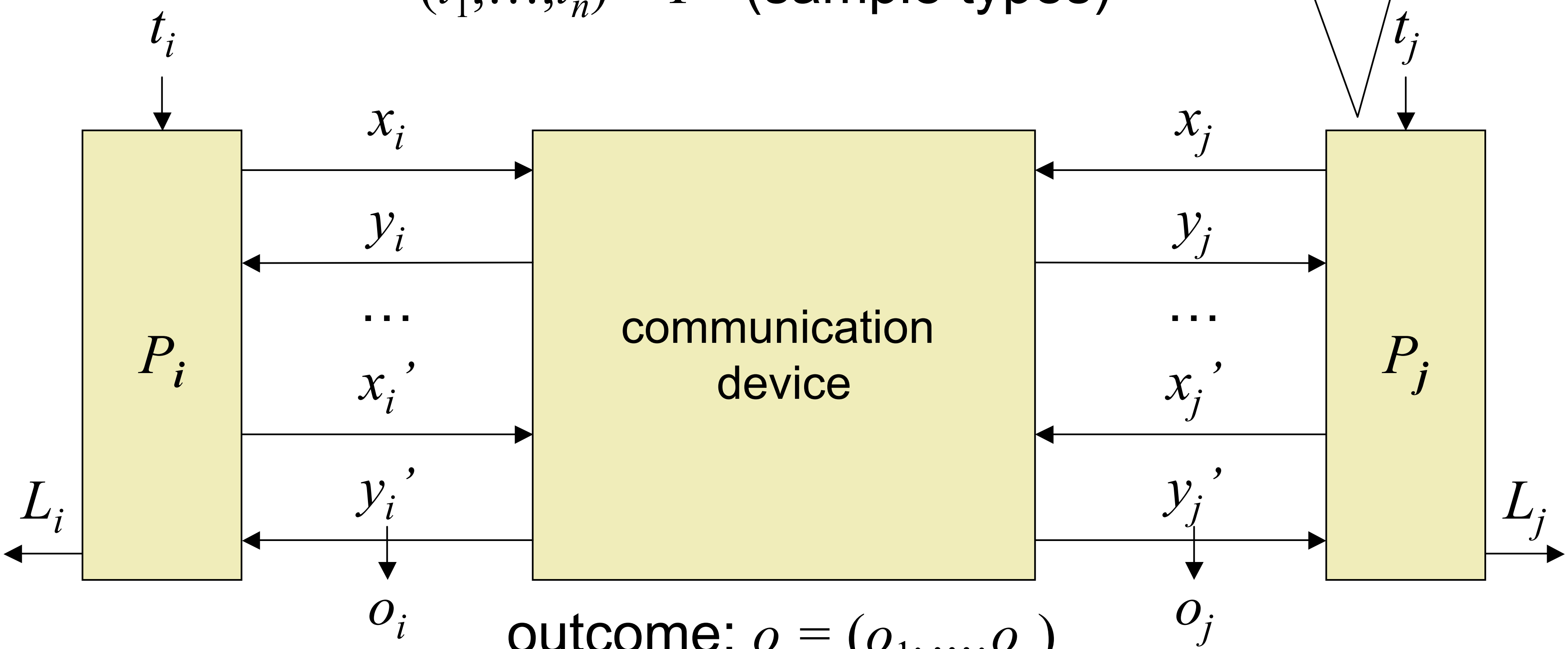
2^{nd} price auctions

formal framework for "composable" Nash-preserving transformations from abstract to concrete settings

Protocol games

strategy $\sigma(k) \in A^{T(k)}$: non-uniform family of circuits, one for each security parameter k

$(t_1,\ldots,t_n) \leftarrow T$ (sample types)

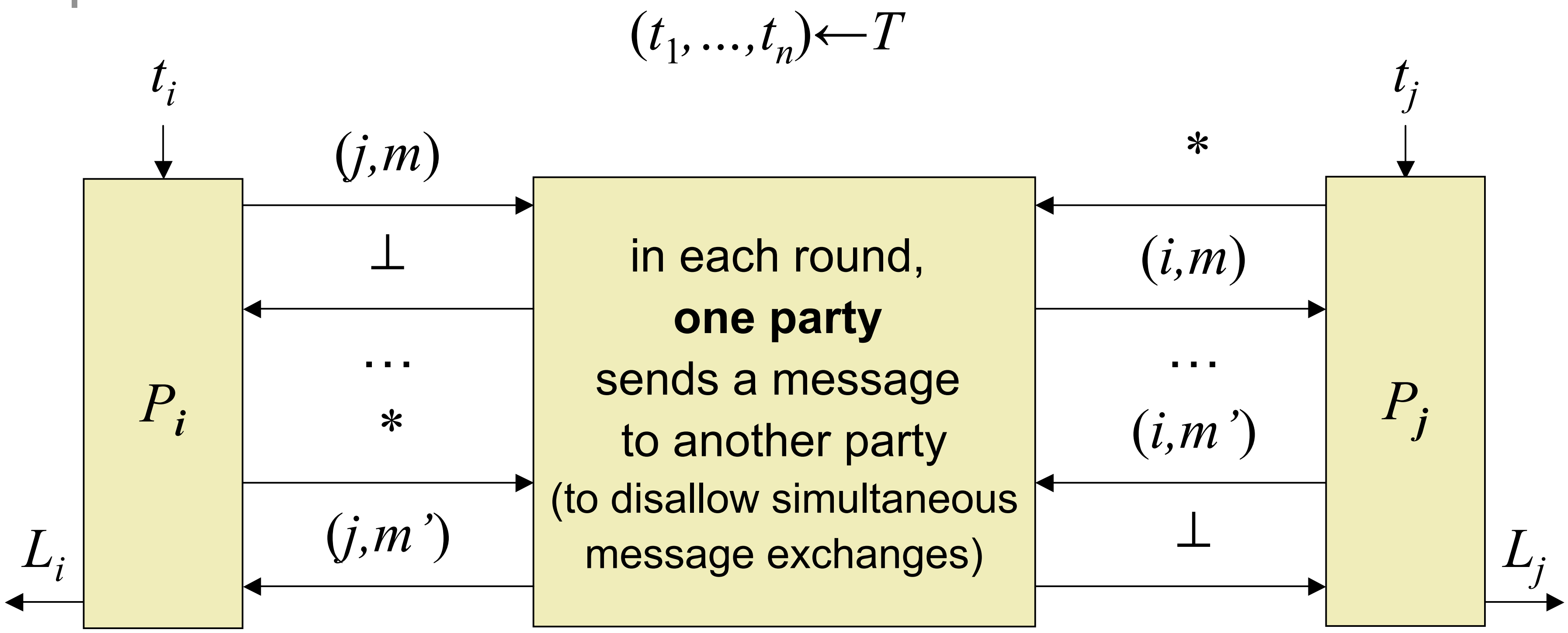


outcome: $o = (o_1, \ldots, o_n)$

local output: $L=(L_1, \ldots, L_n)$

“normal” utility: $u_i(t,o)$, privacy-aware utility: $u_i(t,o,L)$

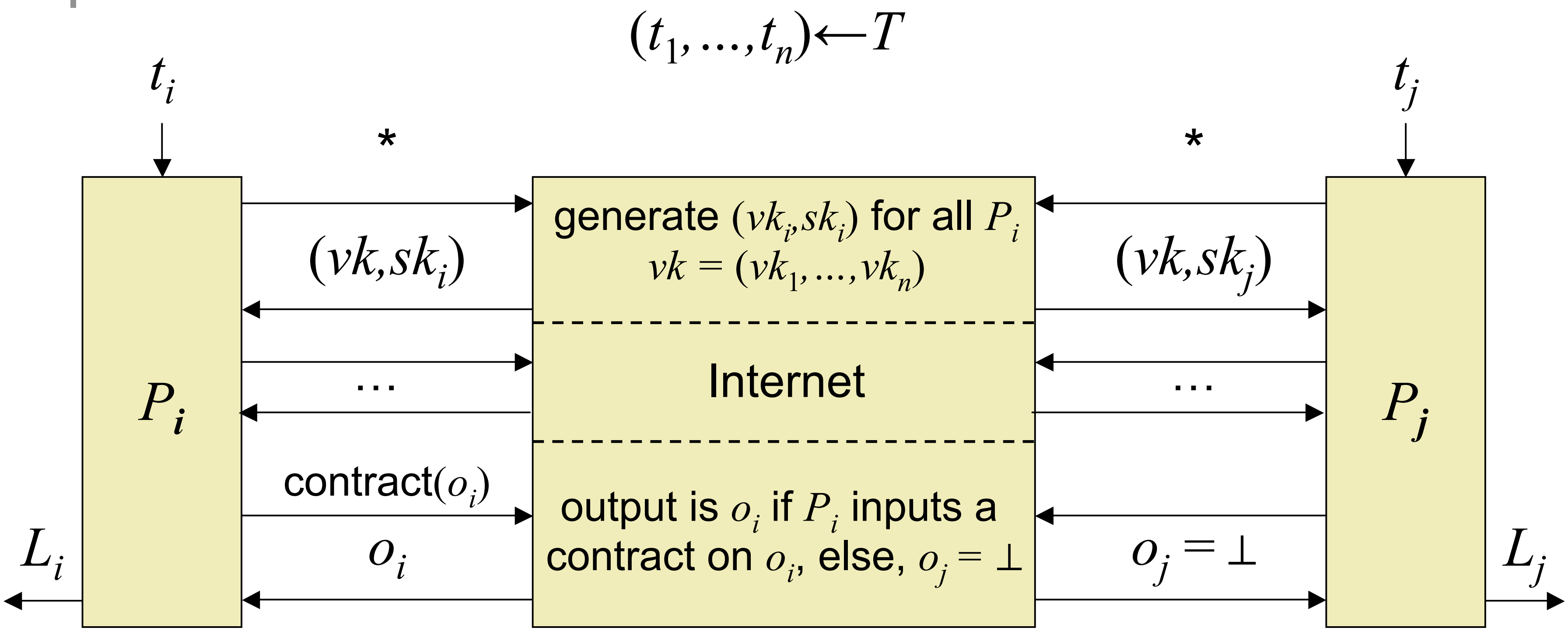
Internet-like communication



what about the protocol outputs?

- defined by **contracts**
  - “winning-state” values that are signed by all parties (using a PKI)

PKI + Internet + Contracts


$(t_1, \ldots, t_n) \leftarrow T$
t_i
t_j
*
*
P_i
P_j
generate (vk_i, sk_i) for all P_i
$vk = (vk_1, \ldots, vk_n)$
(vk, sk_i)
(vk, sk_j)
Internet
…
…
contract(o_i)
o_i
output is o_i if P_i inputs a contract on o_i, else, $o_j = \perp$
$o_j = \perp$
L_i
L_j

we assume that fiscal utility is 0 if $o_j = \perp$

Hybrid utility model

- utility of P_i is a sum of a **fiscal** utility f_i and a **privacy** utility p_i

$$u_i(t,o,L) = f_i(t,o) + p_i(t,L)$$

- we model fiscal preferences using a utility function $f_i(t,o) \in \boldsymbol{R}$
  - its output is polynomial in k
- we model privacy concerns using a utility function $p_i(t,L) \in \boldsymbol{R}$
  - poly-time computable in k
  - it does not significantly value loss of own information
    - e.g., $p_1(t,(\perp,L_2,\ldots,L_n)) - p_1(t,(L_1,L_2,\ldots,L_n)) \leq negl(k)$

we call such a privacy utility **admissible**

different hybrid model [HP08]

Greedy-then-paranoid

- we assume that parties are **greedy-then-paranoid**
  - first and foremost they want the good, but all other things being equal they also value privacy
  - in particular: not willing to always bid b_i=0 just to hide information on their type
- we define **privacy weight**

$$\begin{aligned} p &= (p_1, \ldots, p_n) \\ pw(p_i) &= max_X\, p_i(X) - min_X\, p_i(X) \\ pw(p) &= max_i\, pw(p_i) \end{aligned}$$

BIG BUSINESS
WAR
PROFITS FROM BUILDING MATERIAL
PROFITS FROM PEOPLE'S FOOD
WAR PRICES
PROFITS FROM WAR-PRICED CLOTHING
INTEREST ON WAR LOANS
PROFITS FROM WAR SUPPLIES

Privacy-enhanced ε-Nash

- **deviation incentive** of protocol $\sigma = (\sigma_1, \ldots, \sigma_n)$

  $\varepsilon(k) \;= max\ P_i,\ max\ \sigma_i{}^*(k) \in A^{T(k)}$: $u_i(\ \sigma_i{}^*(k), \sigma_{-i}(k)\ ) - u_i(\sigma)$

- σ is ε-Nash if its deviation incentive is negligible in k for all $T(k)$

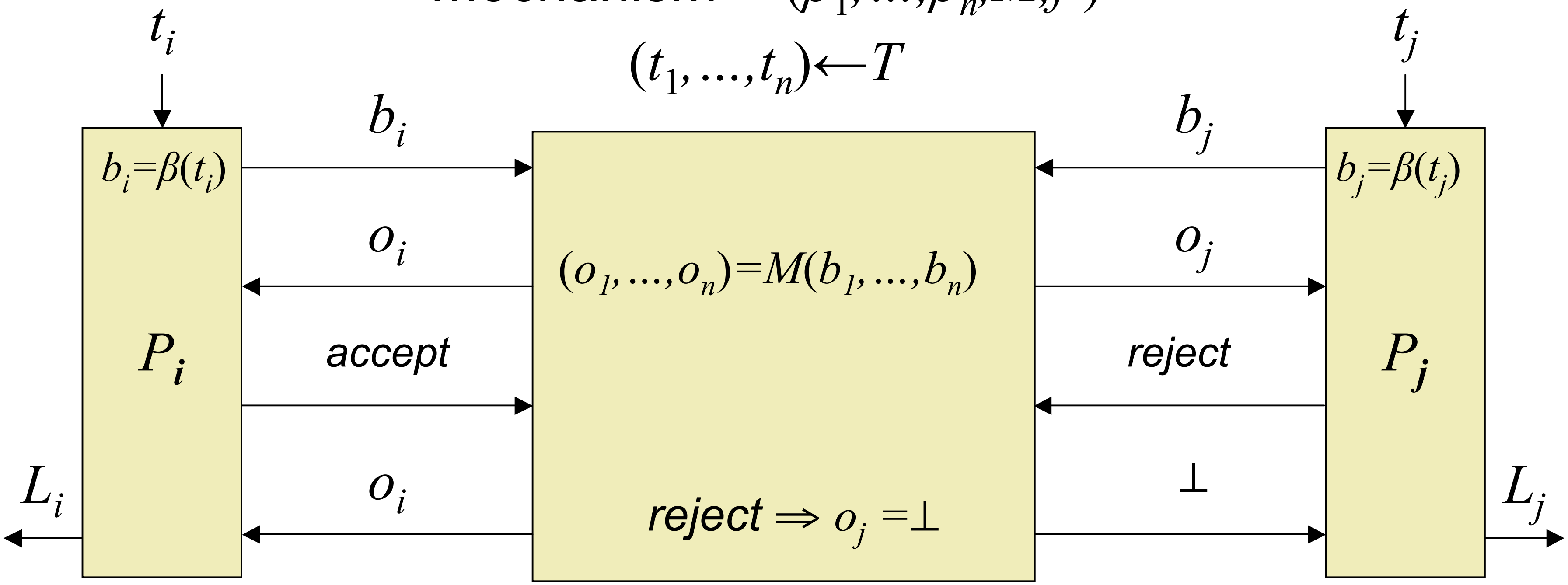
- a mechanism is a **privacy-enhanced** ε-Nash, for privacy weight α and fiscal utilities f, if it is an ε-Nash for

  $$u_i(t,o,L) = f_i(t,o) + p_i(t,L)$$

  for **all** admissible p, $pw(p) \leq \alpha,$ and polynomial strategy spaces A^T

Mediated setting w/ reject

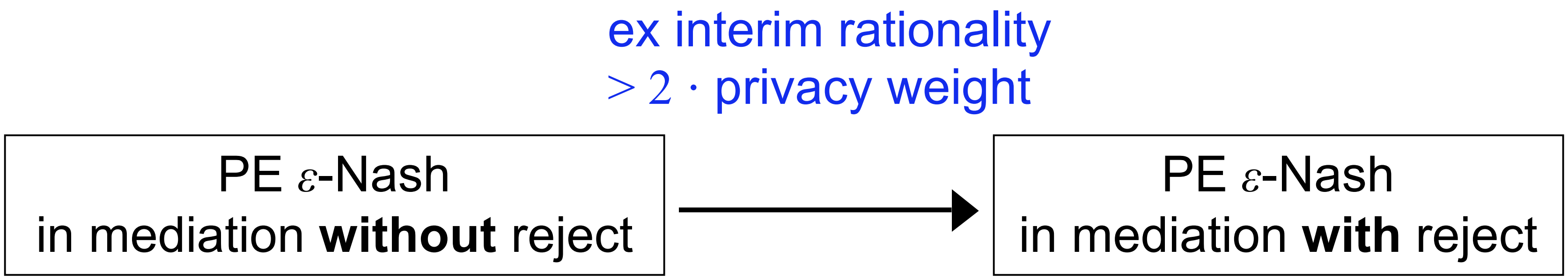
mechanism = $(\beta_1, \ldots, \beta_n, M, f)$



only interested in recommended protocols which play *accept*

Auctions in mediation w/ reject

- 2^{nd} price auctions **are not** PE ε-Nash, $\alpha > 0$
  - rational to bid the maximal price and throw away the contract if you do not like the price
- 1^{st} price auctions **are** PE ε-Nash, suff. small α
  - e.g., a mechanism with strict *ex interim* rationality: after seeing your type your expected utility is never 0



ex interim rationality
$> 2 \cdot$ privacy weight

PE ε-Nash
in mediation **without** reject

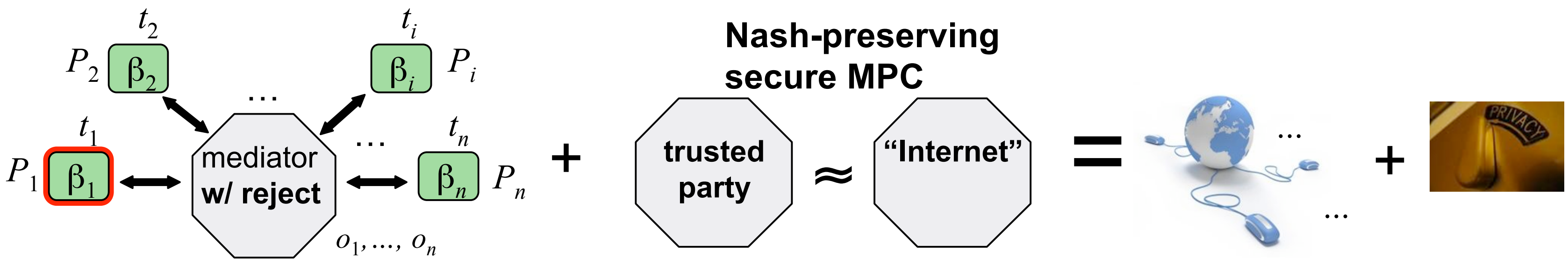
PE ε-Nash
in mediation **with** reject

ex interim rationality
minimum over all P_i
expected utility after seeing t_i

true for any **predictable** mechanism:
expected fiscal utility of a winner P_i
depends only on b_i

Our approach

- general design principle and possibility result



- define appropriate game-theoretic framework
  - protocol games over the Internet w/ hybrid utility model √
  - privacy-enhanced approximate Nash (PE ε-Nash) √
  - contracts and mediation w/ reject √
- design Internet-based Nash-preserving sec-MPC protocol
  - generic Nash-preserving transformation of any mechanism with strict ex interim rationality achieving utility profiles negligibly close to original protocol

Basic idea, a problem and a fix

<u>idea</u> - one winner case

- use sec-MPC to compute an **additive secret sharing** of the contract
- make the shares of the contract public in round robin order
  - if any party withholds share, penalize by also withholding

<u>intuition</u>

- until the last share is made public nobody knows the contract and therefore no party can exclude that it won
- sending ones’ output share is rational by strict *ex interim* rationality

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<u>problem</u>

- the last party will know who won and might not make its share public when it is not the winner

<u>fix</u>

- let the winner hold the last share!

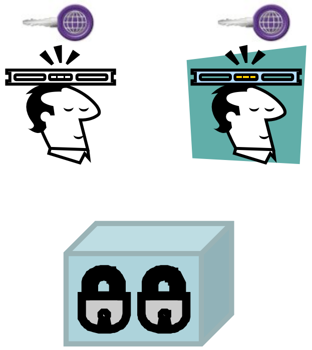
…and another problem

- if by definition the winner is last in the round-robin, then all the n-1 first parties know they did not win
  - makes it rational for them to stay silent!

monetary value for the winner,
but privacy value for other parties

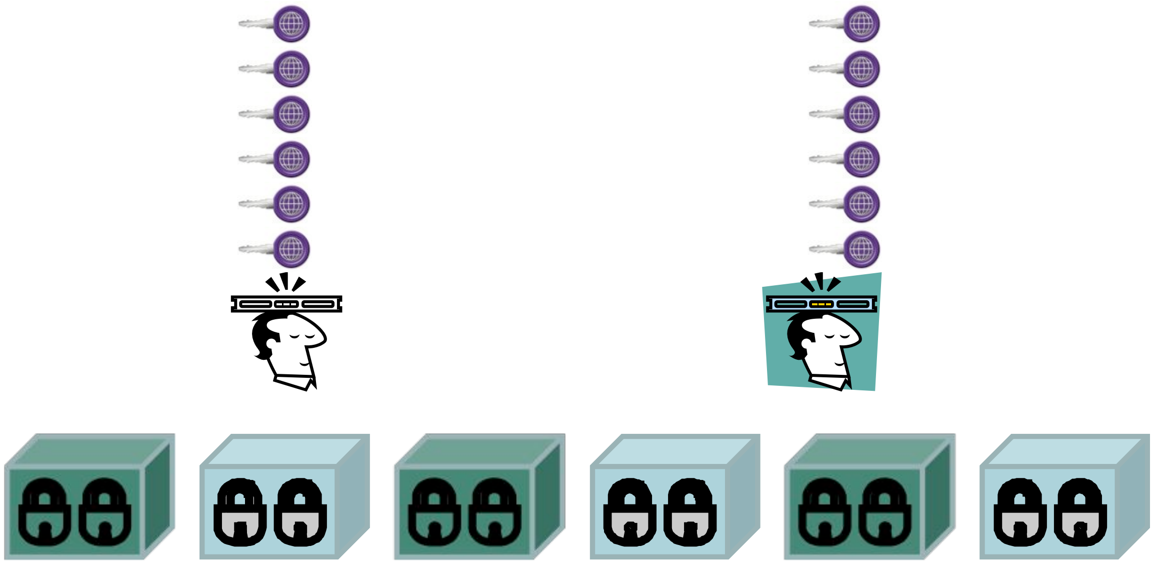
contract

who sends a key first?



Fix: multi-phase protocol

- create many ($O(k)$) boxes (standard trick [HT04,GK06,LT06,GHKL08,KN08])
  - all boxes are empty holding $\perp$, expect the π-th box holding the contract
  - pick π s.t. $Pr[\pi = p] = \tfrac{1}{2}^{p}$; keep π unknown
  - the i-th box stores the output of party $P_{i \bmod n}$
  - unlock i-th box in n rounds, by revealing shares to $P_{i \bmod n}$ in round robin
  - if a party does not reconstruct $\perp$ or a valid contract it stays silent for ever



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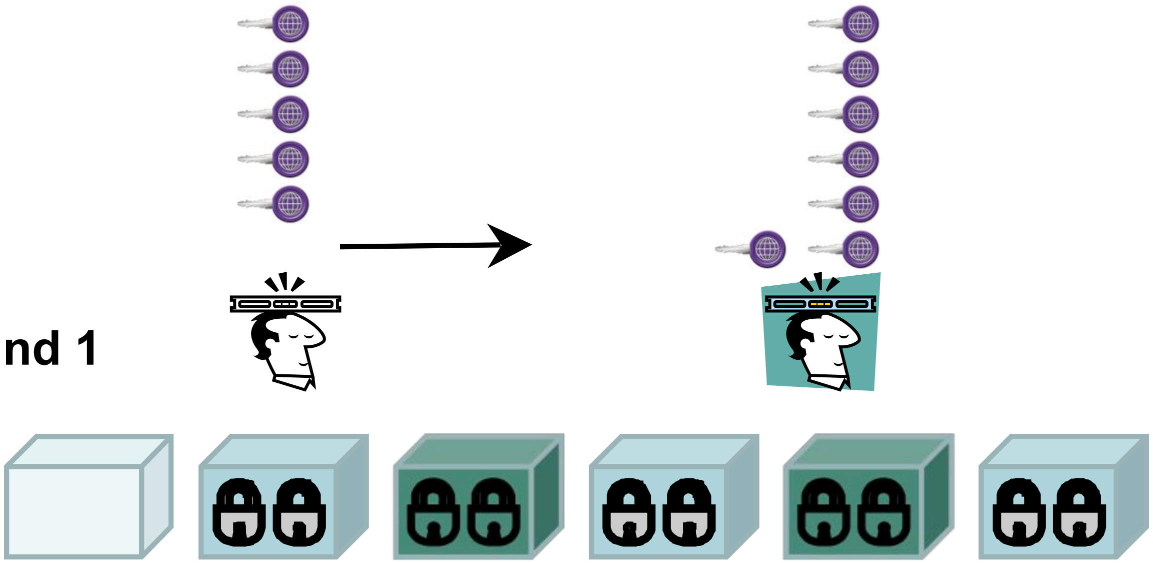
phase 1, round 1



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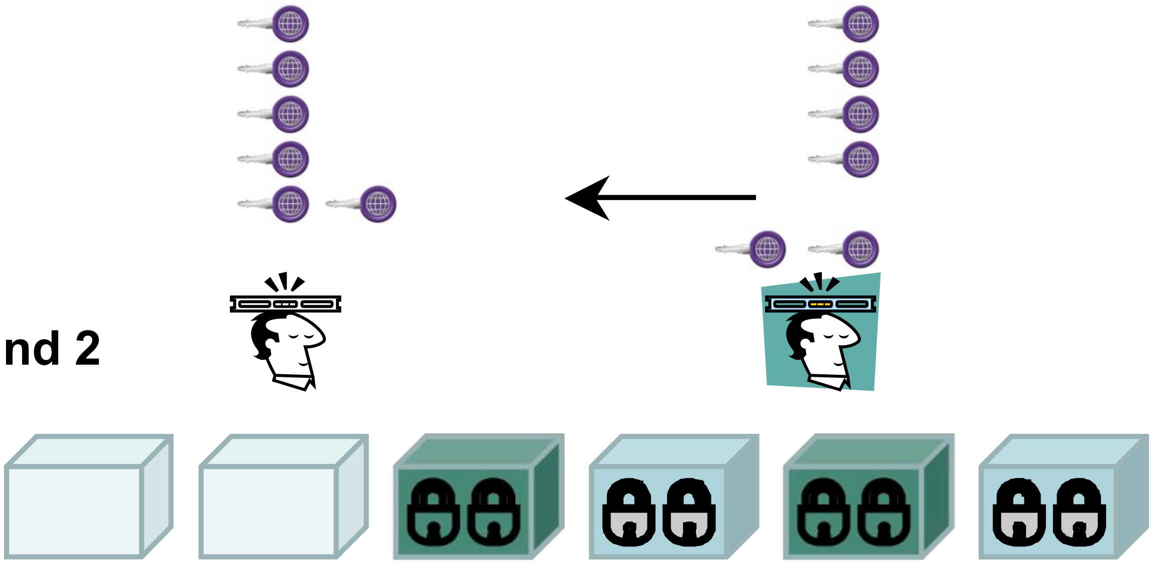
phase 1, round 2



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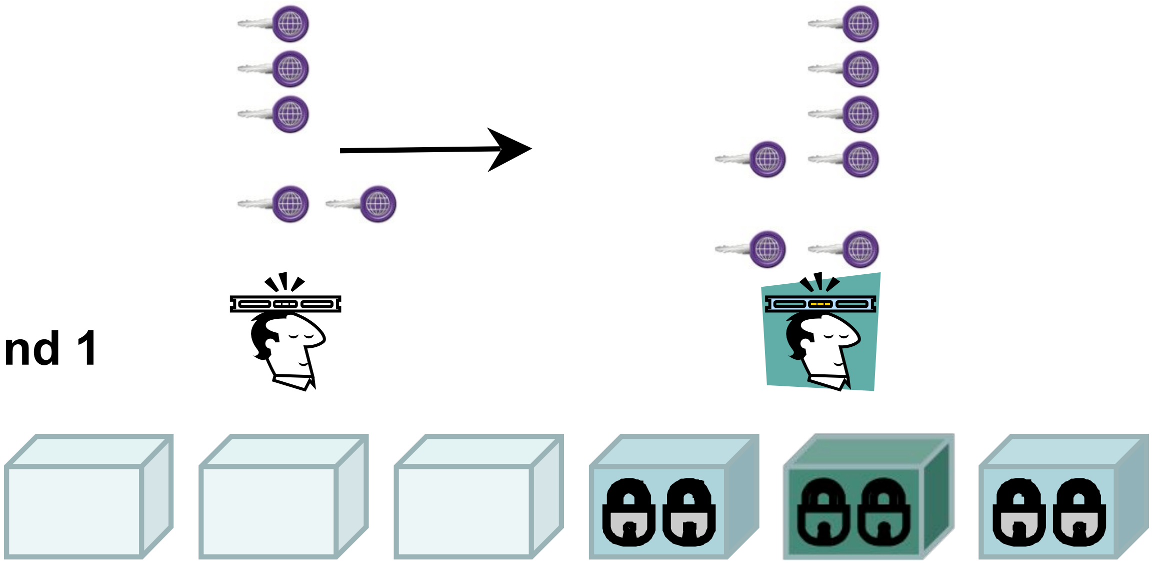

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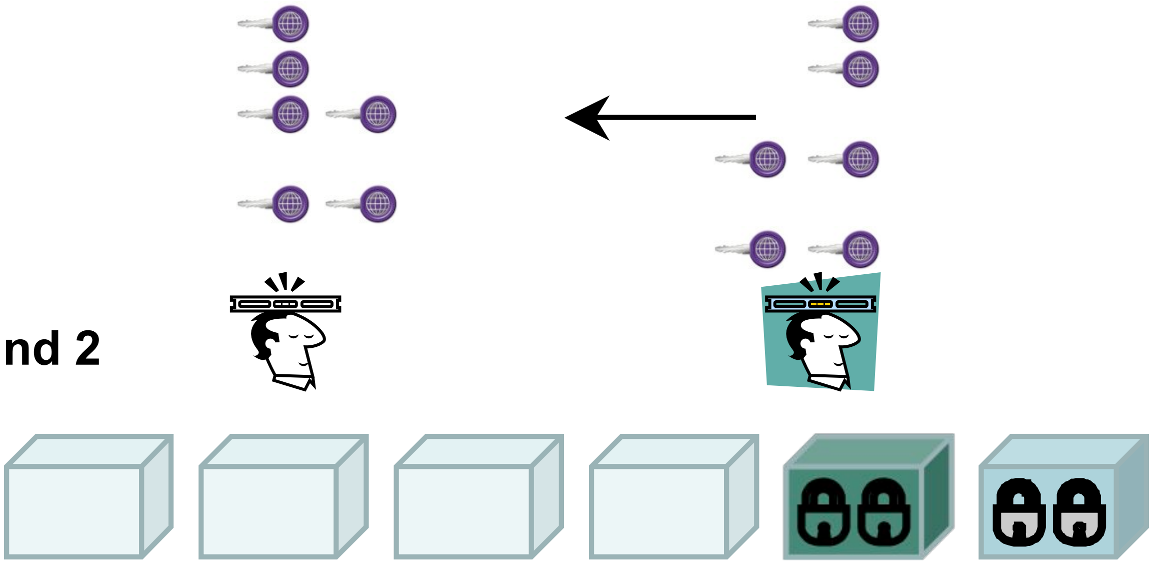

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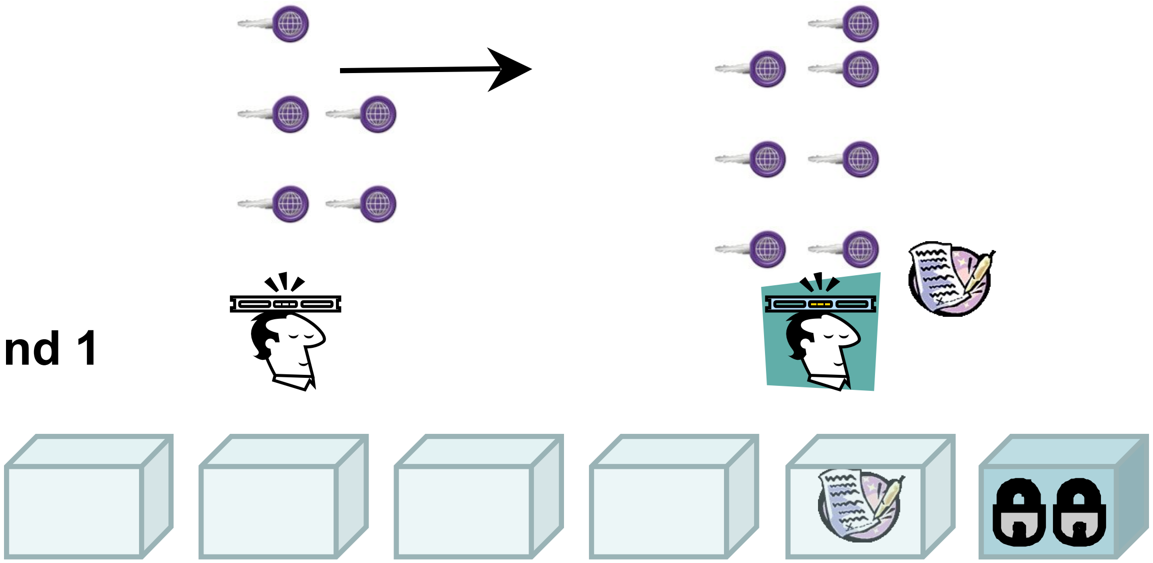
right party
gets contract

phase 3, round 1



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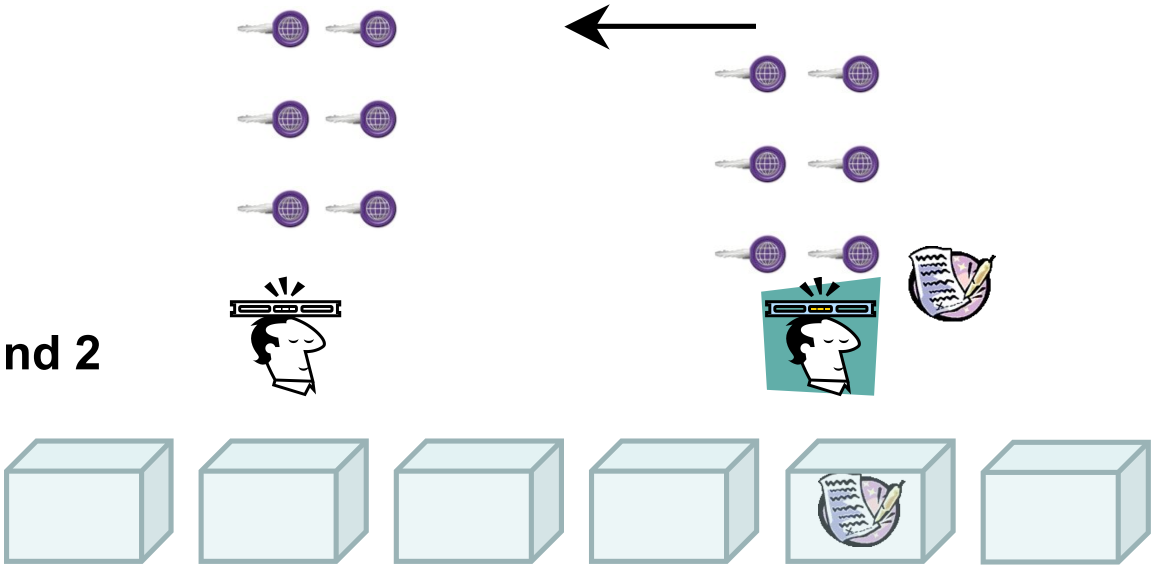


protocol completed

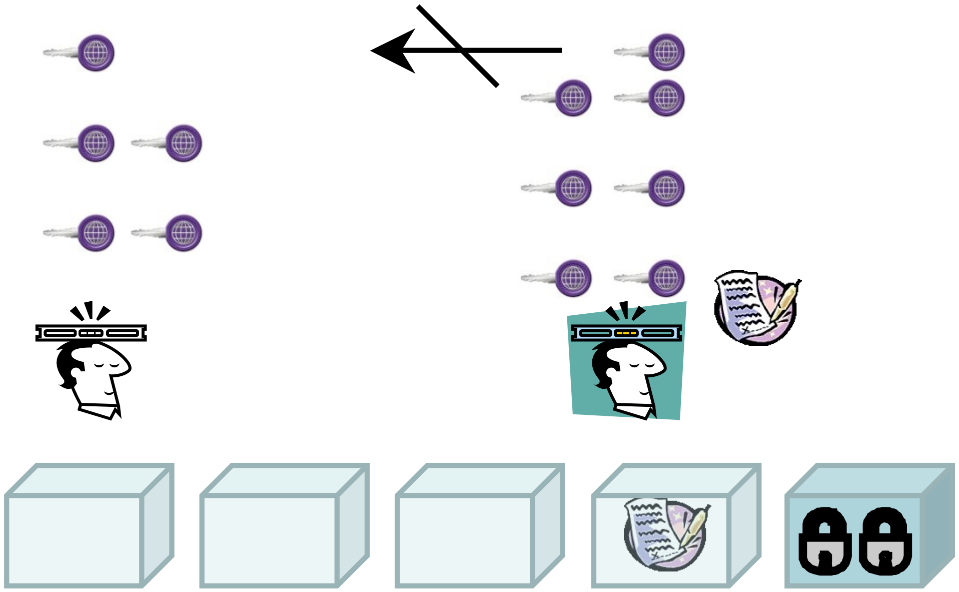
phase 3, round 2

More winners – first attempt

- if there are more winners, all contracts are handed out right after each other (in the same phase)
- what will really happen?
  - the first (e.g., right) winner prevents the information in the second (e.g., left party's) contract from leaking by withholding its shares

right party **does not** send its share to left party

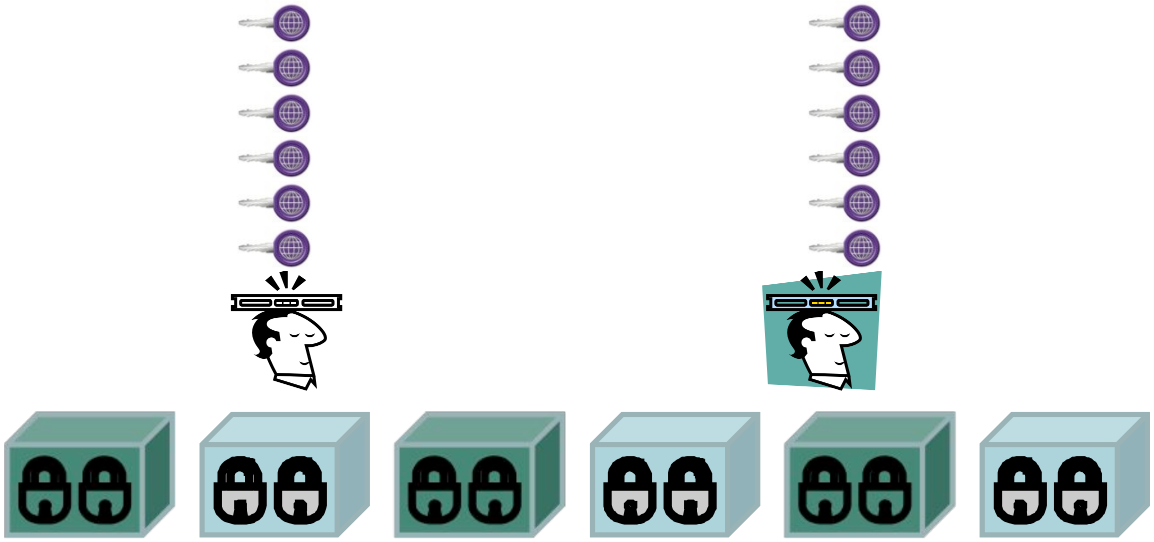
phase 3, **round 3**



More winners – fix

decouple the *information* and the *contract*

- first leak the **information** of the contracts, “P_i won item G at price is p”
- then provide the contracts, the signature on this info, in the **next phase**



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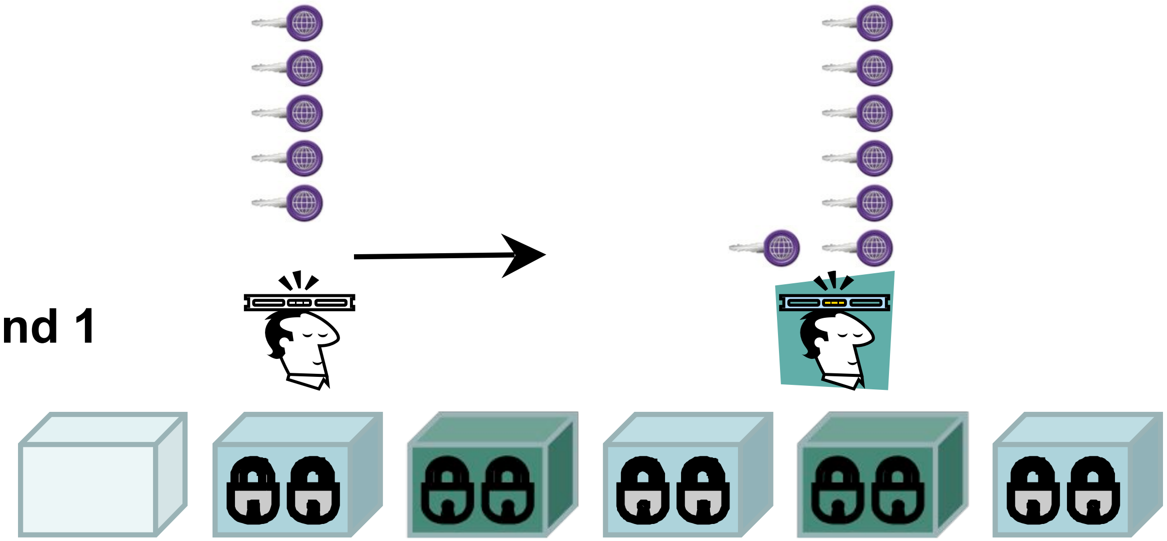

phase 1, round 1

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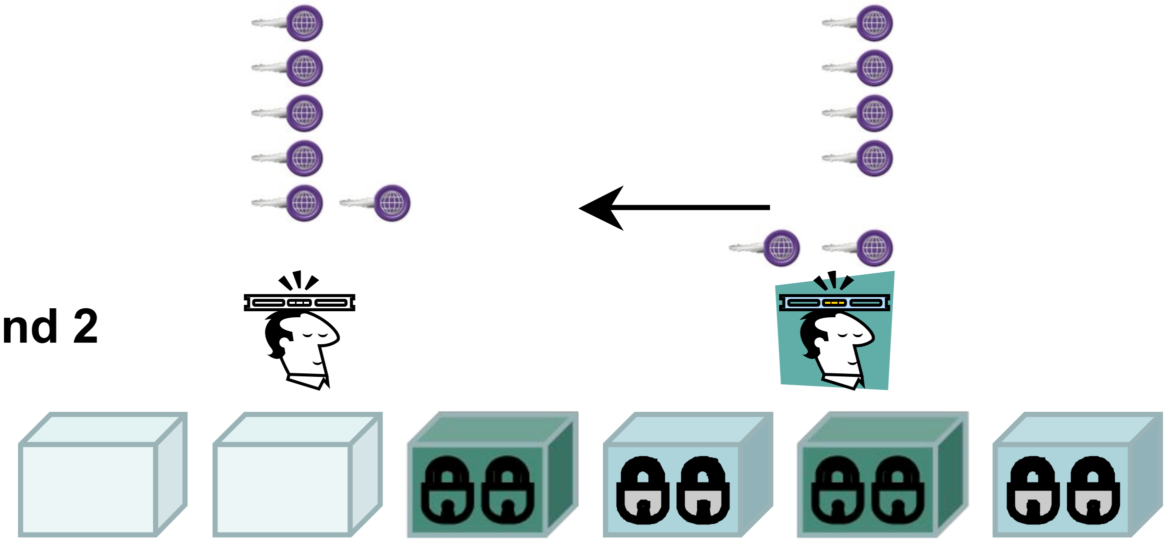

phase 1, round 2

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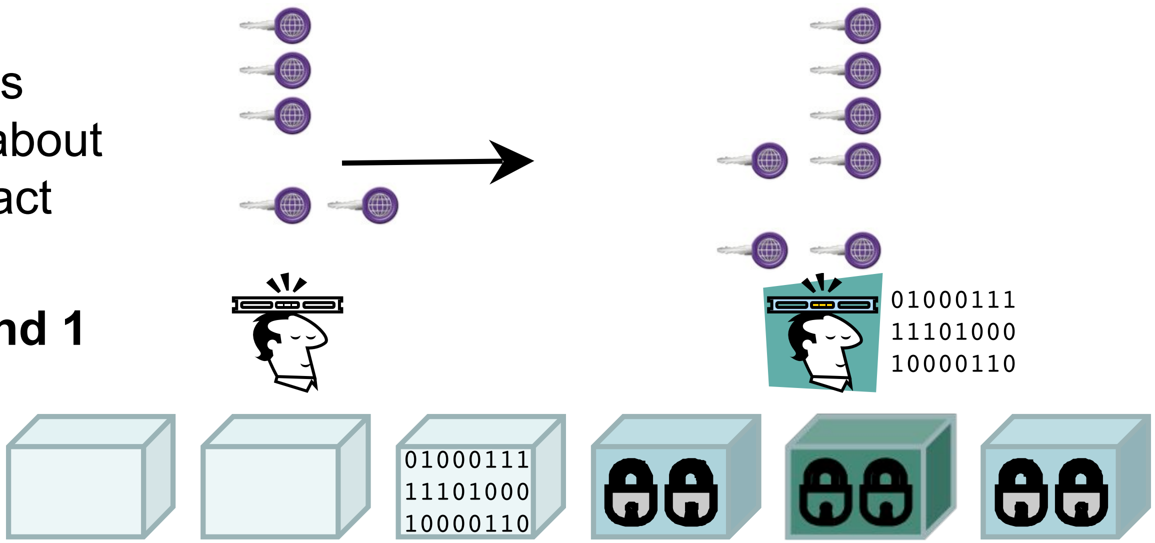
right party gets **information** about winning contract

phase 2, **round 1**



More winners – fix

decouple the *information* and the *contract*

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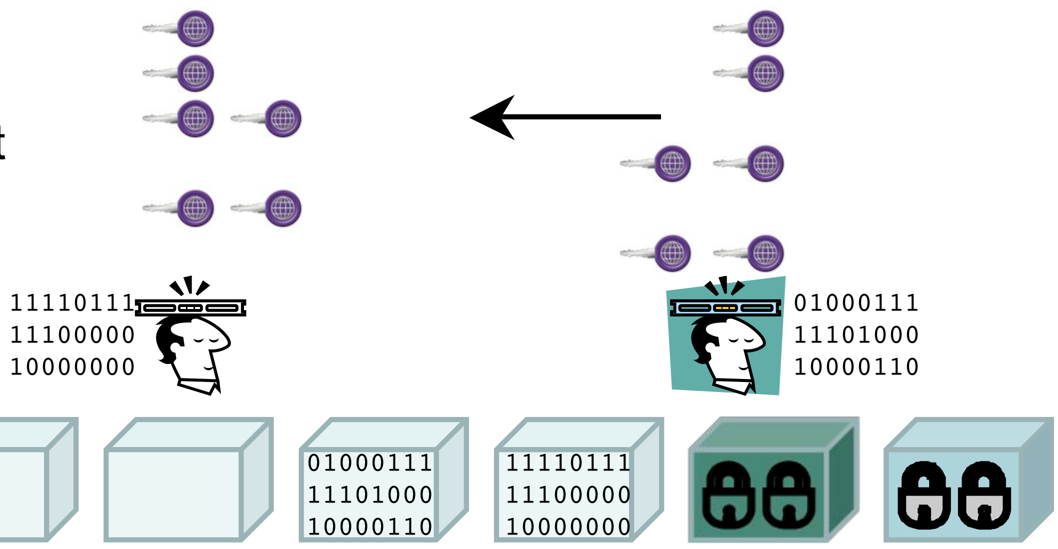
left party gets **information** about winning contract

phase 2, **round 2**



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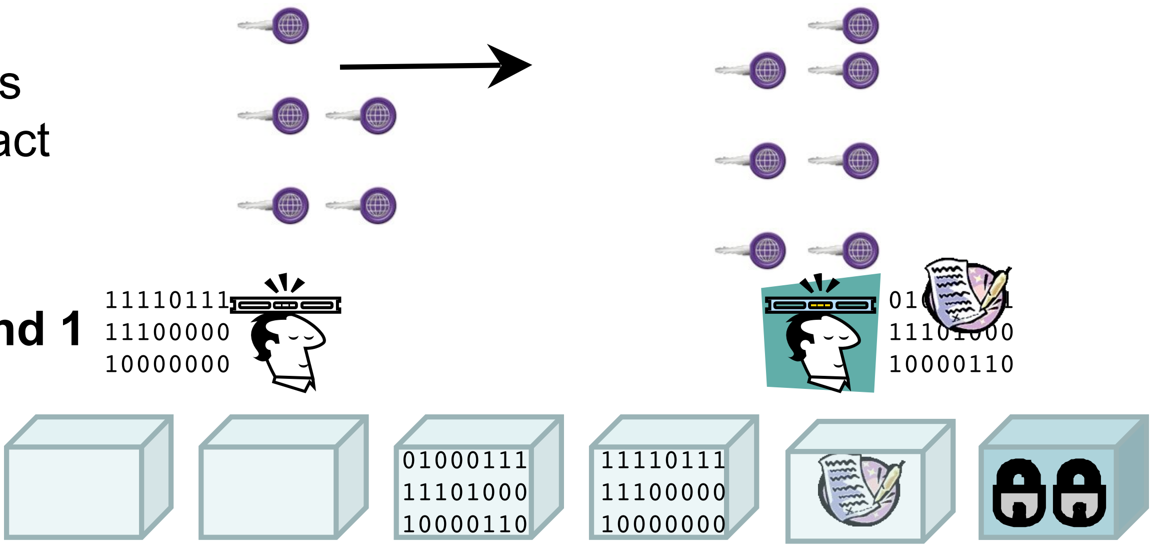
right party gets
winning contract

phase 3, round 1



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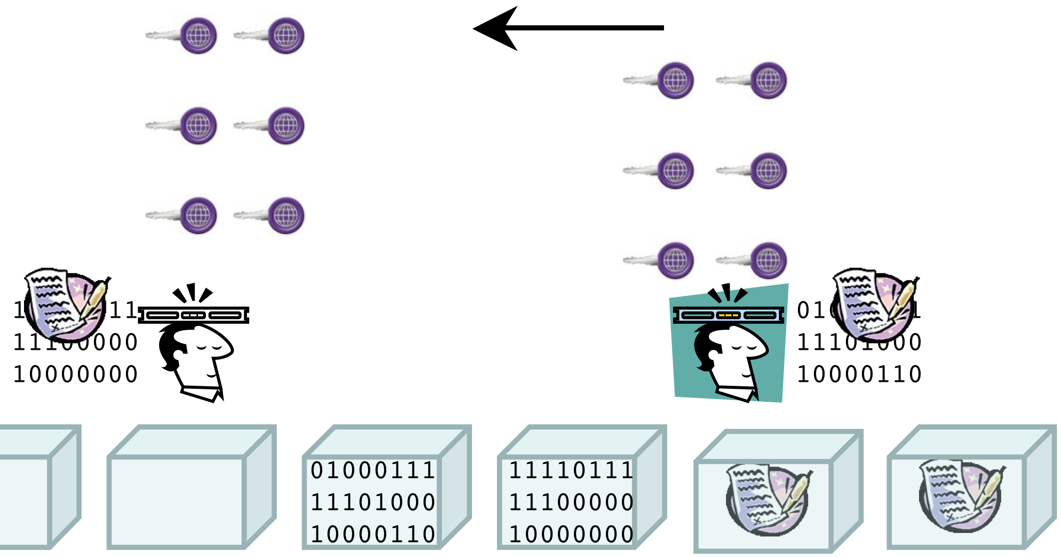
phase 3, **round 2**



Conclusions

- privacy-enhancing auctions
  - first example on practical rational MPC in Internet-like settings
  - inherent limitations (1st Vs. 2nd price auction separation)
  - generic framework for Nash-implementation
- future directions
  - too young area; we are far from having a good understanding
  - towards privacy-aware computational/distributed mechanism design

Thank you