



# Collusion-Free Multiparty Computation in the Mediated Model

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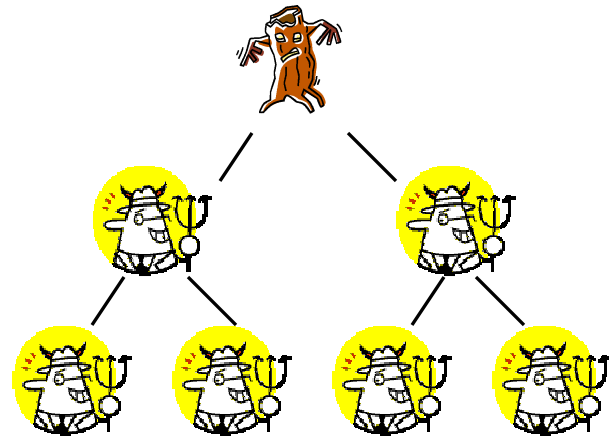
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# Crime



# Organized Crime



Standard Crypto Model:  
Single adversary coordinating all corrupted parties.



# Why Standard Crypto Model Assumes Organized Crime

Intuition: Protect against strongest adversary

On the other hand, unclear how to avoid it in standard communication models.



# How to Coordinate

1. Security requires randomness
2. Randomness enables side channels
3. Side channels imply collusion

ERGO, organized crime.

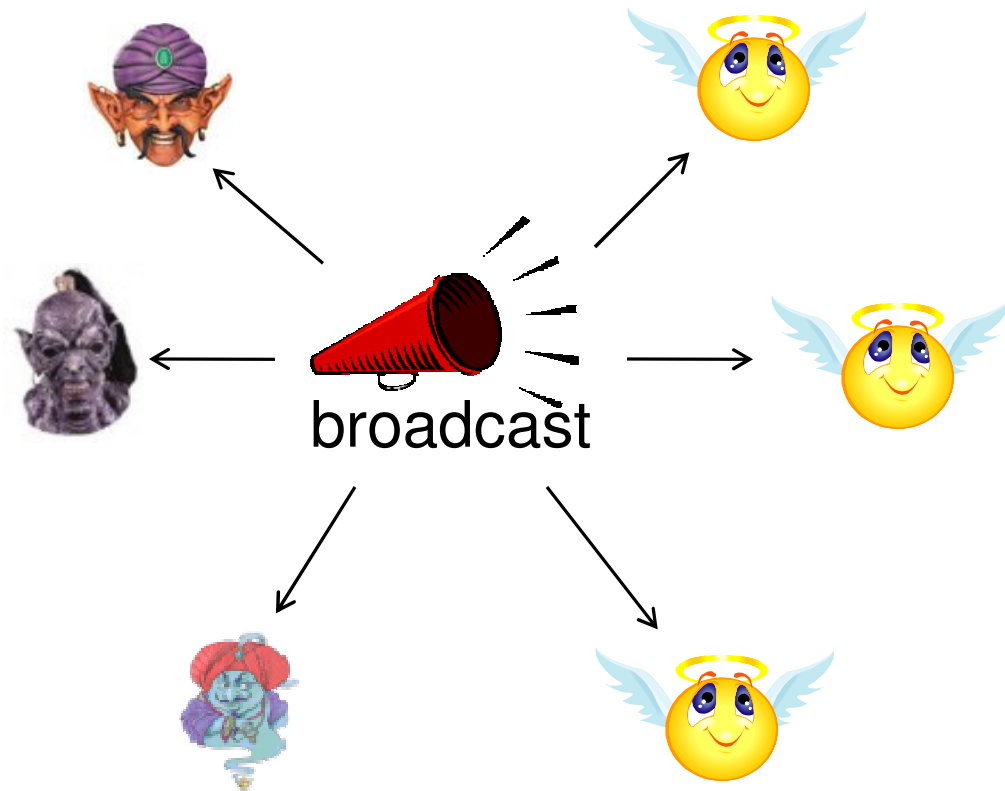


# Collusion-free protocol

“The protocol does not **introduce** any opportunities for parties to collude.”

# Solution Concept

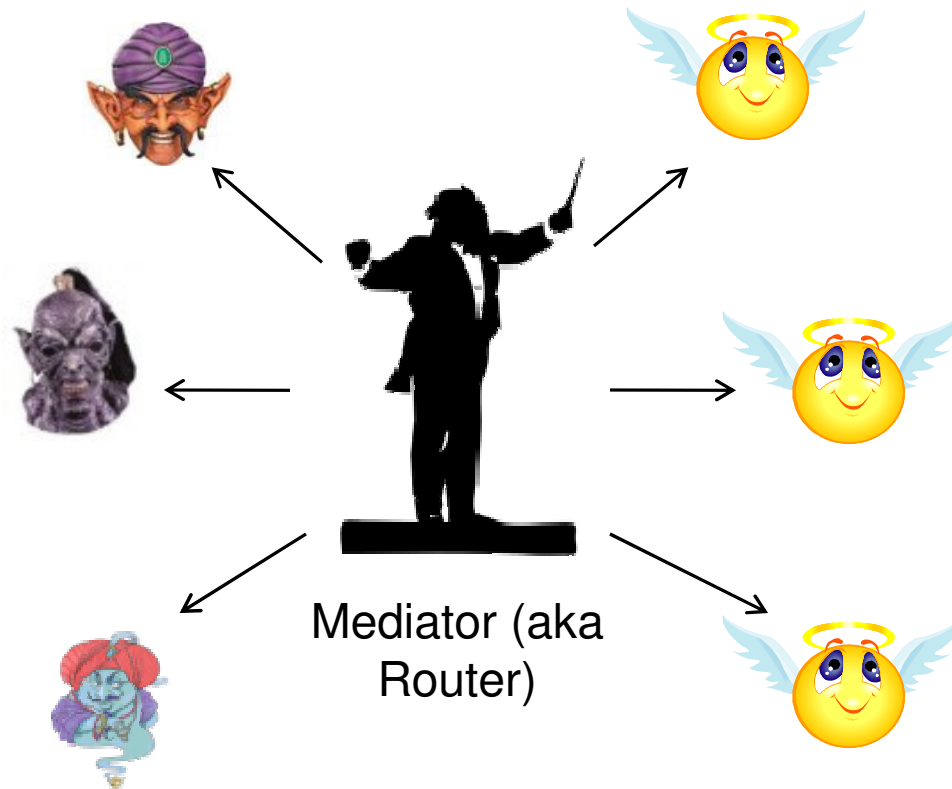
Standard  
Model



Problem: “Randomness enables side channels”

Solution: **Re-Randomize**

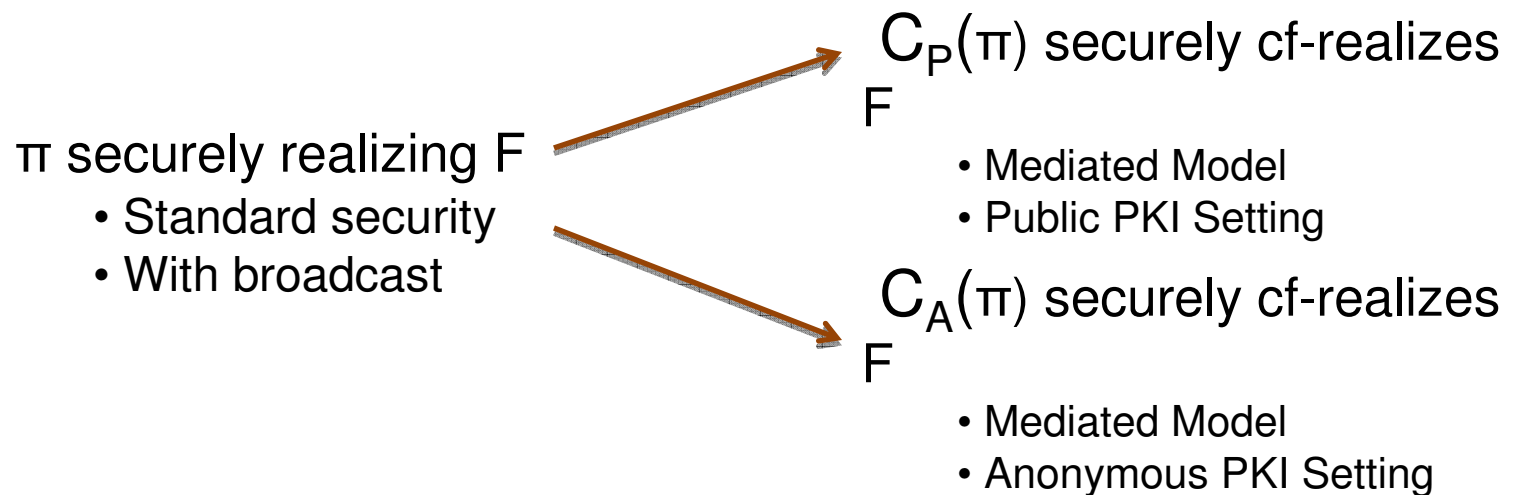
# Mediated Model



**But not a TRUSTED PARTY**

# Main Results

1. Improved definition of Collusion-free
2. Give protocol compilers  $C_P$  and  $C_A$ :



**Result:** Collusion-free computation for any  $n$ -party functionality.

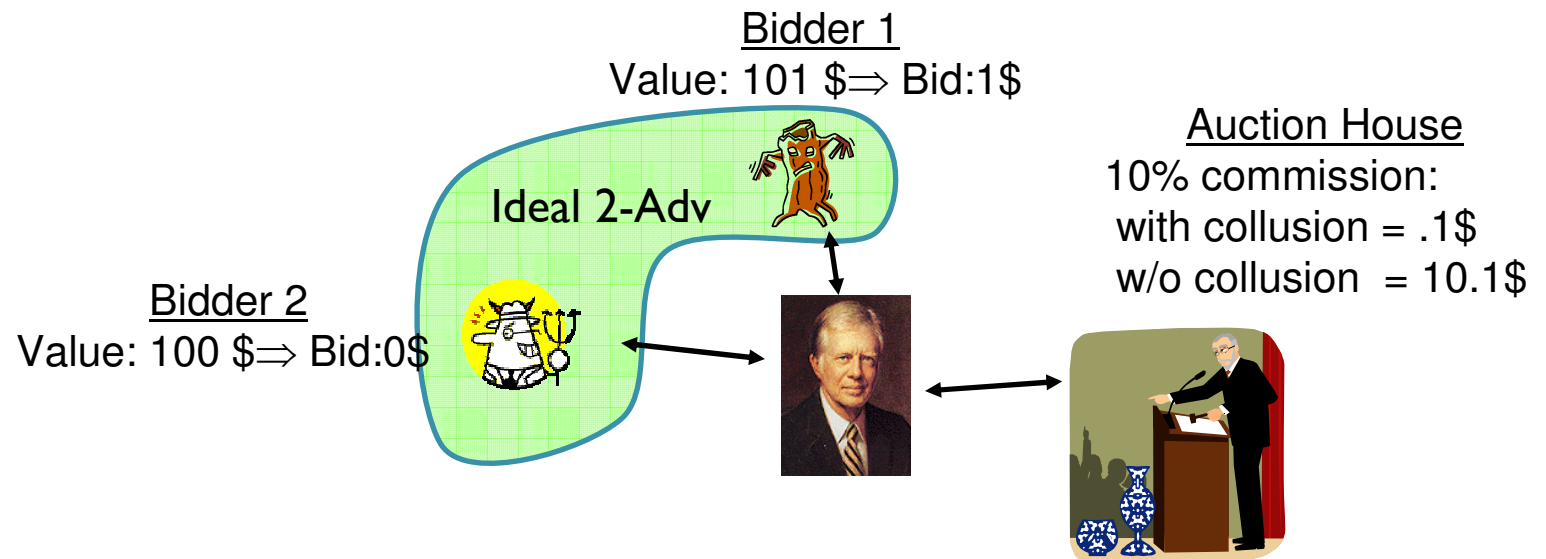


# Motivation: Auction

Parties: n bidders, auction house

Collusion: Bidders decide amongst themselves who is willing to bid the most. Winner bids 1\$, rest bid 0\$.

Result: auction house's commission diminished





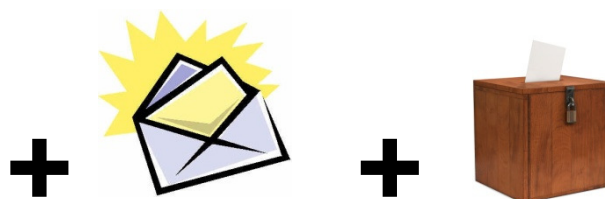
# Motivation: Applications to Game Theory

- Implementing Nash Equilibria
  - Weak Stability: **Unilateral** deviations are irrational.
- Playing Bayesian Games
  - i.e. games with secret input
    - e.g. valuation of an item by a bidder in an auction
- Playing games of Imperfect Information
  - i.e. games in which players do not have full knowledge of the current global state.
    - e.g. hidden cards in opponents hand in poker
- More generally: Playing Mediated Games
  - i.e. games with **isolated** players talking only to a trusted mediator

# Previous Work

Main Goal: Enforce isolation. Avoid steganography.

- Steg.-free Signatures: [S83,D96,S96,BDI+96,BS05]
- Collusion Free MPC: Verifiable Determinism
  - Initiated by Lepinski, Micali, shelat at STOC'05
  - Other works [LMS05b, ILM05, ILM08]
  - Make use of strong physical assumptions



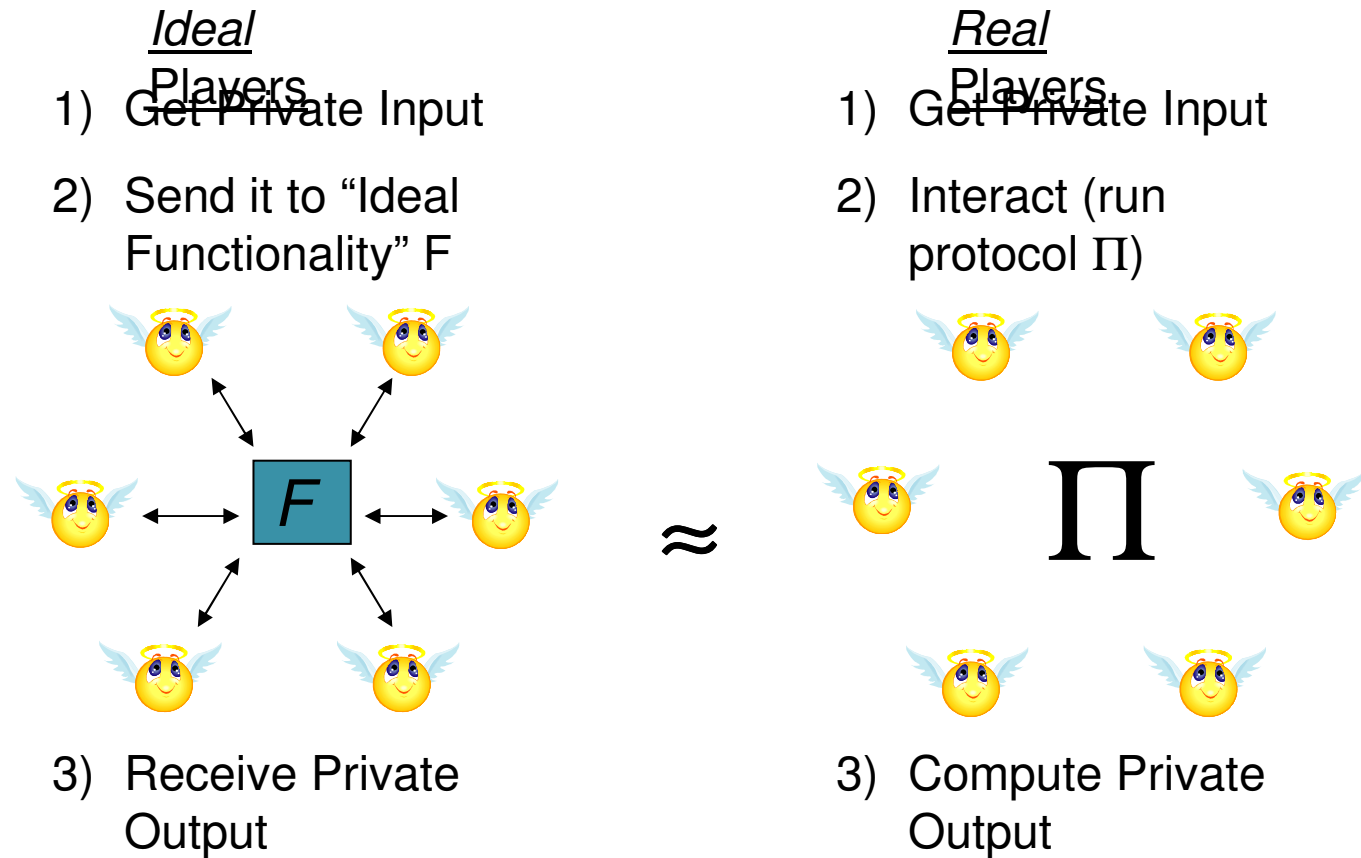
- New Approach: Rerandomization [ASV08]
  - In the Mediated Model
    - Network model still strong assumption
    - But allows for computation with Turing Machines
  - Commitments and Zero Knowledge

# Definitions



# Multiparty Computation

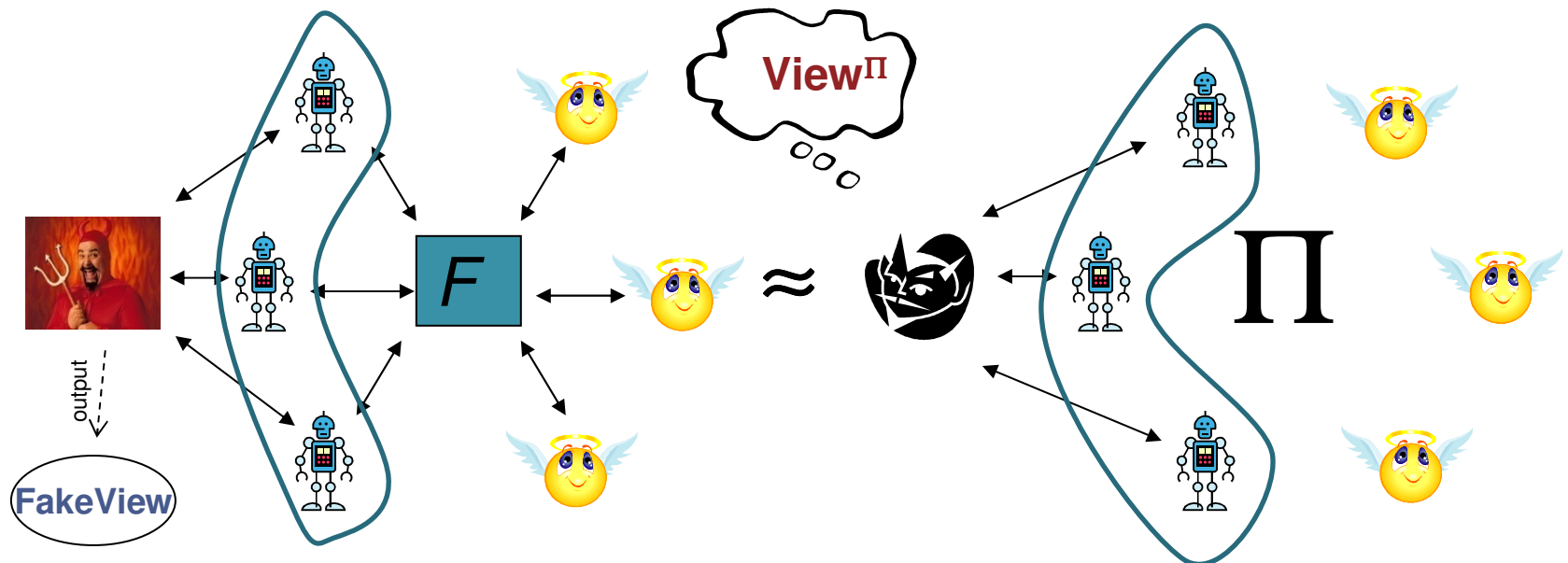
“Protocol  $\Pi$  realizes functionality  $F$ ”



$F$  can be probabilistic, and/or reactive with a secret persistent internal state.

# (Traditional) Monolithic Adversary

- Model Real: All corrupt real parties controlled by a single malicious adversary.
- Model Ideal: All corrupt ideal parties controlled by a single simulator.

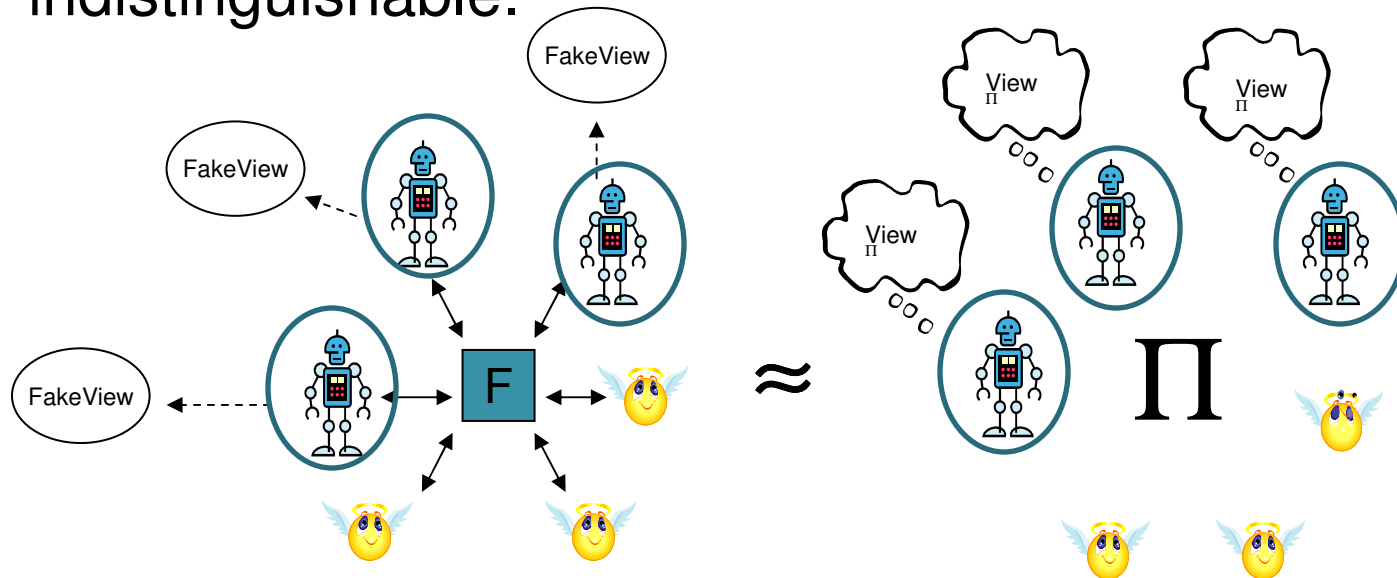


- $\Pi$  is *secure* (power preservation) if for any malicious adversary there exists a simulator that outputs a (fake) view such that:

$$\{\text{FakeView}, \text{Ideal-I/O}\} \approx \{\text{View}^{\Pi}, \text{Real-I/O}\}$$

# Modeling Collusion Free MPC

- Idea: **Corrupt players act independently.** Each has its own simulator. Joint “fake views” still remain indistinguishable.

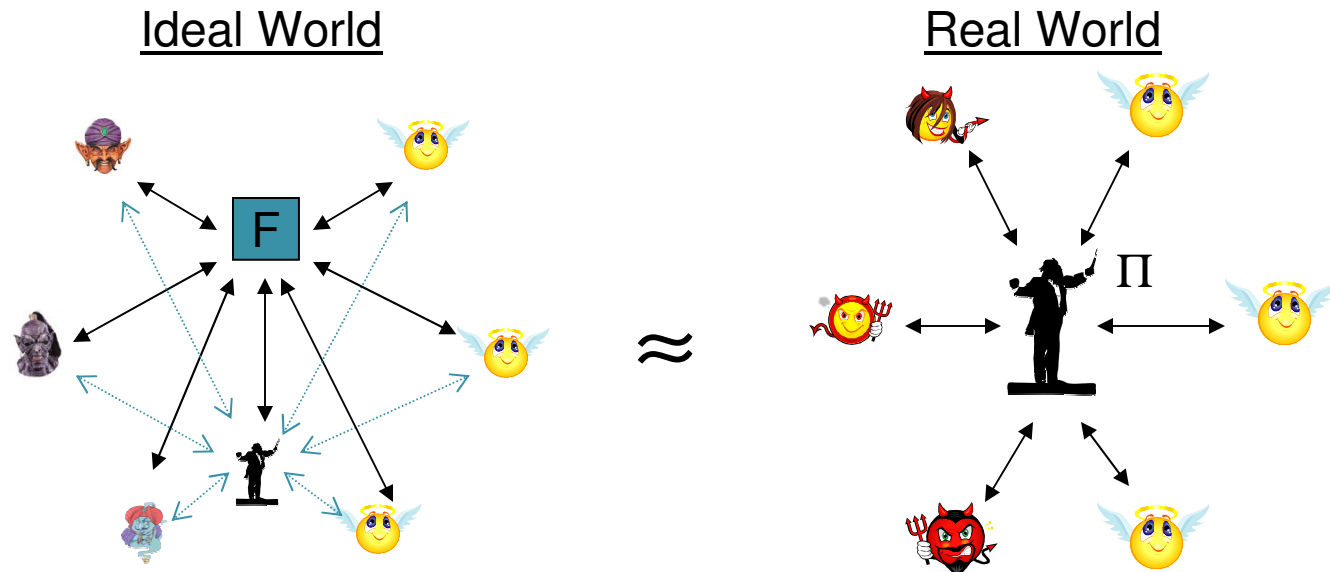


$$\{ \{\text{FakeView}\}, \text{Ideal-I/O} \} \approx \{ \{\text{View}^{\Pi}\}, \text{Real-I/O} \}$$

Anything they can compute together with  $\Pi$  they can also compute with  $F$ .

# The Mediated Model

- New Communication Model
  - Communication channel modeled as turing machine (called *mediator*)
  - The mediator can also have input to F



: Uncorruptable (ideal) functionality



: Honest parties do not use blue communication lines (corrupted ones can)



: Mediator honest  $\Rightarrow$  ideal players separate

Mediator corrupt  $\Rightarrow$  standard security (monolithic adversary)





# Establishing Identities

We explore two settings:

- **Anonymous** Setting: Identities setup after inputs determined
  - Achieves stronger notion of collusion-freeness.
  - Requires more trust in mediator
  - Implementation:
    1. Parties generate key pairs and send their public key to mediator.
    2. For each player the Mediator sends a vector of fresh independent commitment to all public keys.
- **Public PKI** Setting: PKI setup before inputs determined
  - Each player knows the identity (public keys) of all other payers involved in the execution.
  - More practical (realistic).
  - Implementation:
    1. Parties generate keys and send public keys to trusted setup TTP.
    2. TTP redistributes all public keys consistently.

Note: Neither setting requires honest key generation or proof



# Assumptions and Tools

- $\pi$  is n-party protocol
  - Securely computes F.
  - Plain model with broadcast channel
    - W.l.o.g. assume all messages sent via broadcast.
- Primitives
  - Signatures.
  - Perfectly binding Commitments.
- 2-party (bounded) concurrently self-composable protocols.
  - SFE.
  - ZK protocol.

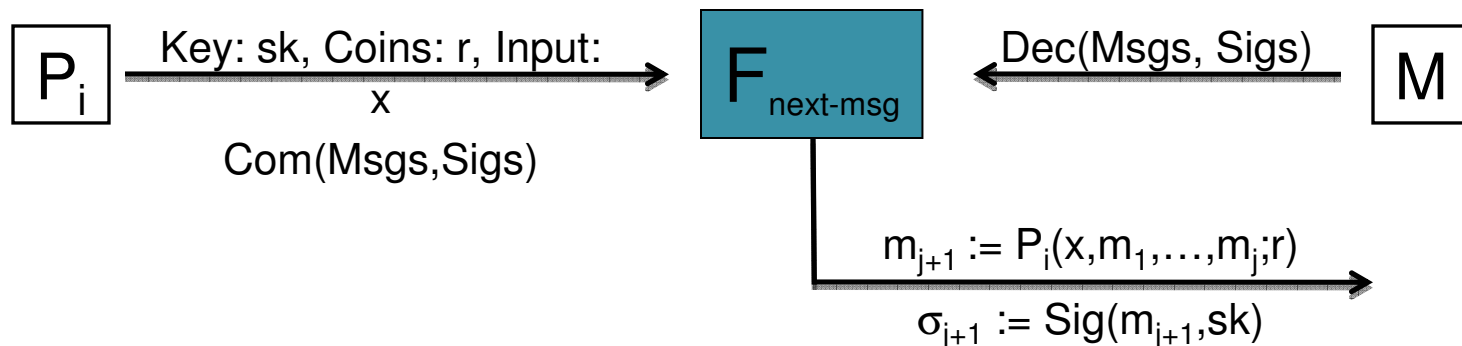
# High Level Idea

- Jointly emulate an execution of  $\pi$ .
  - Mediator maintains list of  $\pi$ -messages received by each player.
  - Players maintain only their random tapes, signing keys, and inputs to  $\pi$ .
  - Emulation proceeds as a sequence of two party computations between a player and the mediator.

- Emulating round  $j+1$  of  $\pi$ .

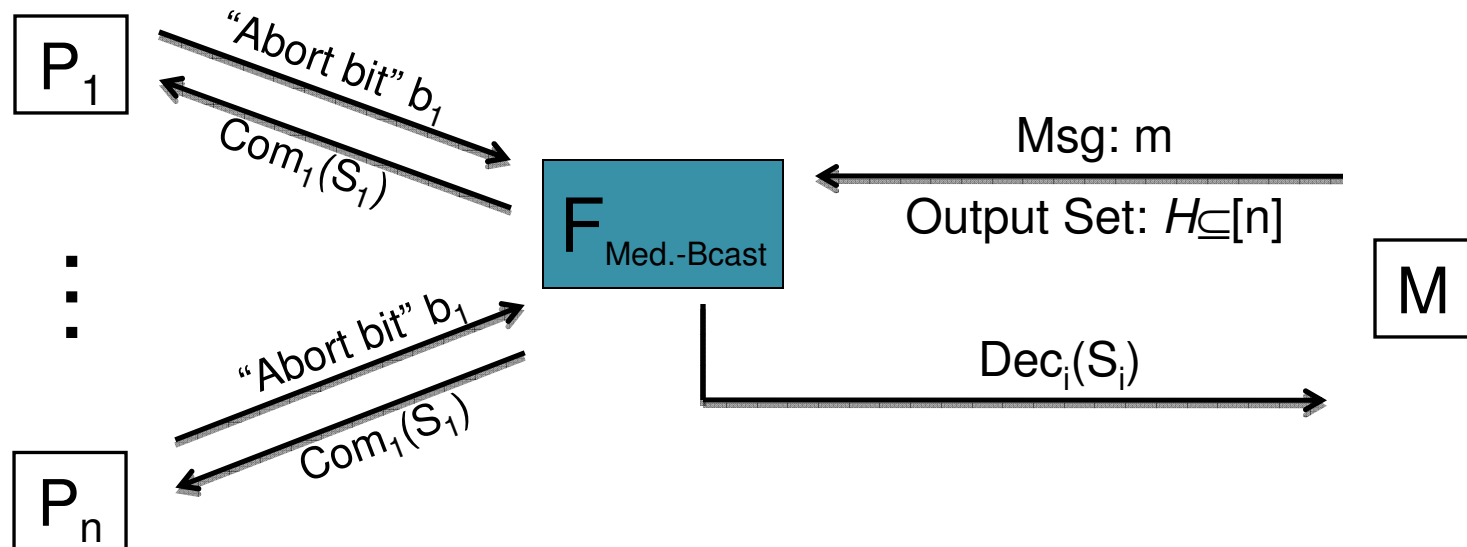
1. Compute message  $m_{j+1}$  of  $\pi$ :

Msgs :=  $(m_1, \dots, m_j)$   
Sigs :=  $(\sigma_1, \dots, \sigma_j)$



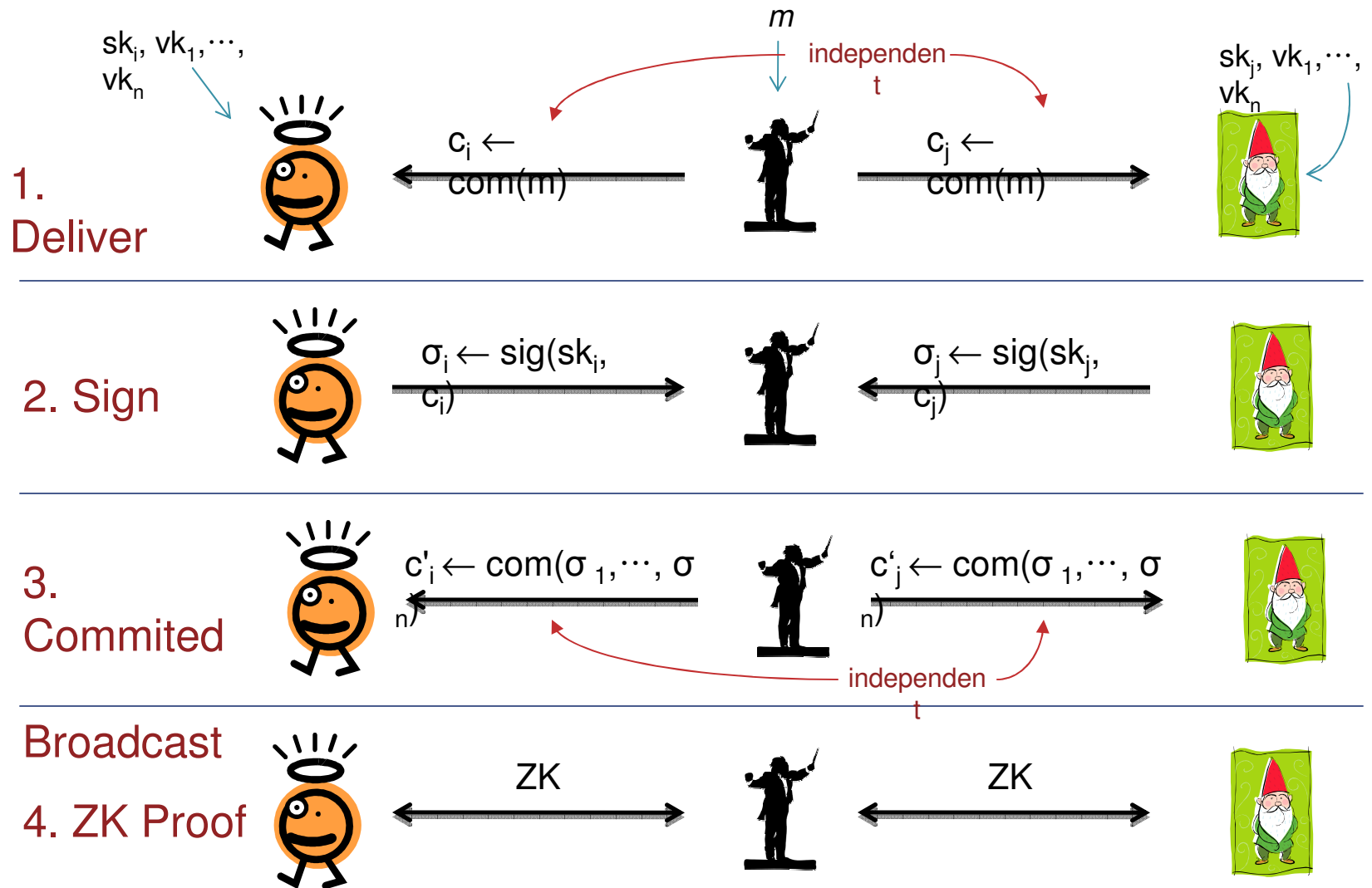
2. Emulate broadcast of  $m'_{i+1} := (m_{i+1}, \sigma_{i+1})$ .

# Mediated Broadcast Functionality



1. If at least one  $P_i$  set  $b_i = 1$  then all  $S_i := \perp$
2. If  $i \notin H$  then  $S_i := \perp$
3. Else  $S_i := m$

# Mediated Broadcast





# Side-channels

- SFE input privacy, Com hiding and ZK properties imply  $\pi$ -messages (nor sigs) ever seen by players.
  - ⇒ Players views remain independent of each other until output is delivered.
- Using aborts to communicate
  - [ASV08] allows  $\log(\# \text{ rounds})$  bits of communication via aborts.
  - This work: 1 bit at end of computation.
    - How: Mediator uses default messages for aborting party and emulation of  $\pi$  continues until output delivery.
    - Result: Round # of abort remains hidden. Only bit communicated is that an abort occurred at some point.



# Honest but Curious Mediator

- $\pi$  secure against passive (eaves dropping) adversary & 2-party SFE's input privacy  
⇒ Mediator learns nothing about I/O of players.
- Mediator removes side channels.  
⇒ Corrupt players can not communicate or coordinate.
- Result: Compiled protocol is a collusion-free secure realization of  $F$ .

# Corrupt Mediators

- Mediator controls scheduling
  - ⇒ Require bounded (by  $n$ ) concurrent security for 2-party SFEs and for ZK.
- $\pi$  secure against active adversary
  - ⇒  $F$  realized faithfully. (Correctness)
  - ⇒ Privacy of honest players maintained.
- Corrupt players can communicate via corrupt mediator.
  - ⇒ Security falls back to standard monolithic adversary security.





# Open Problems

- Efficient constructions (esp. for specific functionalities such as auctions).
- Alternative (yet more realistic) models where similar results are possible.
- Security & Collusion-Freeness under stronger composition.
- Anonymous settings with reduced trust in mediator for setup phase.