Improving the Security of Quantum Protocols via Commit&Open

Ivan Damgård (Aarhus University, DK)

Serge Fehr (CWI, NL)

Carolin Lunemann (Aarhus University, DK)

Louis Salvail (Université de Montréal, CA)

Christian Schaffner (CWI, NL)

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Main Results

Compiler

π

BB84-type protocol

Benign security
against Bob

Unconditional security
against Alice

Commit&Open

(with special properties)

$C^{\alpha}(\pi)$

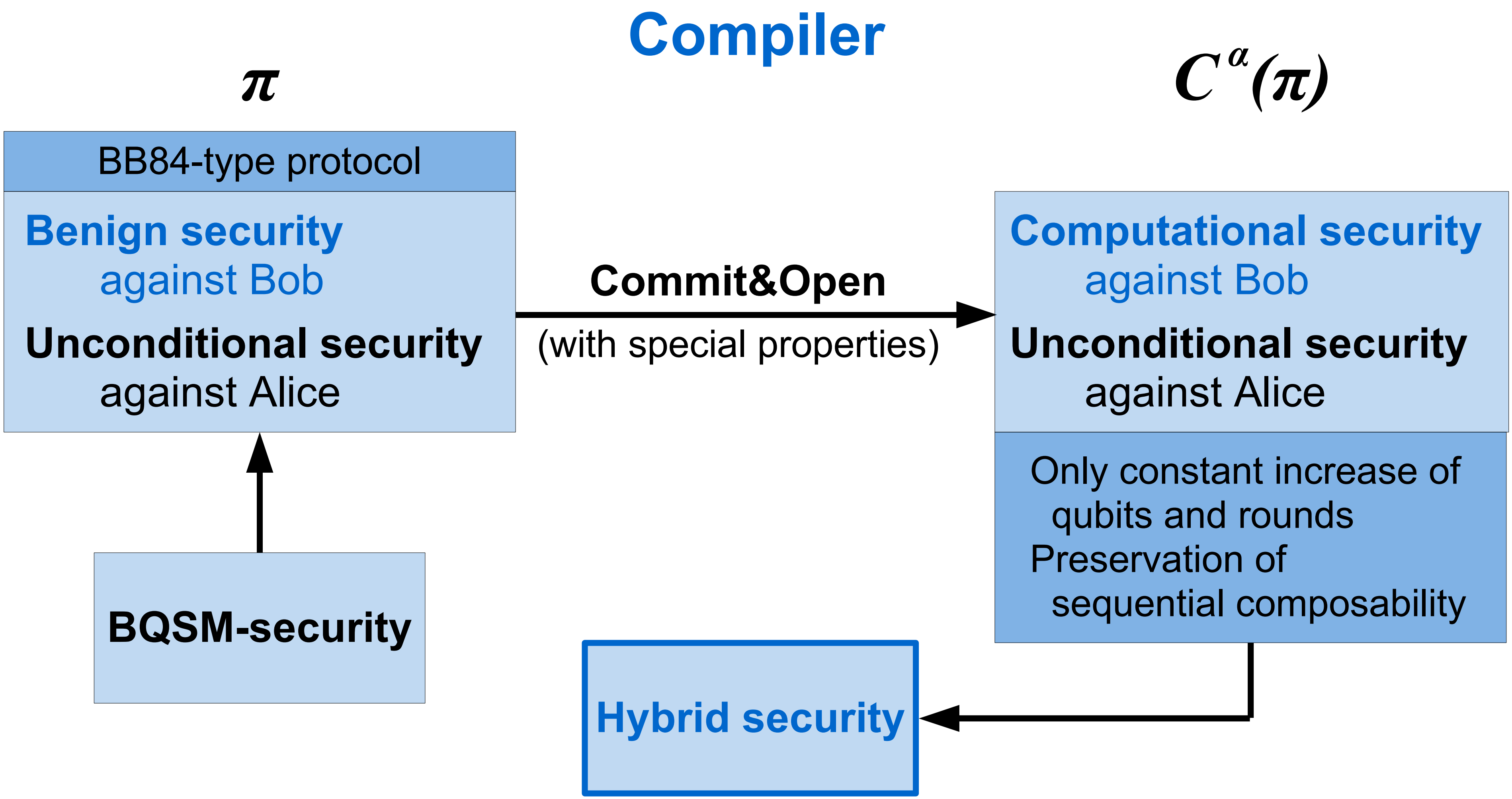
Computational security
against Bob

Unconditional security
against Alice

Only constant increase of qubits and rounds

Preservation of sequential composability

Main Results



Intuition

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Preservation of
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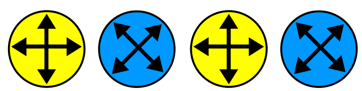
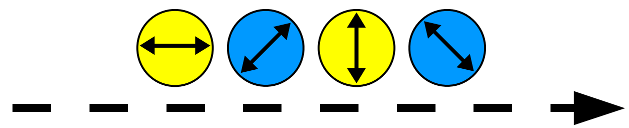
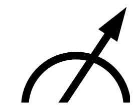
BB84-type protocols

preparation (quantum)

$x \in_R \{0,1\}^n$ 0 0 1 1

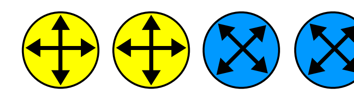
$\theta \in_R \{+,\times\}^n$ 





$\hat{\theta} \in_R \{+,\times\}^n$

$\hat{\theta}$

$\hat{\theta}$ 

$\hat{x}$ 0 R R 1

post-processing (classical)

arbitrary classical messages

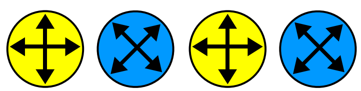
and local computations

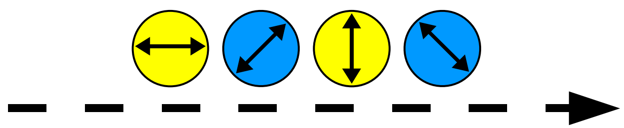
Notation: C. Schaffner

BB84-type protocols

preparation (quantum)

$x \in_R \{0,1\}^n$ 0 0 1 1

$\theta \in_R \{+,\times\}^n$ 



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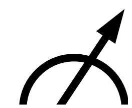
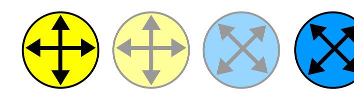
 $\hat{\theta}$

$\hat{\theta}$ 

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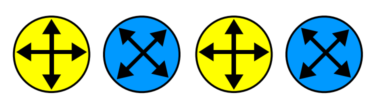
post-processing (classical)



arbitrary classical messages

and local computations

Security

- Bob measures in **random bases**:
  - He knows x_i whenever $\theta_i = \hat{\theta}_i$.
  - For $\theta_i \neq \hat{\theta}_i$ his uncertainty is high (privacy amplification).
- We must ensure that Bob measures most of his qubits **before** Alice announces further information (e.g. her bases).

BB84-type protocols

preparation (quantum)

$x \in_R \{0,1\}^n$ 0 0 1 1

$\theta \in_R \{+,\times\}^n$

$\hat{\theta}$

post-processing (classical)

θ

arbitrary classical messages

and local computations

θ

$\hat{x}$ 0 0 1 1

Security

- Bob measures in **random bases**:
  - He knows x_i whenever $\theta_i = \hat{\theta}_i$.
  - For $\theta_i \neq \hat{\theta}_i$ his uncertainty is high (privacy amplification).
- We must ensure that Bob measures most of his qubits **before** Alice announces further information (e.g. her bases).
- Security against **benign** Bob ('almost' honest in preparation phase).
- Unconditional security against dishonest Alice.

Improvement

Compiler

π

BB84-type protocol
Benign security against Bob **Unconditional security** against Alice

Commit&Open

(with special properties)

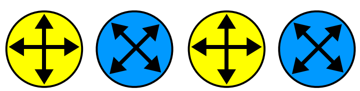
$C^{\alpha}(\pi)$

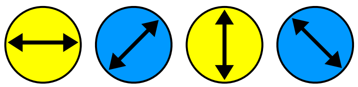
Computational security against Bob **Unconditional security** against Alice
Only constant increase of qubits and rounds Preservation of sequential composability

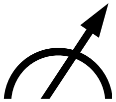
Security

preparation (quantum)

$x \in_R \{0,1\}^m$ 0 0 1 1

$\theta \in_R \{+,\times\}^m$ 



 $\hat{\theta}$ $\hat{x}$

verification (classical)

$c_i = commit[\hat{x}_i,\, \hat{\theta}_i,\, r_i]$ ←

T →

open c_i $\forall i \in T$ ←

$T \subset \{1,\ldots,m\},\ |T| = \alpha m$

for all $i \in T:\ x_i = \hat{x}_i$

whenever $\theta_i = \hat{\theta}_i$

$n = (1-\alpha)m$

post-processing (classical)

arbitrary classical messages ↔ and local computations

Commit&Open

- Idea already in 1-2 QOT [BBCS91].
- **Intuition**: If Bob passes the measurement test, he must have measured most of his qubits (also in the remaining subset).
- Partial results for QOT,
  e.g. [Yao95, Mayers96, CDMS04].
- **Formal characterization** of what Commit&Open achieves in a quantum world ⇒ **Benignity**

Commit&Open

⇒ Computational Security

- Commitment can only be **computationally binding**.
- **Standard reduction** from computational security of protocol to computational binding property of commitment would require **rewinding**.
- Quantum rewinding is only possible in limited settings [Watrous06].

Benignity

- Bob treats the qubits **'almost' honestly** in preparation phase.
- Two conditions are satisfied after preparation phase:

  where $x|_I \stackrel{?}{=} (x_i)_{i \in I}$; $d_H(\theta, \hat{\theta}) := \left| \{ i : \theta_i \neq \hat{\theta}_i \} \right|$; $\beta \geq 0$

  - *Bob's* ***quantum storage*** *is small:*

    $$\boxed{H_0(\rho_B) \leq \beta n}$$

  - *There exists a* $\hat{\theta}$ *, such that the* ***uncertainty*** *about* x_i *is (essentially) 1 whenever* $\theta_i \neq \hat{\theta}_i$ *:*

    $$\boxed{H_\infty\left(X|_I \,\middle|\, X|_{\bar{I}} = x|_{\bar{I}}\right) \geq d_H\left(\theta|_I, \hat{\theta}|_I\right) - \beta n}$$

    for any $I \subseteq \{1, \ldots, n\}$*; for any fixed* θ*,* $\hat{\theta}$*,* $\hat{x}$*; for any* $x|_{\bar{I}}$

Computational Security

- Simulation-based proof in the **common-reference-string model**.
- Commitment scheme with special properties and secure against quantum adversaries (e.g. [Regev05]).
- **Keyed dual-mode commitment scheme**
  - Unconditionally binding key **pkB**.
  - Unconditionally hiding key **pkH**.
  - **Indistinguishability of keys** (also for quantum algorithms).

Indistinguishability

$$
\begin{aligned}
& out[C^{\alpha}(\pi)]_{A,B'} \\
= \ & out[C^{\alpha}{}_{pkH}(\pi)]_{A,B'} \\
\approx_{\mathrm{q}} \ & out[C^{\alpha}{}_{pkB}(\pi)]_{A,B'} \\
= \ & out[\pi]_{A_o,B'_o}
\end{aligned}
$$

Indistinguishability

$$out[C^{\alpha}(\pi)]_{A,B'}$$

$$\approx_q$$

$$out[\pi]_{A_o,B'_o}$$

General Compiler

Main Theorem:

If the original protocol π is ***unconditionally secure*** *against a* ***β-benign adversary****,*

then the compiled protocol $C^{\alpha}(\pi)$ is (quantum-) ***computationally secure*** *against* ***any adversary***

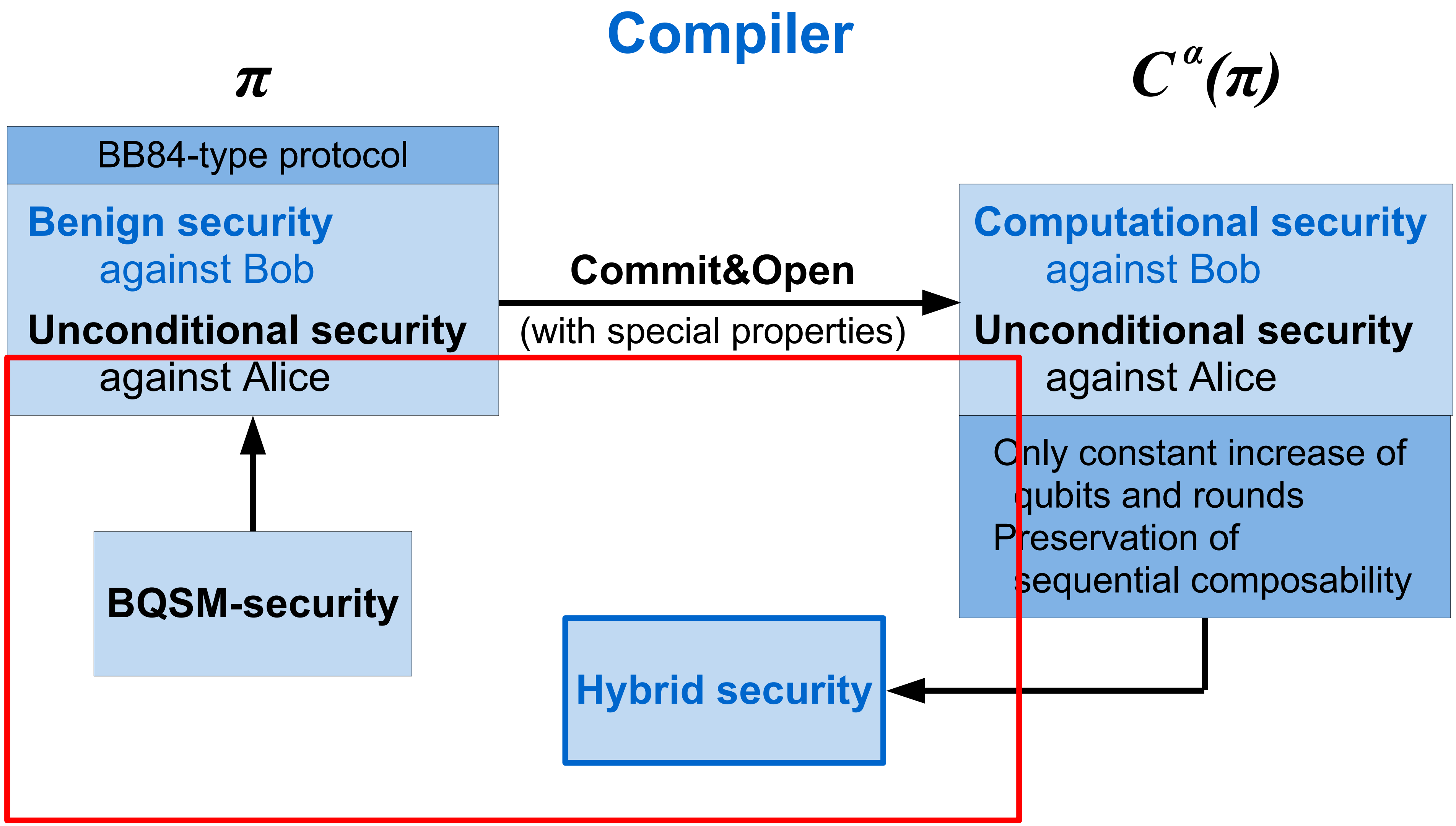
for const. $0 < \alpha < 1$, $0 < \beta$.

Unconditional security against Alice is maintained.

General Compiler

- Benignity is (relatively) **weak assumption**.
- Compilation only requires an increase of qubits and rounds by a **constant factor**.
- Compilation preserves **sequential composability** [FS09].

Hybrid Security



Hybrid Security

Bob needs large quantum memory **and** large quantum computing power.

Theorem:

*If π is **unconditionally secure** against **γ -BQSM Bob**,*

*then $C^{\alpha}(\pi)$ is **computationally secure** against*

a dishonest Bob

*and **unconditionally secure** against*

$\gamma(1-\alpha)$ -BQSM Bob

for const. $0 < \alpha < 1$, $0 < \gamma < 1$.

Unconditional security against Alice is maintained.

Summary

- **General compiler** to additionally achieve computational security.
- Characterization of commit&open in quantum settings (**benignity**).
- Protocols with **hybrid security**, e.g. QOT [BBCS91] and QID [DFSS07].
- Hybrid security against **man-in-the-middle attacks** for QID.
- Extensions for **noisy** quantum communication.

- *Full Version:* arXiv: 0902.3918
- *Quantum-Secure Coin-Flipping and Applications* (Damgård and Lunemann; to appear at Asiacrypt'09, arXiv: 0903.3118)
- *Sampling in a Quantum Population, and Applications* (Bouman and Fehr; arXiv: 0907.4246)

Thank You!