

# Position Based Cryptography

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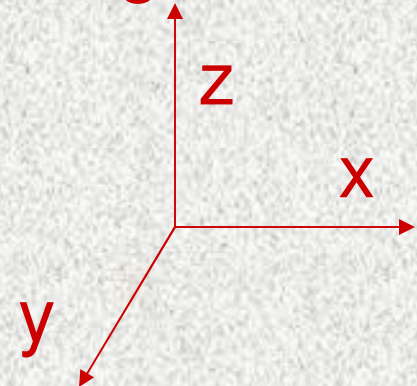
# What constitutes an identity?

- Your public key
- Your biometric
- Email ID
- How about where you are?

PK



abc@gmail.com





# Geographical Position as an Identity

sk

US Military Base  
in USA

$Enc_{sk}(m)$

sk

US Military Base  
in Iraq



# Geographical Position as an Identity

**US Military Base  
in USA**

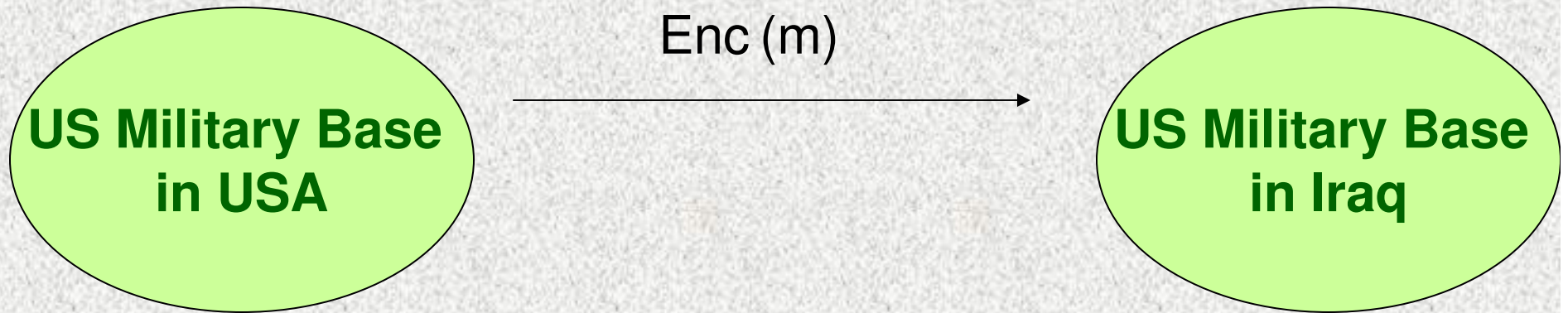
**US Military Base  
in Iraq**

- **We trust physical security**
- **Guarantee that those inside a particular geographical region are good**





# Geographical Position as an Identity



**Only someone at a particular geographical position can decrypt**

# Other Applications

- *Position-based Authentication*: guarantee that a message came from a person at a particular geographical position
- *Position-based access control*: allow access to resource only if user is at particular geographical position

Many more....



# Problem (informally)

- A set of verifiers present at various geographical positions in space
- A prover present at some geographical position  $P$

**GOAL:** Exchange a key with the prover if and only if prover is in fact at position  $P$

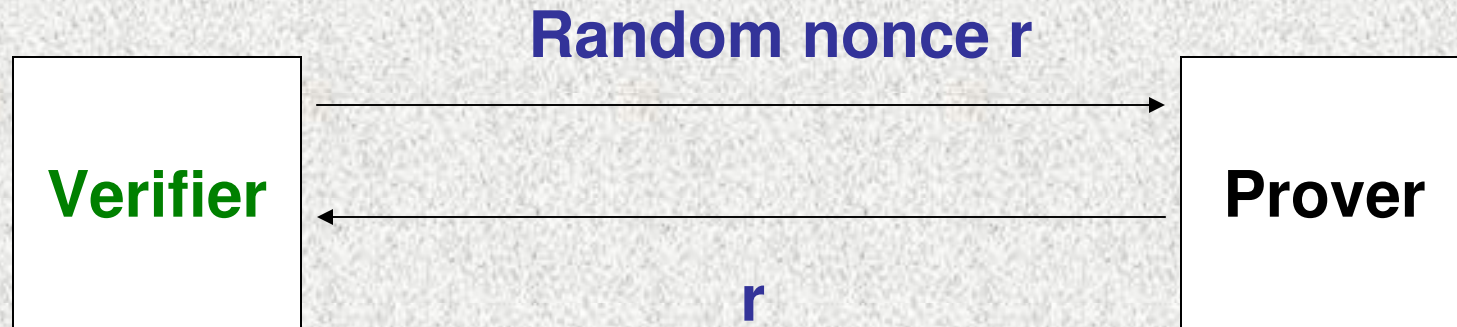
# Secure Positioning

- Set of verifiers wish to verify the position claim of a prover at position  $P$
- Run an interactive protocol with the prover at  $P$  to verify this
- Studied in wireless security  
[SSW03, B04, SP05, CH05, CCS06]



# Previous Techniques for Secure Positioning

All messages travel at speed of light  
Radio waves, GPS....

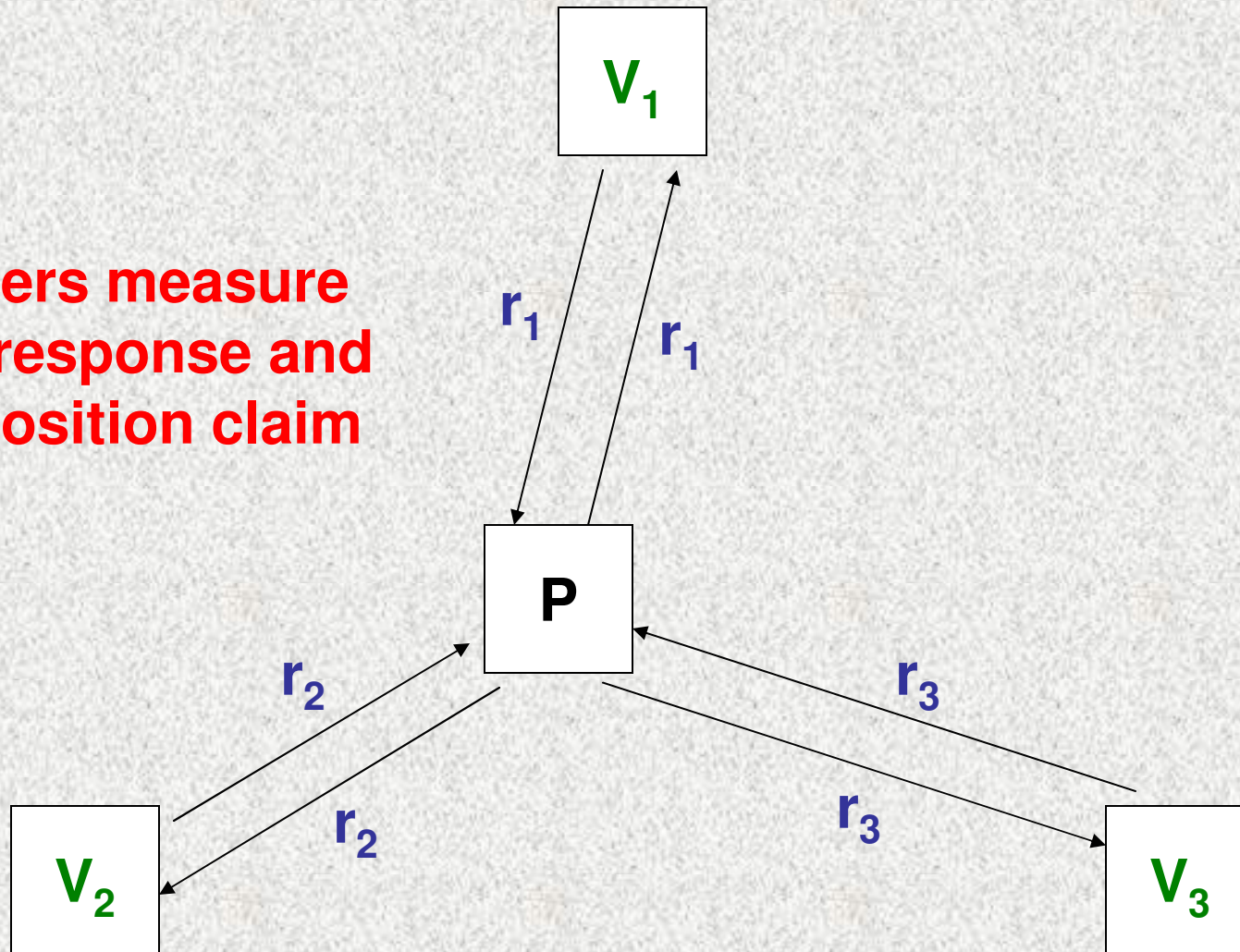


**Time of response**

**Prover cannot claim to be closer to the verifier than he actually is**

# Triangulation [CH05]

**3 Verifiers measure  
Time of response and  
verify position claim**

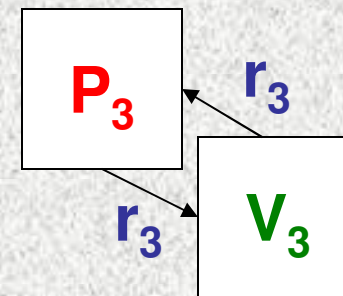
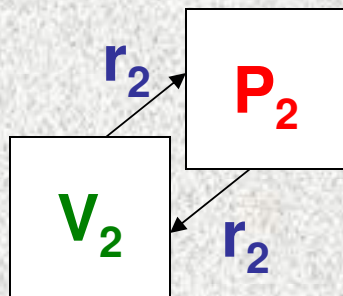
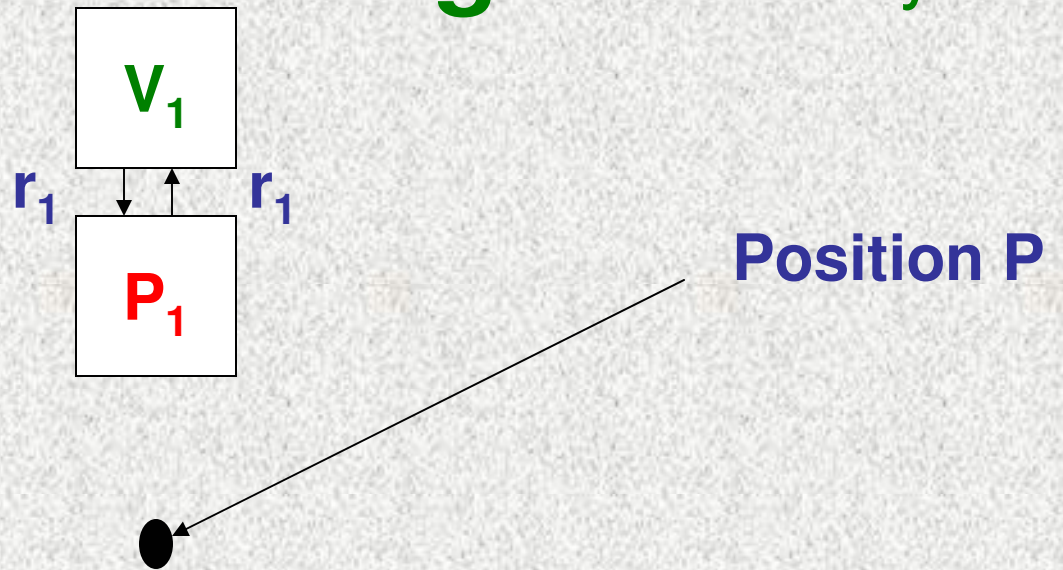




# Triangulation [CH05]

Attack with multiple scalar **single** adversary

$P_i$  can delay response to  $V_i$  as if it were coming from  $P$



# Talk Outline

- ❑ Vanilla Model

- ❑ Secure Positioning

- Impossible in vanilla model
- Positive information-theoretic results in the Bounded Retrieval Model

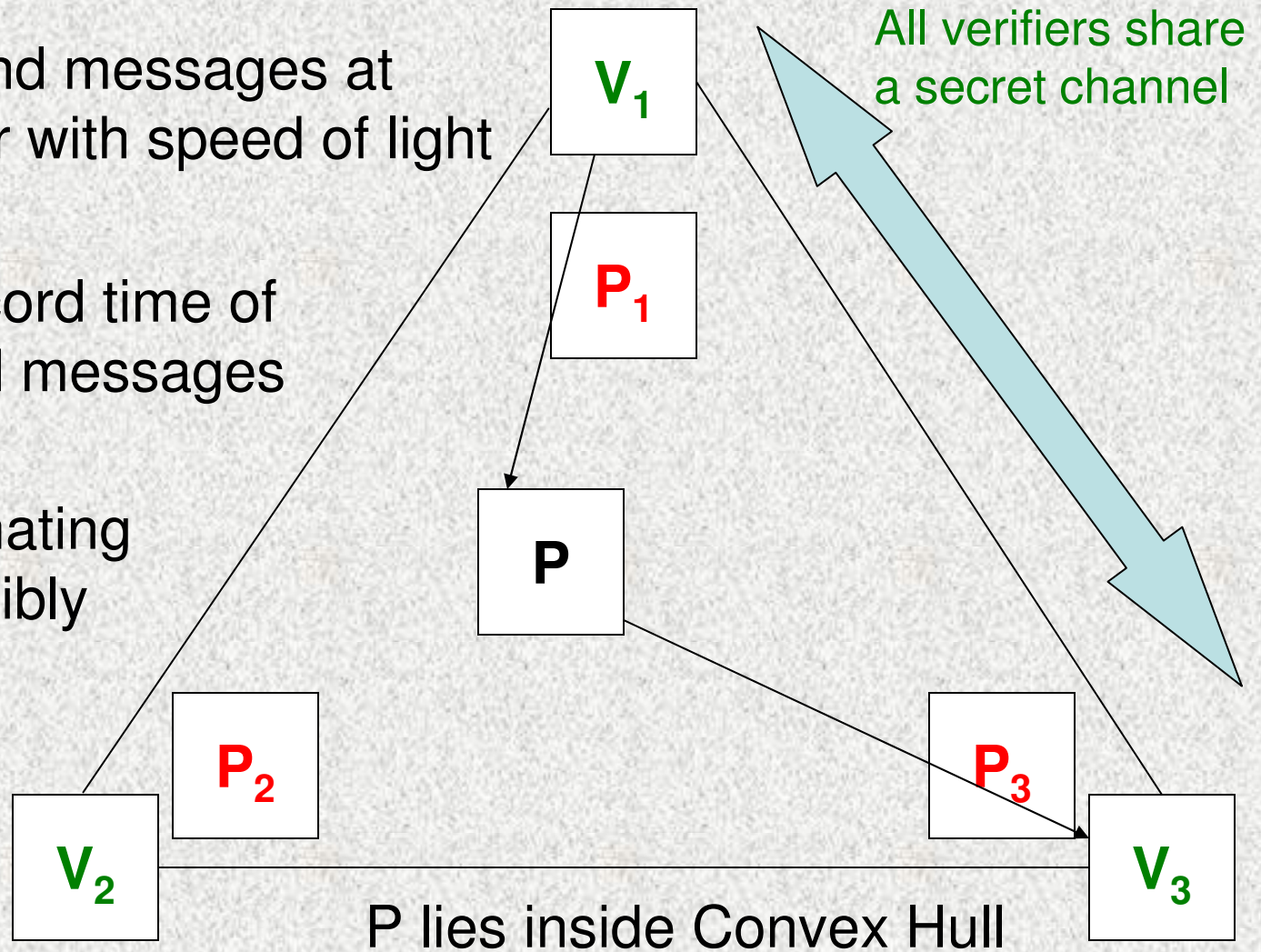
- ❑ Position-based Key Exchange

- Positive information-theoretic results in the BRM



# Vanilla Model

- Verifiers can send messages at any time to prover with speed of light
- Verifiers can record time of sent and received messages
- Multiple, coordinating adversaries, possibly computationally bounded



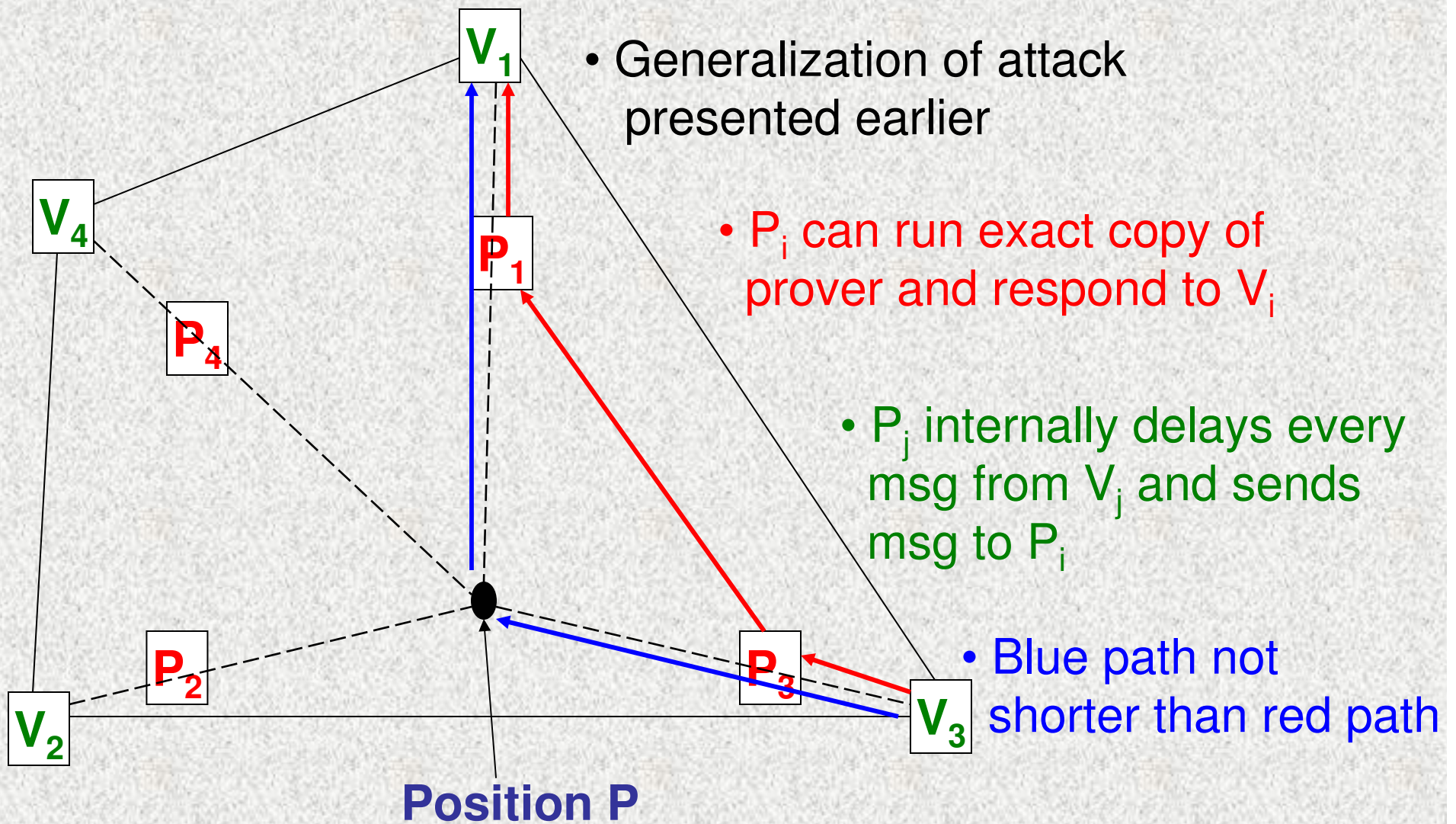
# Lower Bound

**Theorem:** *There does not exist any protocol to achieve secure positioning in the Vanilla model*

**Corollary:** *Position-based key exchange is impossible in the Vanilla model*



# Lower Bound – Proof sketch



# Lower bound implications

- Secure positioning and hence position-based cryptography is impossible in Vanilla model (even with computational assumptions!)
- Search for alternate models where position-based cryptography is possible?



# CONSTRUCTIONS & PROOFS

# Bounded Retrieval Model (BRM)

[Maurer'92, Dziembowski06, CLW06]

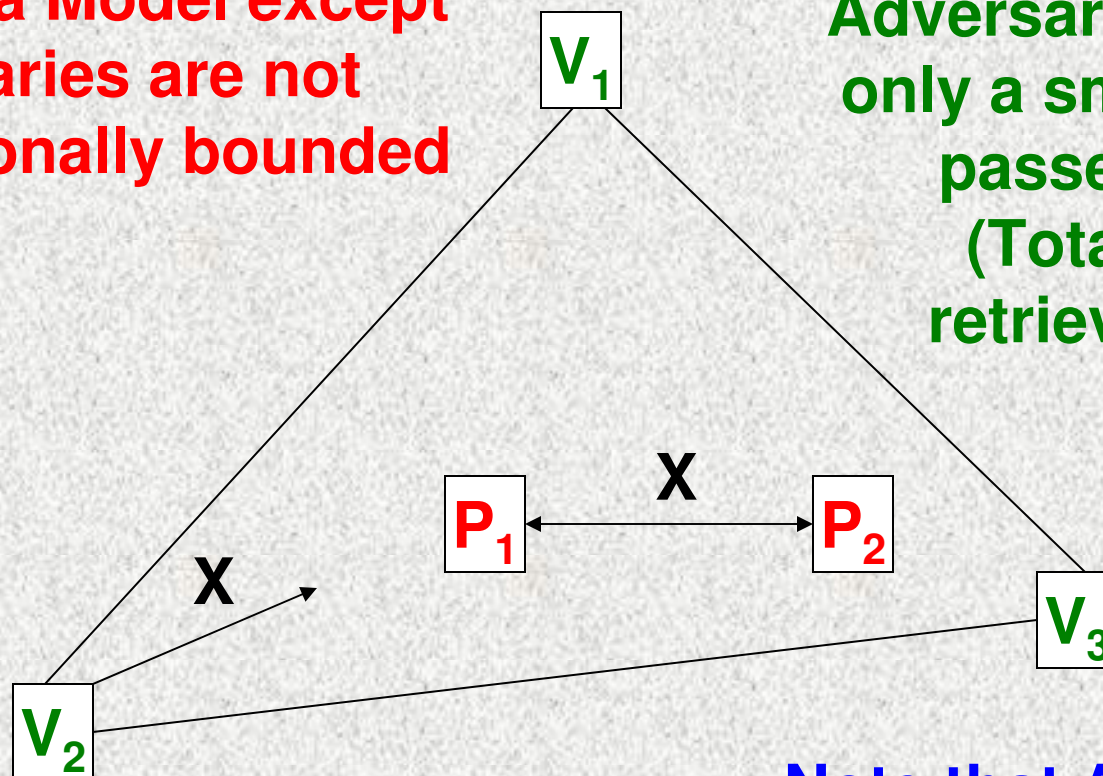
- Assumes long string  $X$  (of length  $n$  and high min-entropy) in the sky or generated by some party
- Assumes all parties (including honest) have retrieval bound  $\beta n$  for some  $0 < \beta < 1$
- Adversaries can retrieve any information from  $X$  as long as the total information retrieved is bounded
- Several works have studied the model in great detail



# BRM in the context of Position-based Cryptography

Like Vanilla Model except  
Adversaries are not  
computationally bounded

Adversaries can store  
only a small  $f(X)$  as  $X$   
passes by...i.e.  
(Total  $|f(X)| <$   
retrieval bound)



Verifiers can broadcast  
HUGE  $X$

Note that Adversaries  
can NOT “reflect”  $X$   
(violates BRM framework)

# To make things more clear

- **Computation is instantaneous – modern GPS perform computation while using speed of light assumption**  
(relaxation → error in position)
- **Huge X travels in its entirety when broadcast and not as a stream**  
(again, relaxation → error in position)



# Physically realizing BRM

- Seems reasonable that an adversary can only retrieve small amount of information as a string passes by
- Verifiers could split  $X$  and broadcast the portions on different frequencies.
- Adversary cannot listen on all frequencies

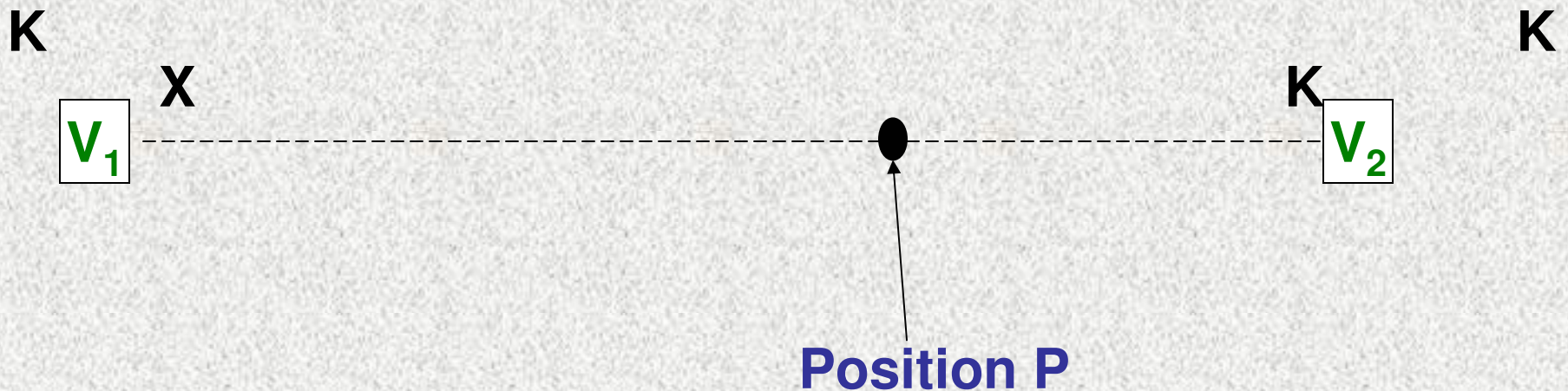
# BSM/BRM primitives needed

- Locally computable PRG from [Vad04]
- PRG takes as input string  $X$  with high min-entropy and short seed  $K$
- $\text{PRG}(X,K) \approx \text{Uniform}$ , even given  $K$  and  $A(X)$  for arbitrary bounded output length function  $A$



# Secure Positioning in 1-Dimensional Space

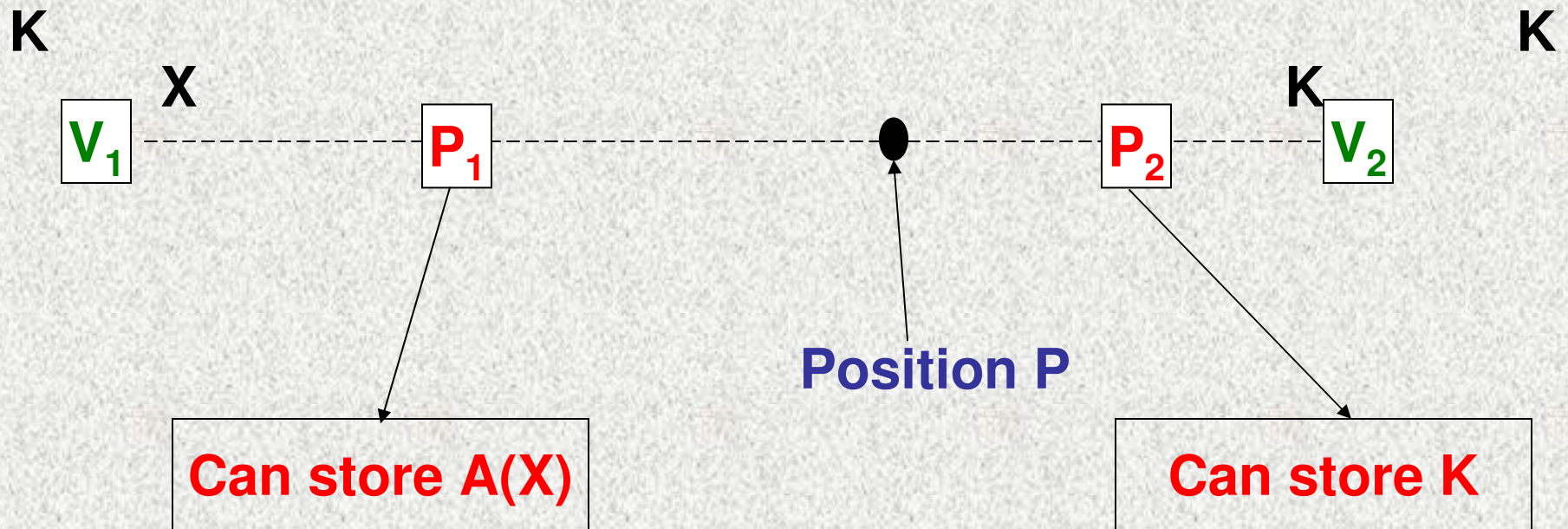
$\text{PRG}(X,K)$



- Correctness of protocol follows from**
- $V_1$  measures time of response and accepts if response is correct**
  - $V_1$  can compute  $\text{PRG}(X,K)$  when broadcasting  $X$**
  - Response of prover from  $P$  will be on time**

# Secure Positioning in 1-Dimensional Space

## Proof Intuition



- $P_1$  closer to  $V_1$  than  $P$ , but has only  $A(X)$  and  $K$
- $P_2$  can compute  $\text{PRG}(X, K)$ , but farther away from  $V_1$  than  $P$



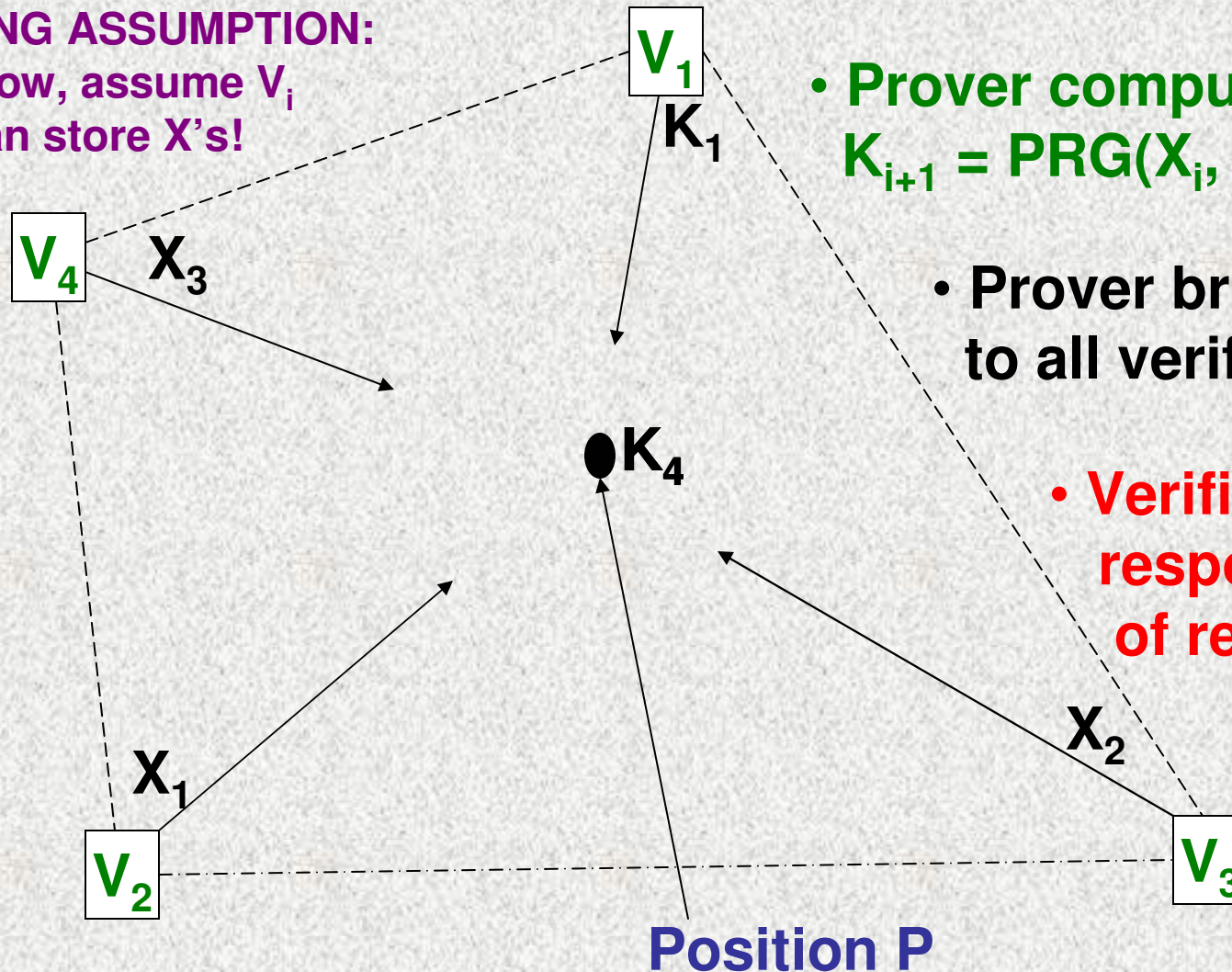
# Secure Positioning in 3-Dimensional Space

- First, we will make an UNREASONABLE assumption...
- Then show how to get rid of it!

# Secure Positioning in 3-Dimensional Space

**CHEATING ASSUMPTION:**

For now, assume  $V_i$   
can store  $X$ 's!



- Prover computes  $K_{i+1} = \text{PRG}(X_i, K_i)$ ,  $1 \leq i \leq 3$

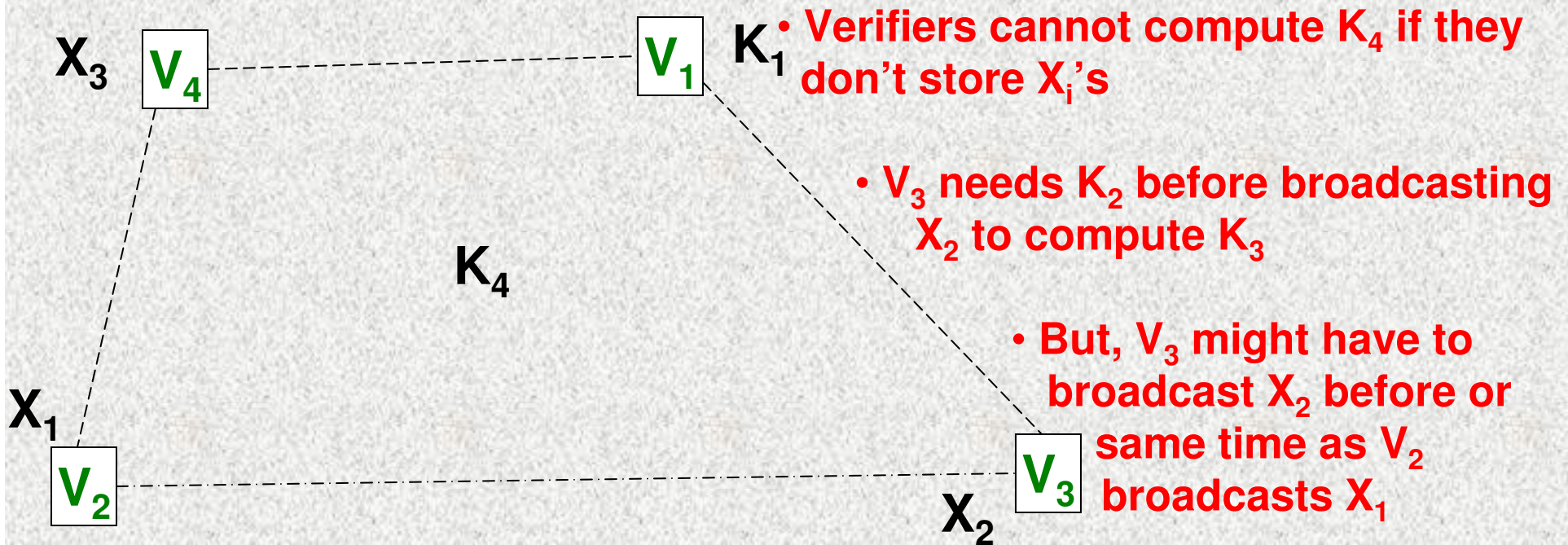
- Prover broadcasts  $K_4$  to all verifiers

- Verifiers check response & time of response



# Secure Positioning in 3-Dimensional Space

- Security will follow from security of position based based key exchange protocol presented later
- What about correctness??



# Secure Positioning in 3-Dimensional Space

## ELIMINATING CHEATING:

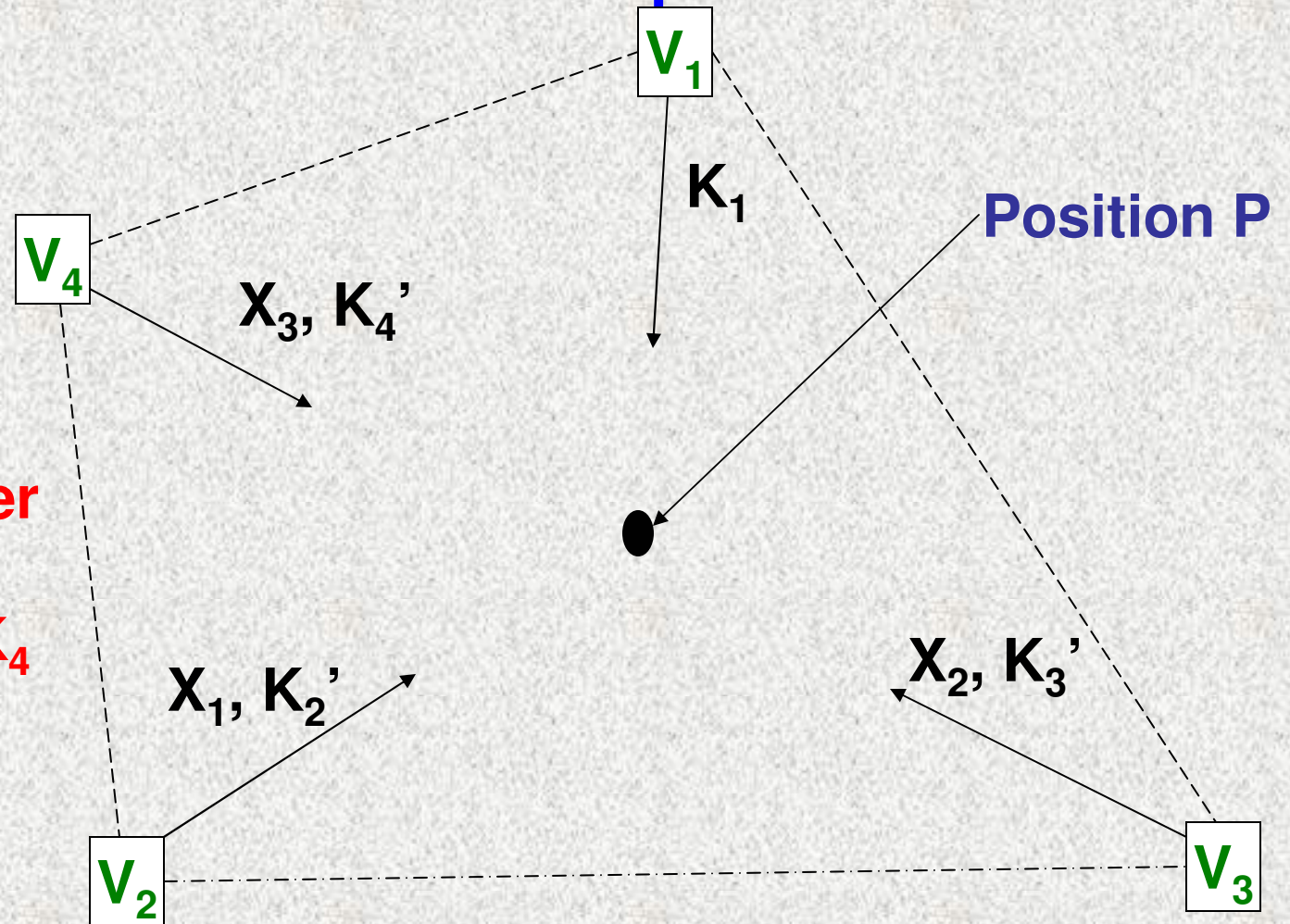
### Protocol when Verifiers cannot store $X_i$ 's

- $V_1, V_2, V_3, V_4$  pick  $K_1, K_2, K_3, K_4$  at random before protocol
- Now, Verifiers know  $K_4$ ; they must help prover compute it
- $V_1$  broadcasts  $K_1$
- $V_2$  broadcasts  $X_1$  and  $K_2' = \text{PRG}(X_1, K_1) \text{ xor } K_2$
- $V_3$  broadcasts  $X_2$  and  $K_3' = \text{PRG}(X_2, K_2) \text{ xor } K_3$
- $V_4$  broadcasts  $X_3$  and  $K_4' = \text{PRG}(X_3, K_3) \text{ xor } K_4$

Verifiers secret share  $K_i$ s and broadcast one share according to  $X_i$ s



# Secure Positioning in 3-Dimensional Space



• Note that prover can compute  $K_4$  and broadcast  $K_4$

# Secure Positioning: Bottom line

- We can do secure positioning in 3D in the bounded retrieval model
- We can obtain a protocol even if there is a small variance in delivery time when small positioning error is allowed

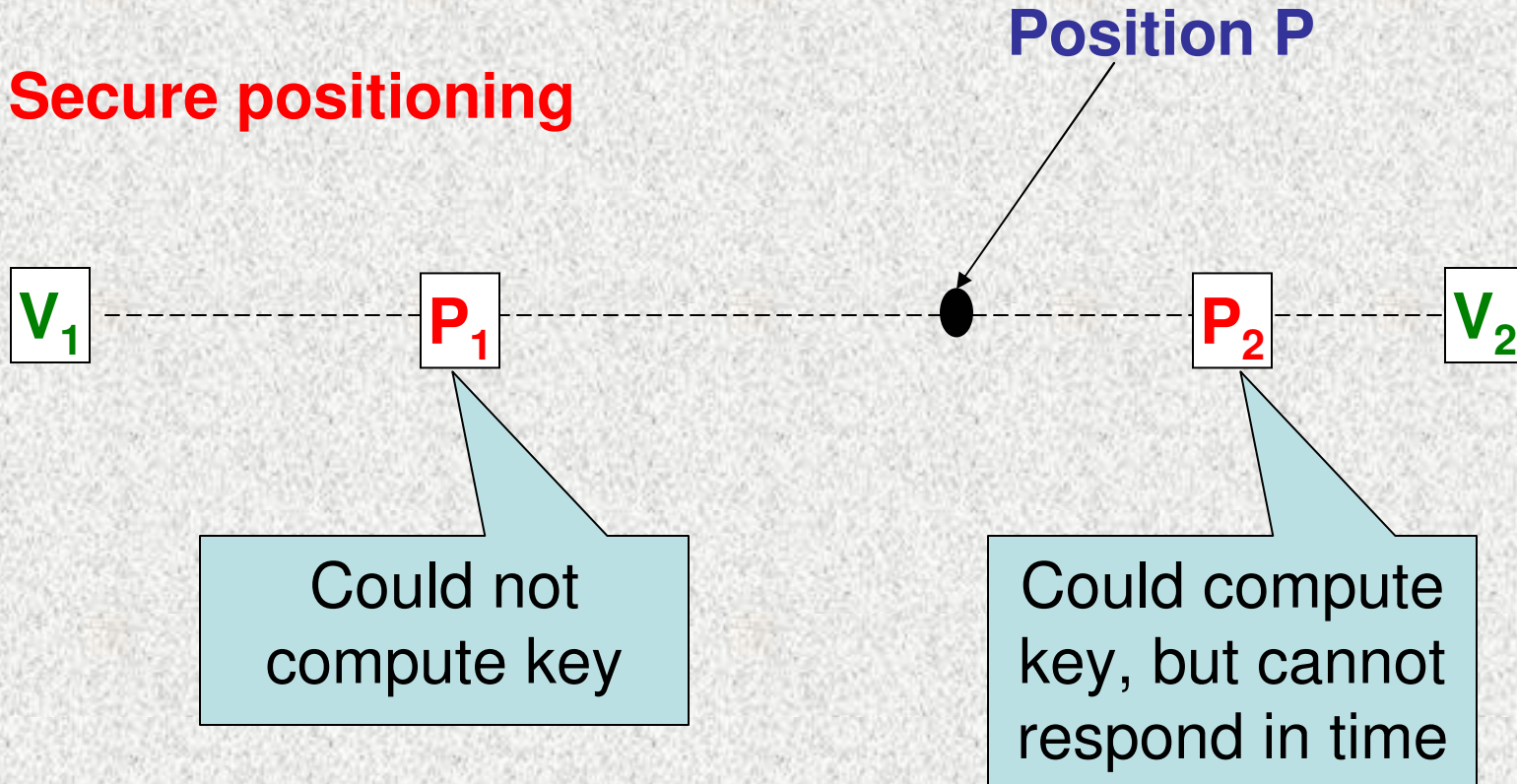


What else can we do in this model?

What about key agreement?

# Information-theoretic Key Exchange in 1-Dimensional Space

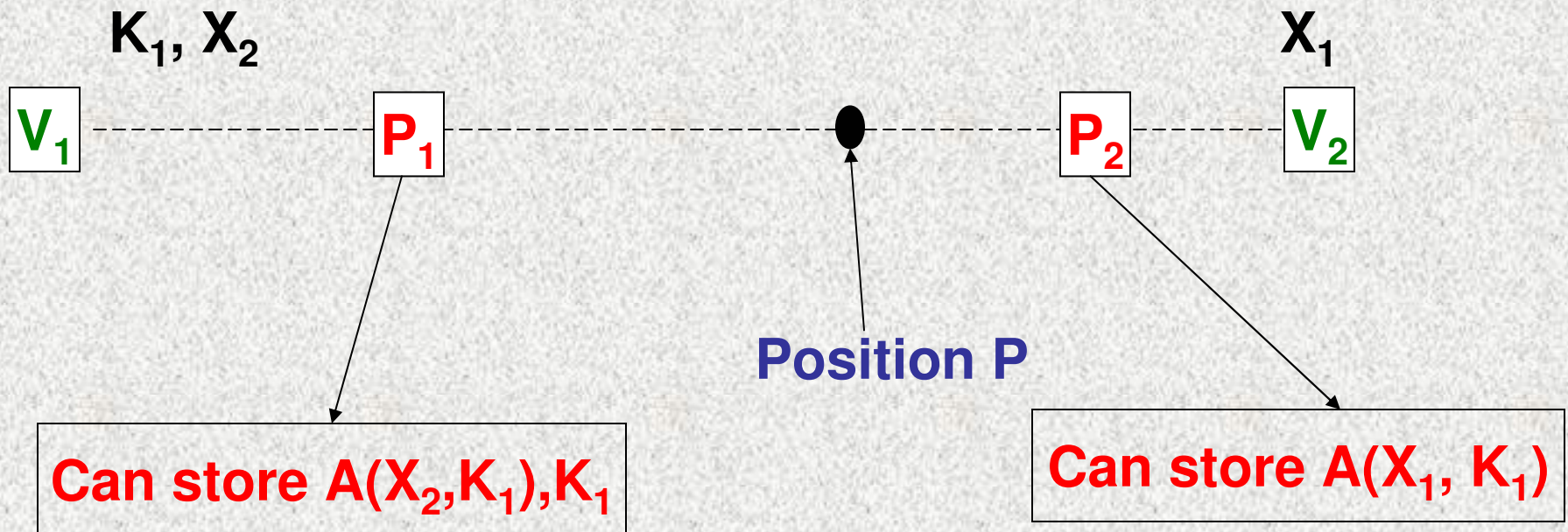
**Secure positioning**





# Information-theoretic Key Exchange in 1-Dimensional Space

$$K_3 = \text{PRG}(X_2, \text{PRG}(X_1, K_1))$$

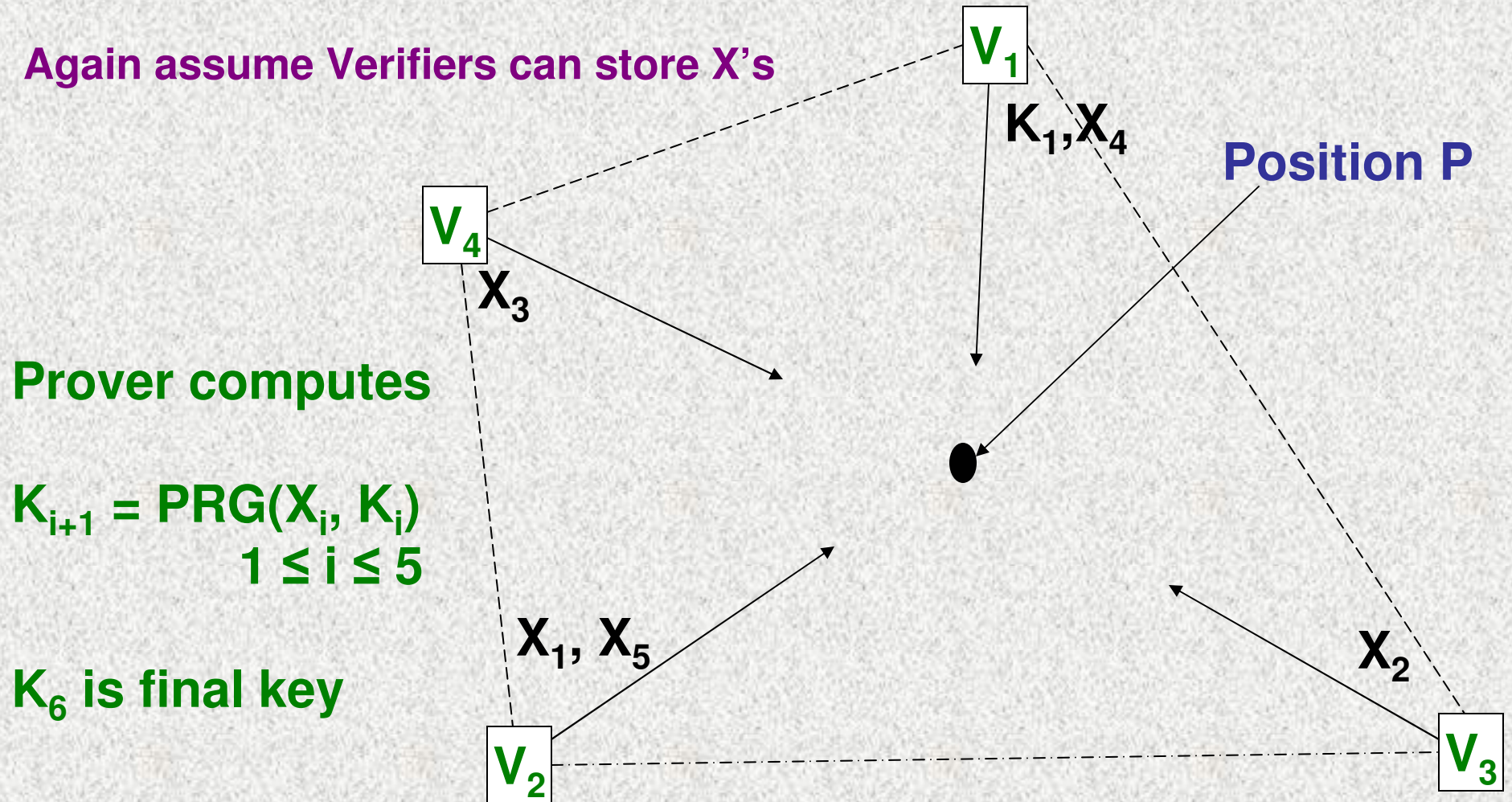


Seems like no adversary can compute  $\text{PRG}(X_2, K_2)$

Intuition works!!

# Information-theoretic Key Exchange in 3-Dimensional Space

Again assume Verifiers can store  $X$ 's



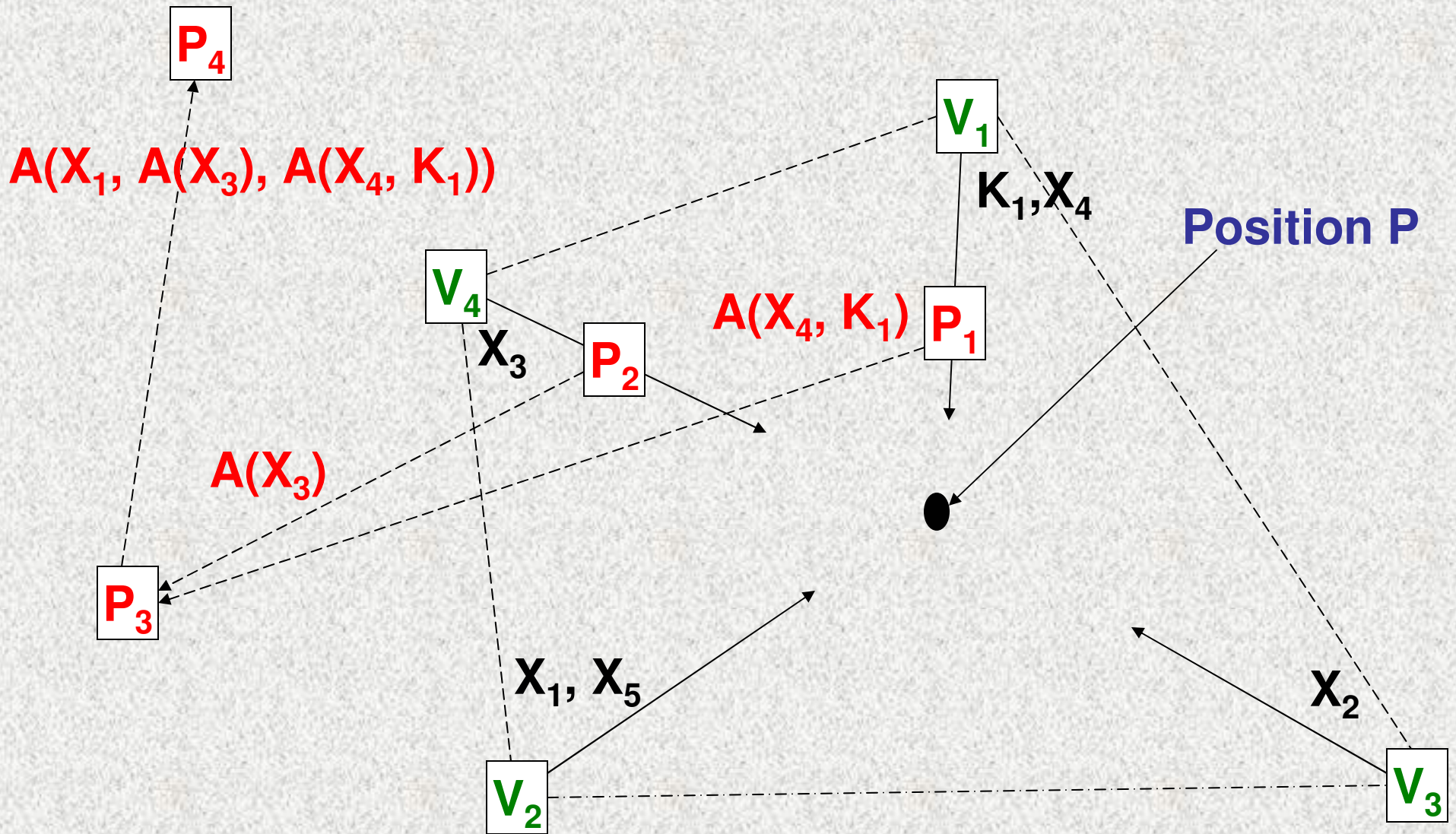
Prover computes

$$K_{i+1} = \text{PRG}(X_i, K_i) \\ 1 \leq i \leq 5$$

$K_6$  is final key



# Subtleties in proof

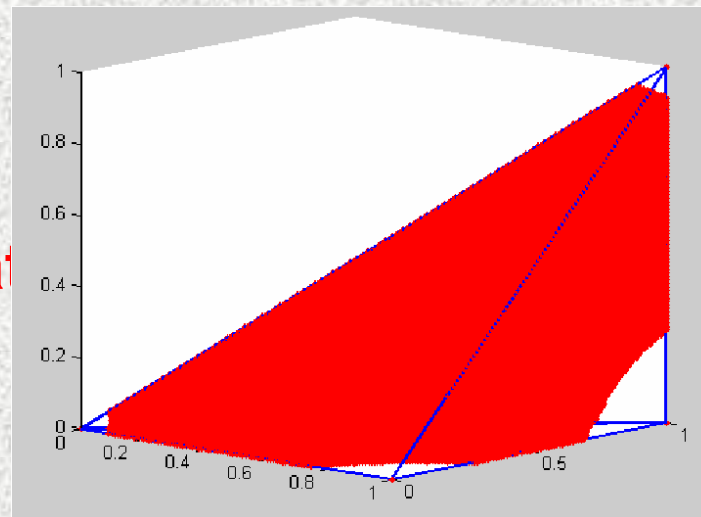


# Proof Ideas

## Part 1: Geometric Arguments

- A lemma ruling out any adversary simultaneously receiving all messages of the verifiers
  - Characterizes regions within convex hull where position-based key exchange is possible

- Combination of information that can be obtained



- To characterize what positions can

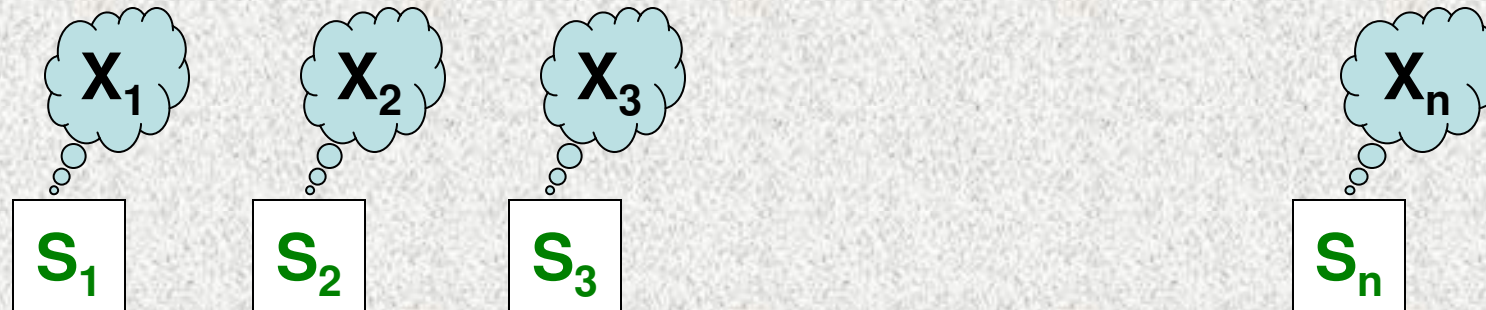


# Proof Ideas

## Part 2: Extractor Arguments

- **Build on techniques from Intrusion-Resilient Random Secret Sharing scheme of Dziembowski-Pietrzak [DP07]**
- **Show a reduction of the security of our protocol to a (slight) generalization of [DP07] allowing multiple adversaries working in parallel**

# A REMINDER: Intrusion-Resilient Random Secret Sharing Scheme (IRRSS) [DP07]



- $K_1$  is chosen at random and given to  $S_1$
- $S_i$  computes  $K_{i+1} = \text{PRG}(X_i, K_i)$  and sends  $K_{i+1}$  to  $S_{i+1}$
- $S_n$  outputs key  $K_{n+1}$

**Bounded adversary can corrupt a sequence of players (with repetition) as long as sequence is valid**

**Valid sequence does not contain  $S_1, S_2, \dots, S_n$  as a subsequence**

**Eg: If  $n = 5$ ; 13425434125 is invalid, but 134525435 is valid**

**Then,  $K_{n+1}$  is statistically close to uniform**



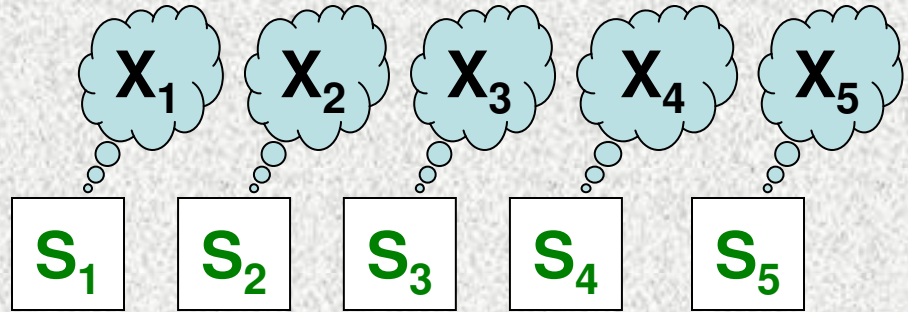
# Reduction to IRRSS

$A(X_1, A(X_3), A(X_4, K_1))$

$P_3$

$K_1, X_4$

$V_1$



$P_1$ : corrupts  $S_4$

$P_2$ : corrupts  $S_3$

$P_3$ : corrupts  $S_4, S_3, S_1$

$A(X_4, K_1)$

$P_1$

$A(X_3)$

$P_2$

$X_3$

$V_4$

$X_2$

$V_3$

$X_1, X_5$

$V_2$

All adversaries given  $K_1$  for free

# Reduction to IRRSS

- For every adversary  $\mathbf{A}$  that receives information only from a verifier (not from other adversaries), we show a *corresponding* adversary  $\mathbf{B}$  for [DP07] with valid corruption sequence  $\mathbf{C}$ .
- If the *corresponding* adversary for  $\mathbf{A}$  has an invalid corruption sequence in [DP07], then  $\mathbf{A}$  must have received info from all verifiers simultaneously (Not possible by geometric lemma)
- Given two adversaries  $\mathbf{A}_1$  and  $\mathbf{A}_2$  with *corresponding* adversaries  $\mathbf{B}_1$  and  $\mathbf{B}_2$  (in [DP07]) and sequences  $\mathbf{C}_1$  and  $\mathbf{C}_2$ , show how to get *corresponding* adversary  $\mathbf{B}$  for  $\mathbf{A}_1 \cup \mathbf{A}_2$  with corruption sequence  $\mathbf{C}$ .



# Conclusions

- WE HAVE SHOWN IN THE PAPER:
  - Position based Key Exchange in BRM for entire convex hull region (but computational security)
  - Protocol for position based Public Key Infrastructure
  - Protocol for position based MPC
- OPEN:
  - Other models?  
(Quantum: [C–Fehr–Goyal–Ostrovsky'09])
  - Other applications of position-based crypto?

**Thank you**

**Full version available at**

**<http://eprint.iacr.org/2009/364>**