Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Information Security
& Cryptography

Computational Indistinguishability Amplification: Tight Product Theorems for System Composition

Ueli Maurer **Stefano Tessaro**

ETH Zurich

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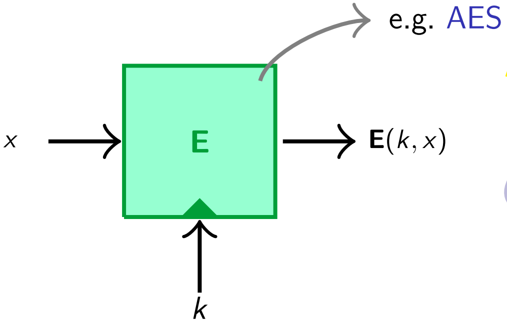
August 18th, 2009

Motivation: Block Ciphers

Block cipher



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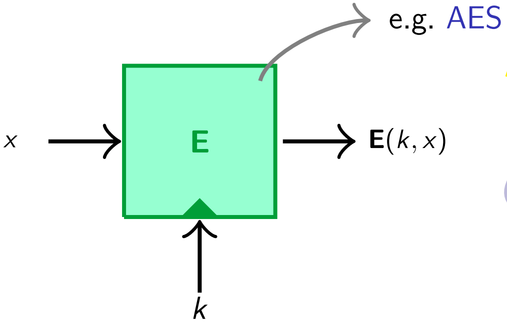
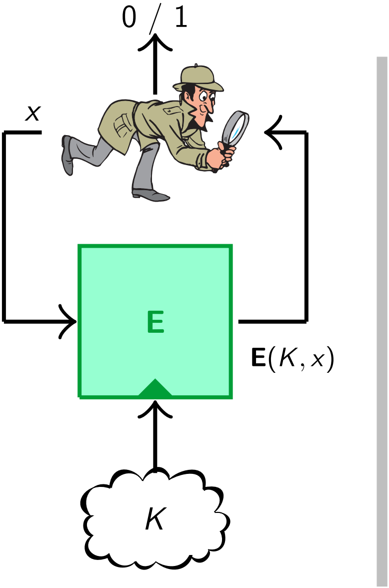


Security definition: **Computational Indistinguishability**

Motivation: Block Ciphers – Pseudorandom Functions



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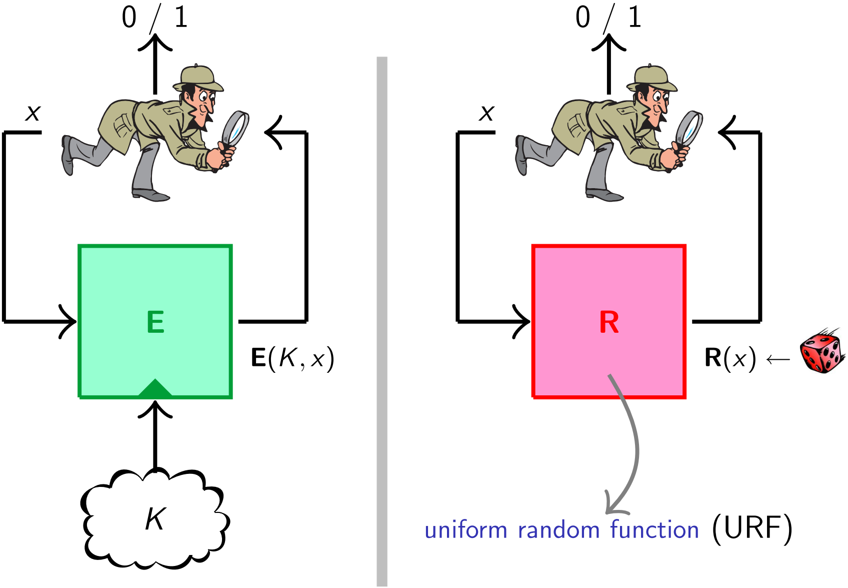

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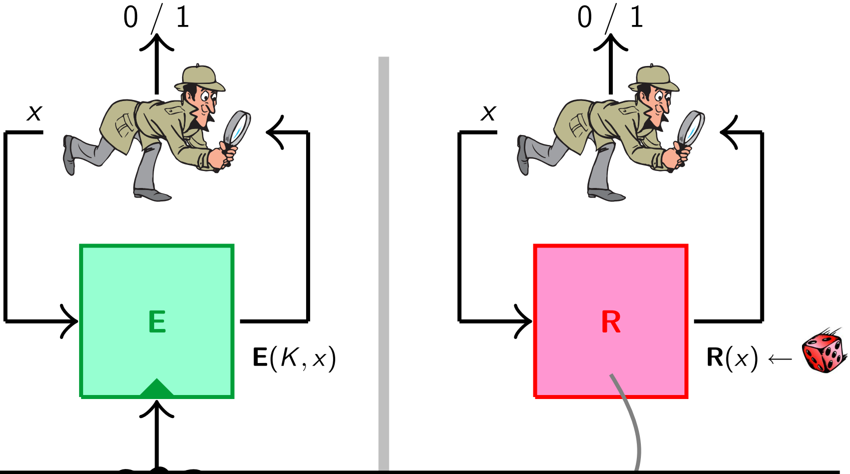
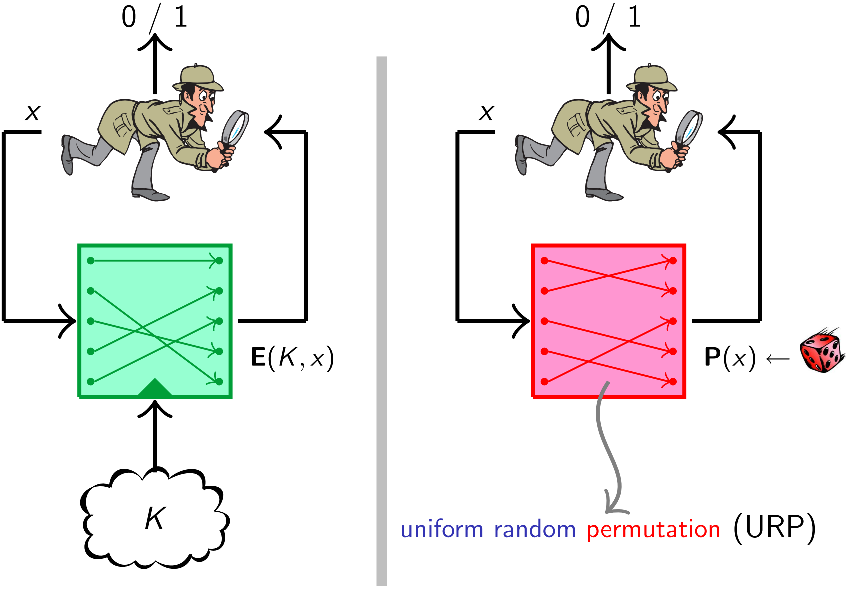


E **PRF**: $\forall$ efficient **D**:

$$\Delta^{\mathbf{D}}(\mathbf{E}_K, \mathbf{R}) = \left|\Pr[\mathbf{D}(\mathbf{E}_K) = 1] - \Pr[\mathbf{D}(\mathbf{R}) = 1]\right| = \text{negl}$$

Motivation: Block Ciphers – Pseudorandom Permutations



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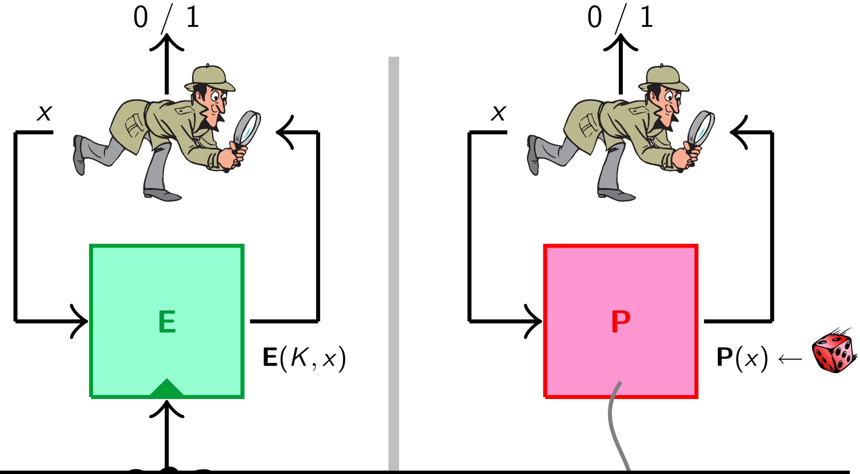

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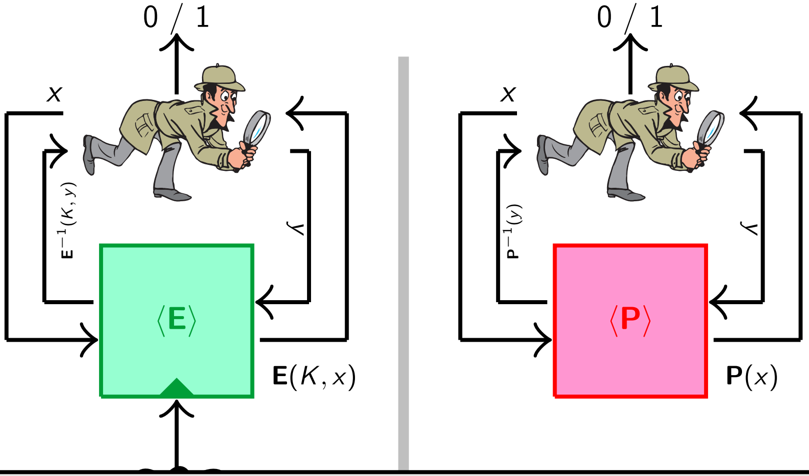

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Weakening Security Assumptions

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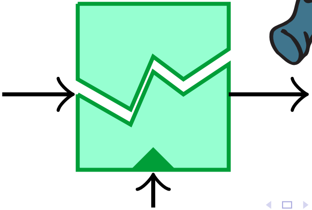

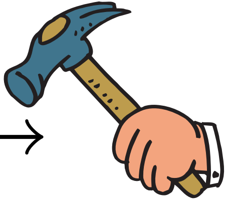

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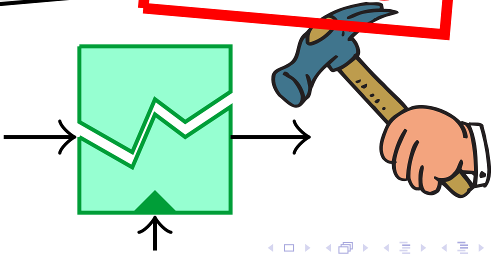
STRONG



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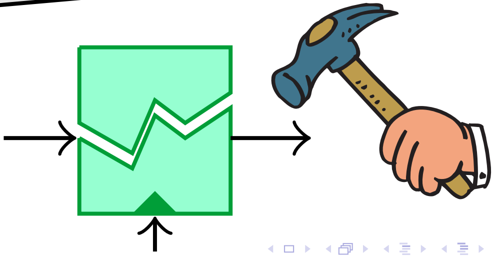

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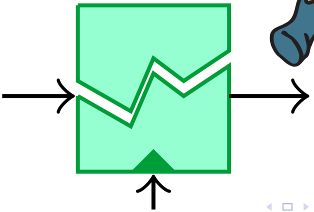

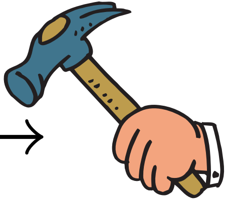

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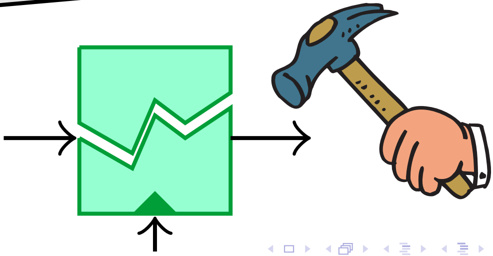

Weakening Security Assumptions

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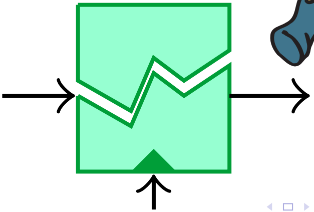

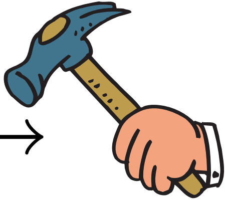

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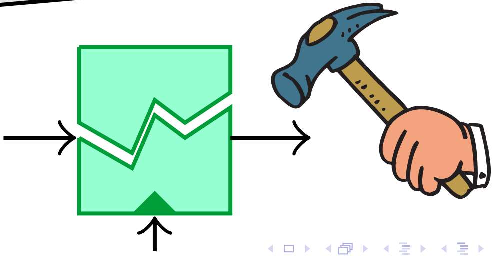

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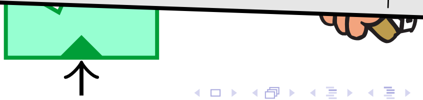

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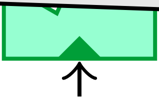
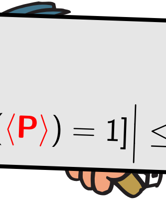
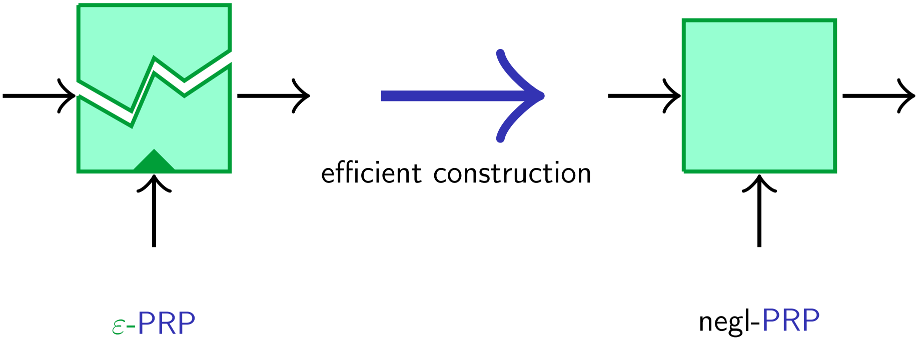
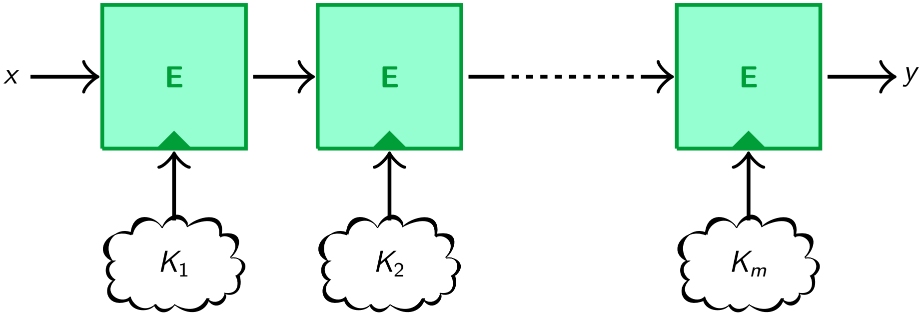




Security Amplification



Cascaded Encryption



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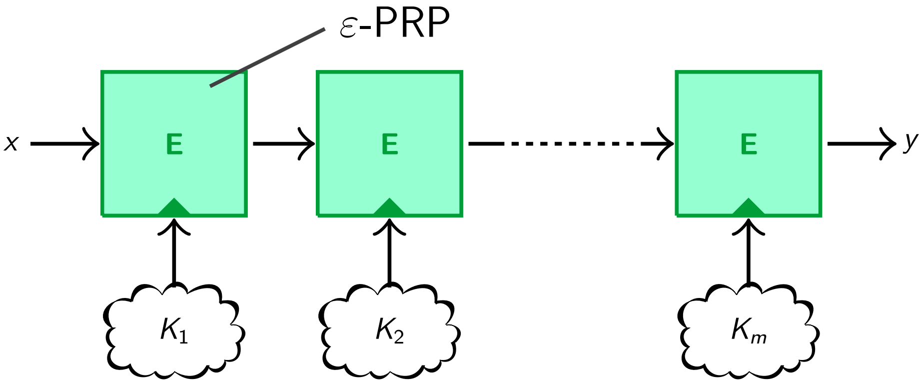

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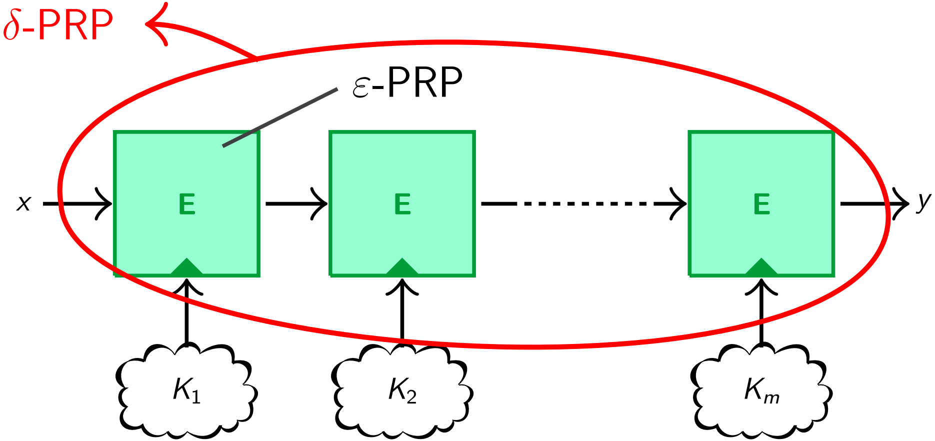
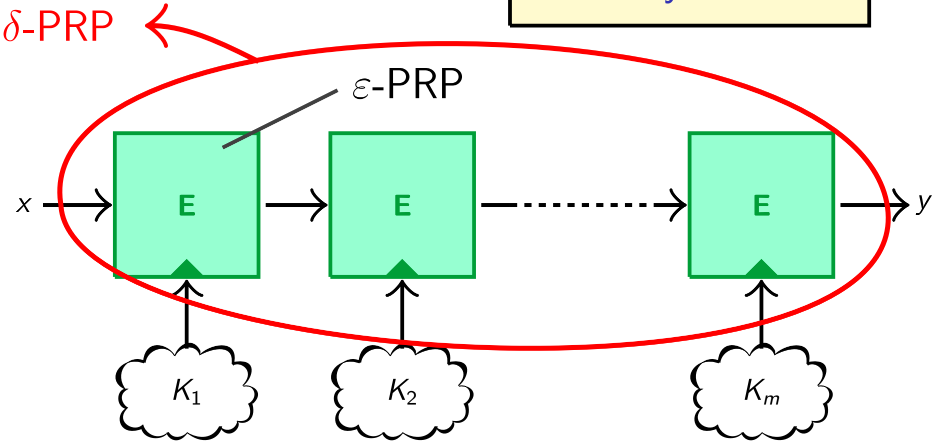


Cascaded Encryption

Ideally: $\delta \approx \varepsilon^m$



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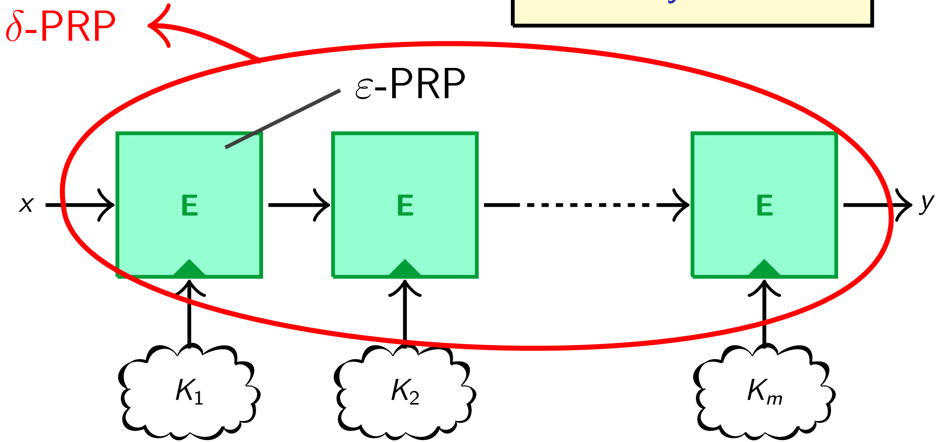

ideal/IT settings [BR06,MG09,V98,MPR07]

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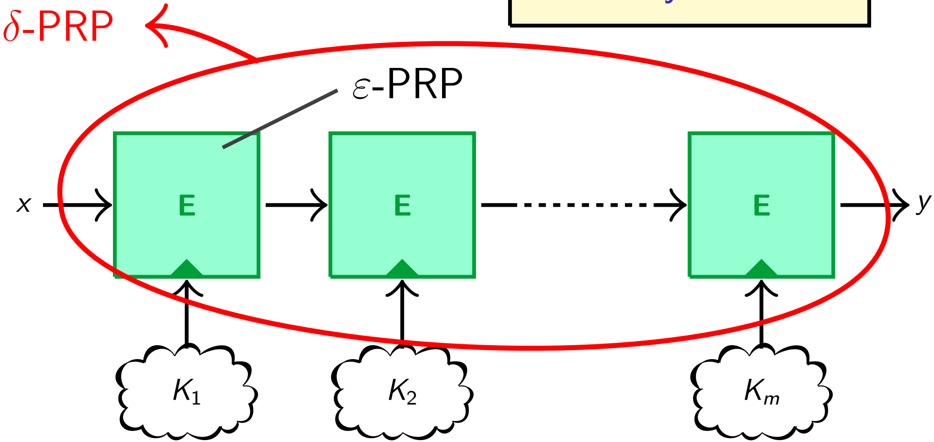
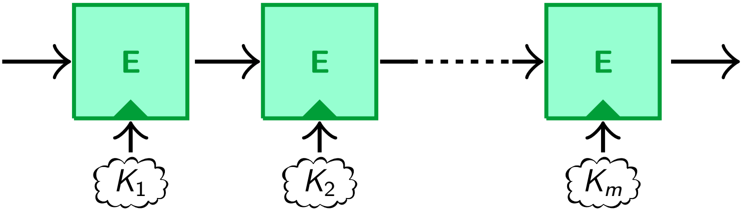


ideal/IT settings [BR06,MG09,V98,MPR07]

[LR86,M99]: small $m \implies$ **no** security amplification

This Paper – A Preview



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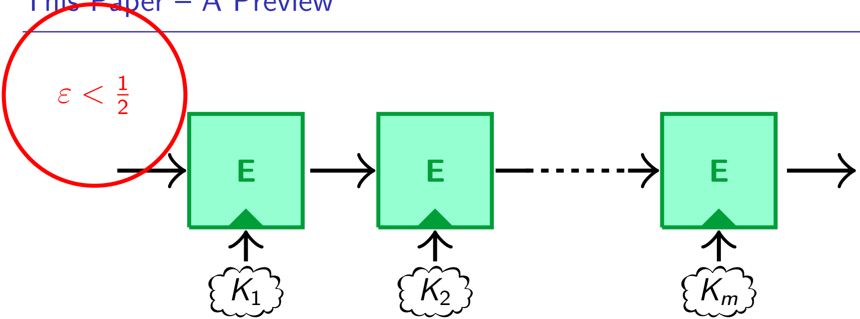
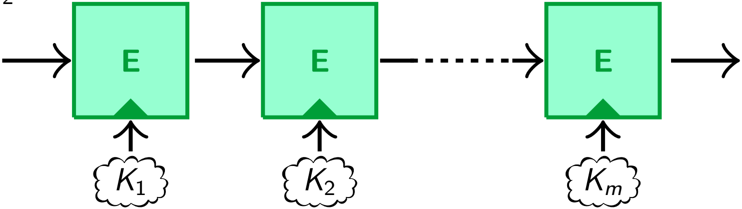


This Paper – A Preview

$$\delta = 2^{m-1} \cdot \varepsilon^m + \text{negl}$$

$\varepsilon < \frac{1}{2}$



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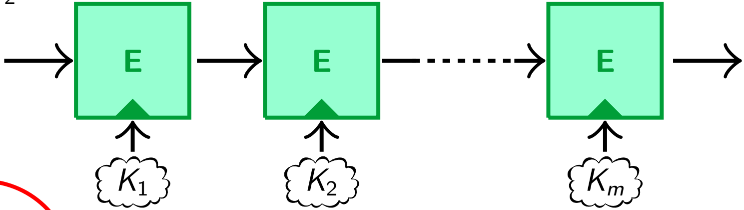

$\frac{1}{2} \leq \varepsilon < 1$

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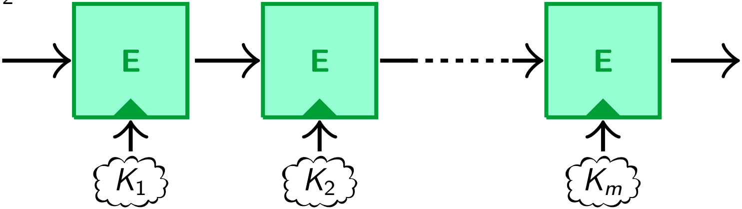

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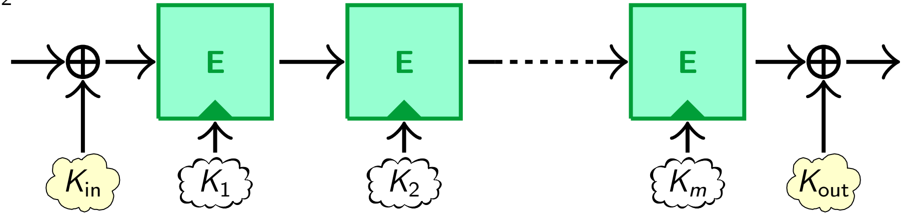

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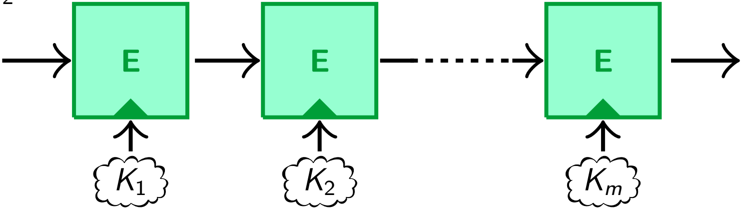

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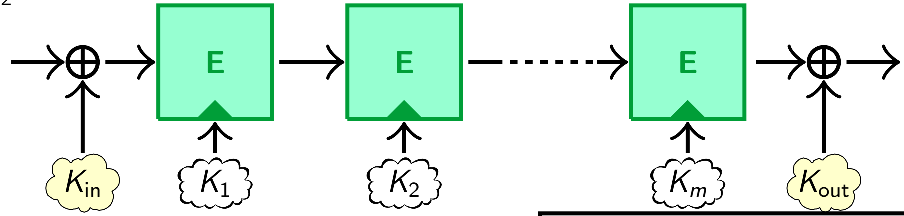

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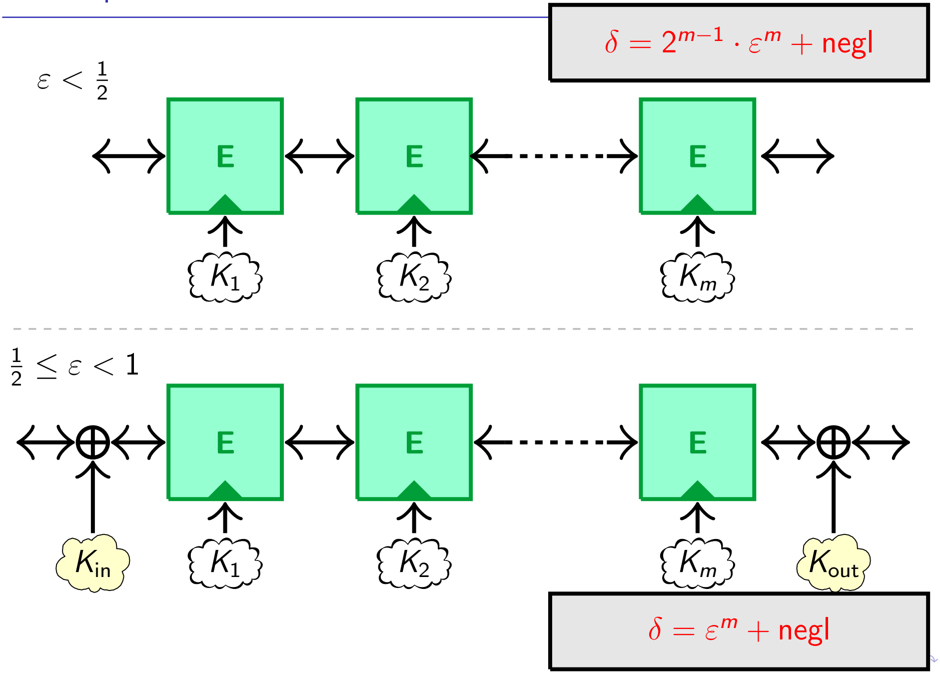

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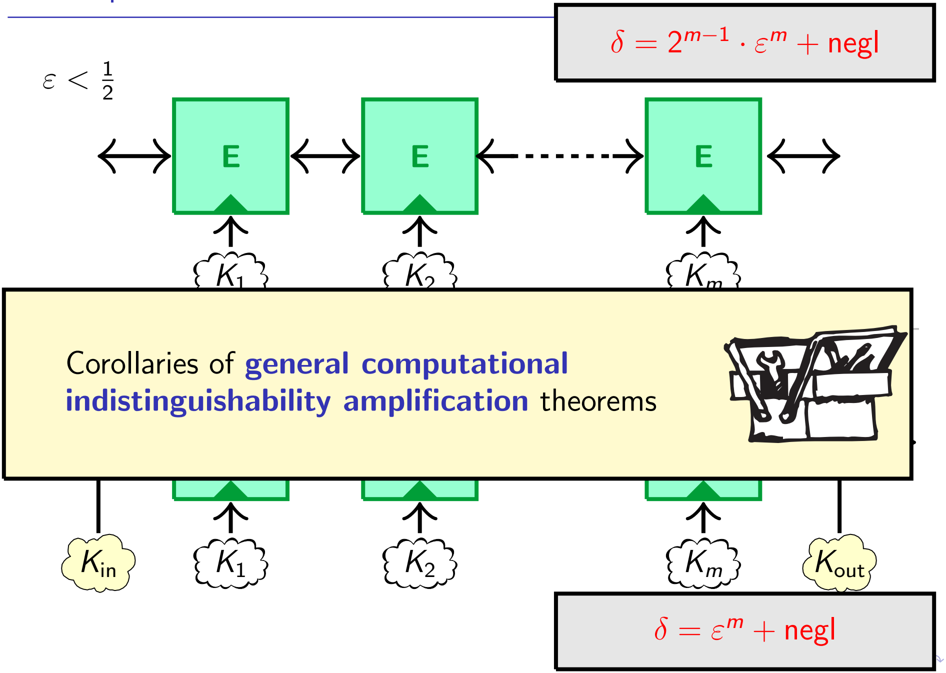
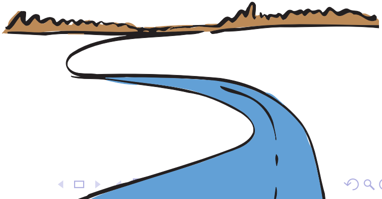


Corollaries of **general computational indistinguishability amplification** theorems

Outline

1. Generalizing Yao's XOR Lemma
2. Neutralizing Constructions
3. Strong Indistinguishability Amplification
4. Concluding Remarks



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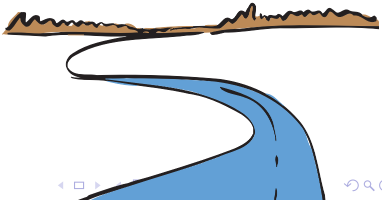
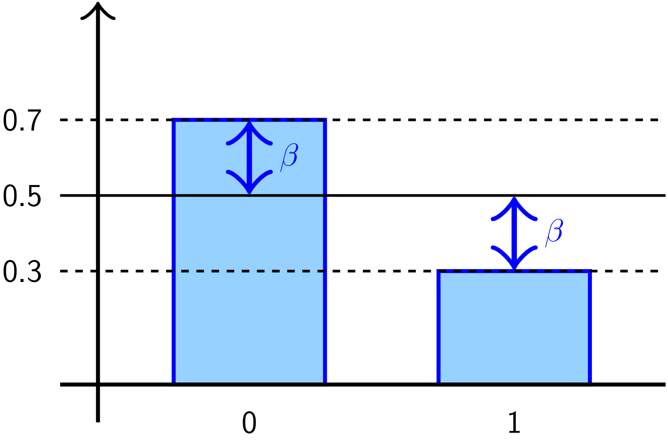


Biased Bits

x	**0**	**1**
$\Pr[B = x]$	0.7	0.3



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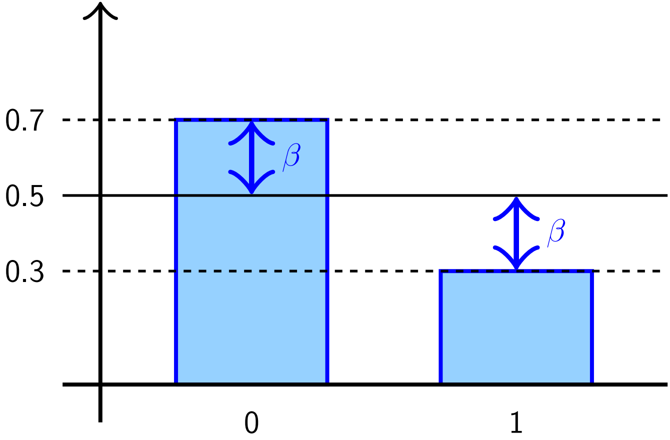

Biased Bits

$B \in \{0, 1\}$

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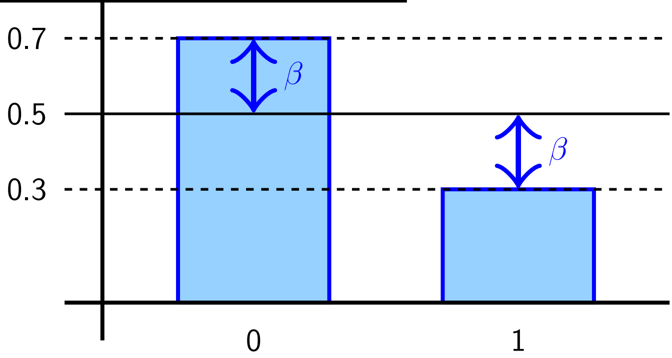

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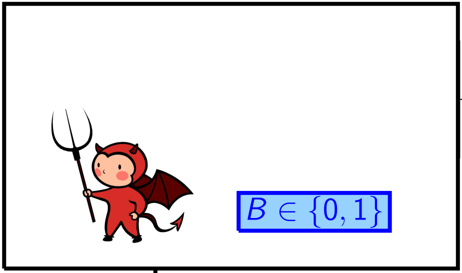

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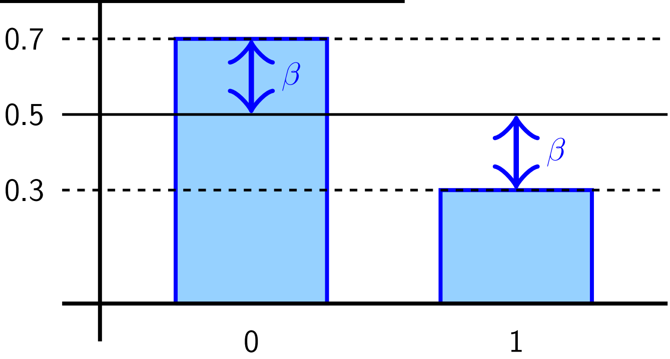

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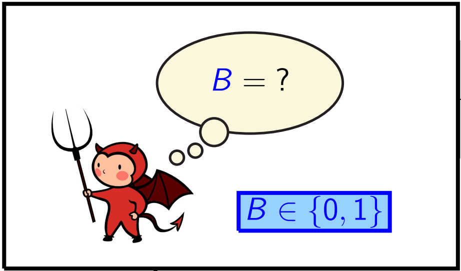

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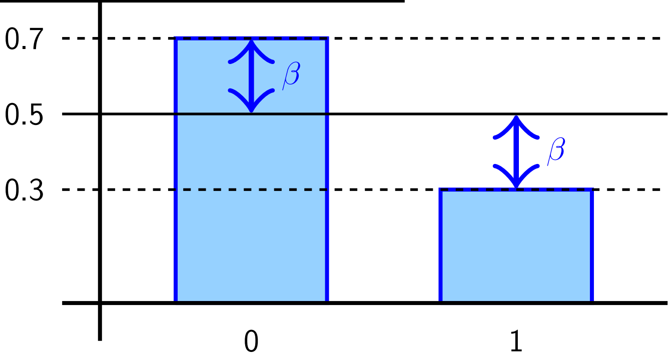

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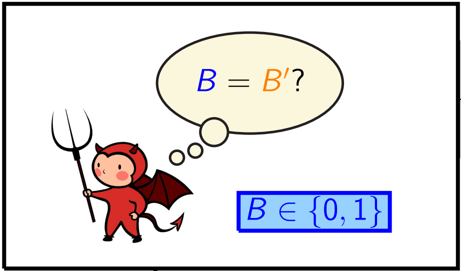

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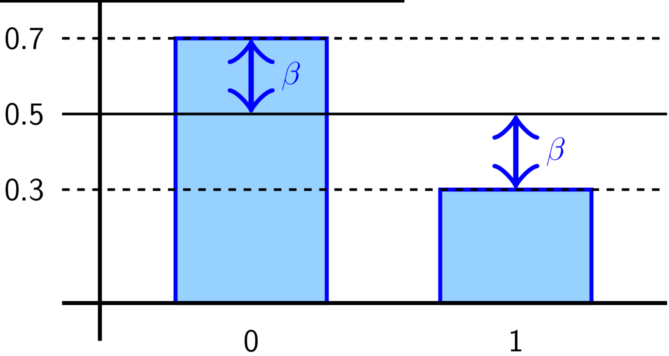

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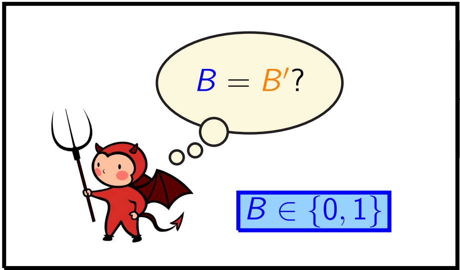

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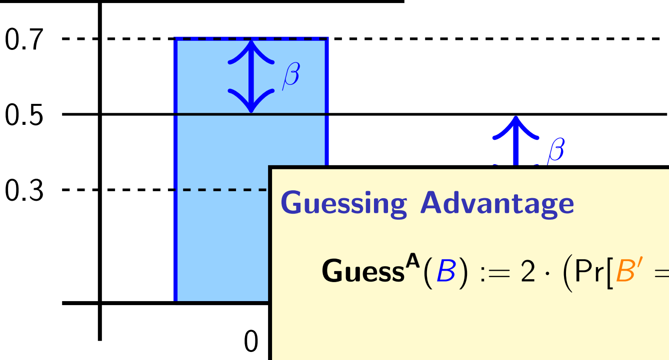
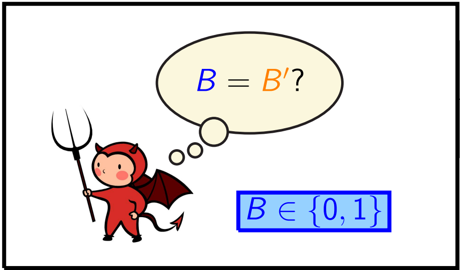
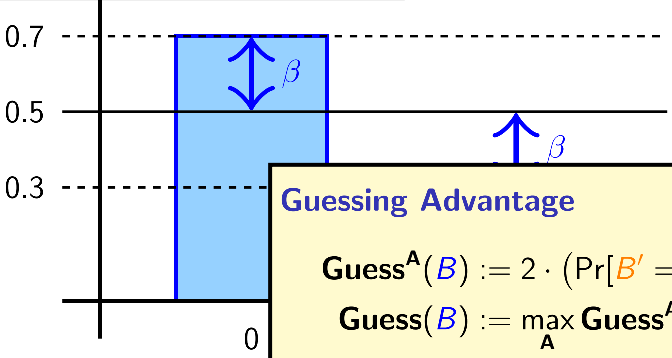


Guessing Advantage

$$\mathbf{Guess}^{\mathbf{A}}(B) := 2 \cdot \left(\Pr[B' = B] - \frac{1}{2}\right)$$



0	1
0.7	0.3



Guessing Advantage

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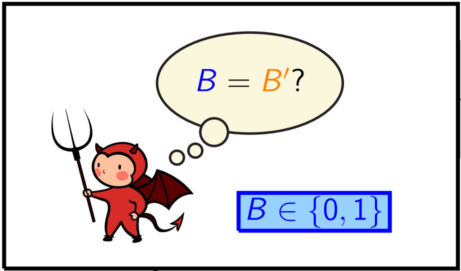
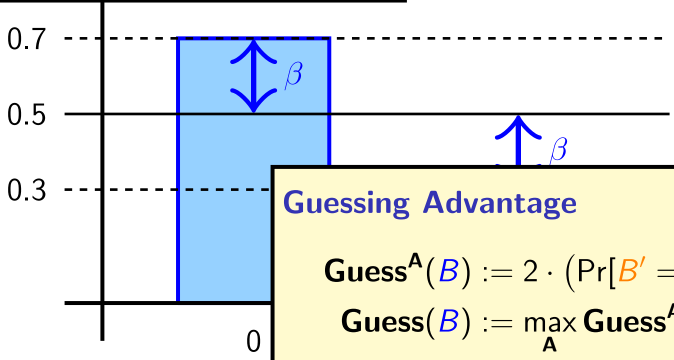
Biased Bits



0	1
0.7	0.3



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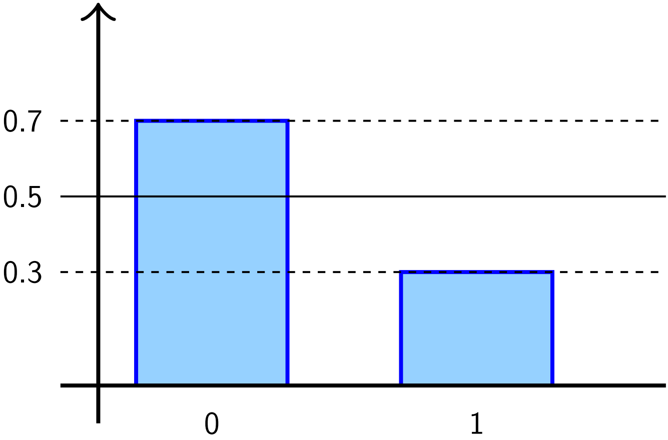
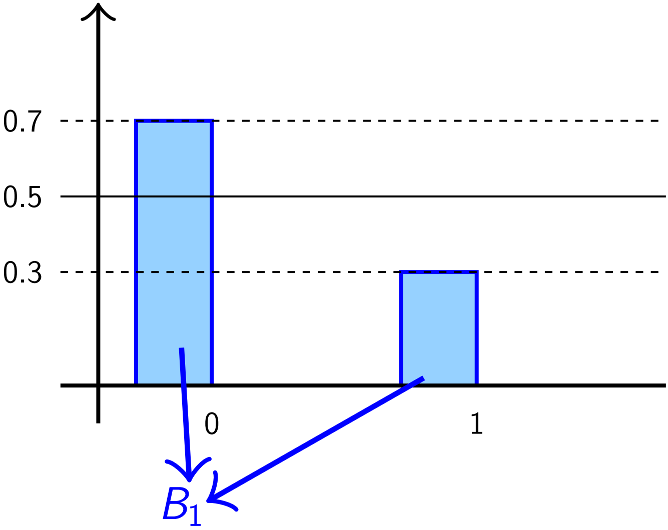
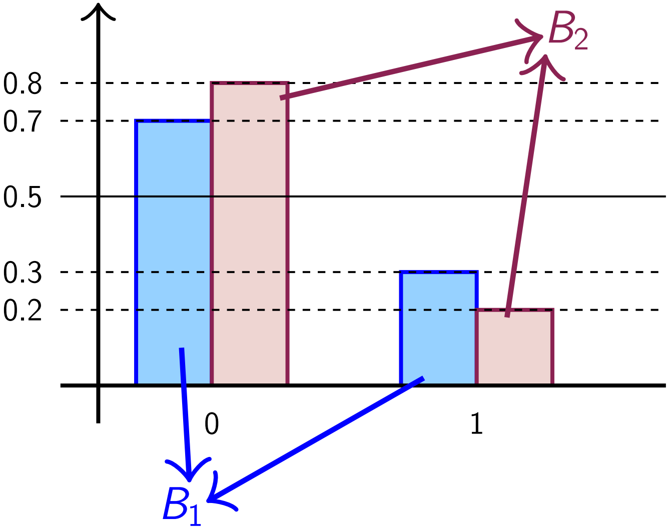
Biased Bits – XOR



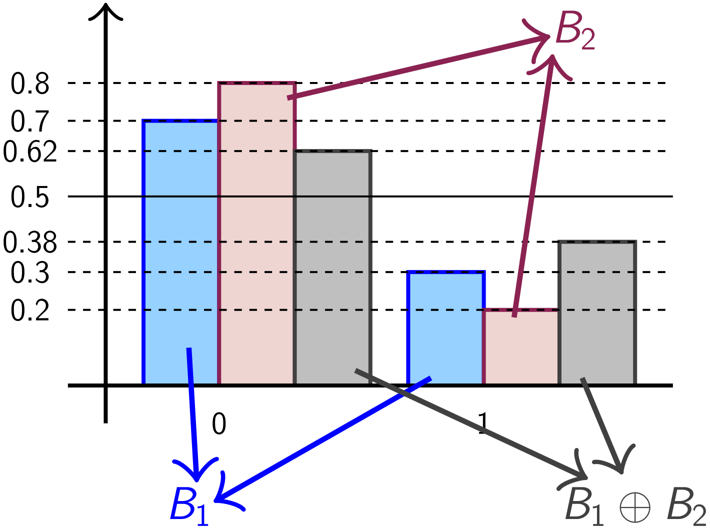
Biased Bits – XOR



Biased Bits – XOR

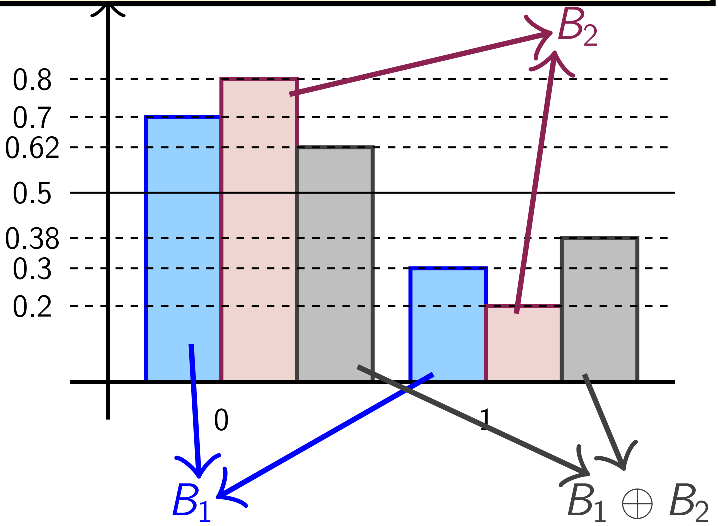


Biased Bits – XOR



Biased Bits – XOR

Theorem. $\textbf{Guess}(B_1 \oplus B_2) = \textbf{Guess}(B_1) \cdot \textbf{Guess}(B_2)$



Yao's XOR Lemma

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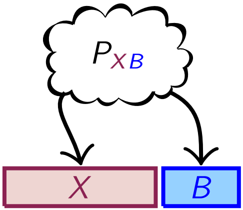

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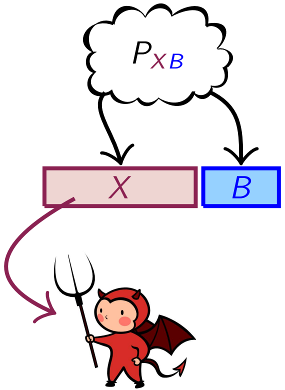

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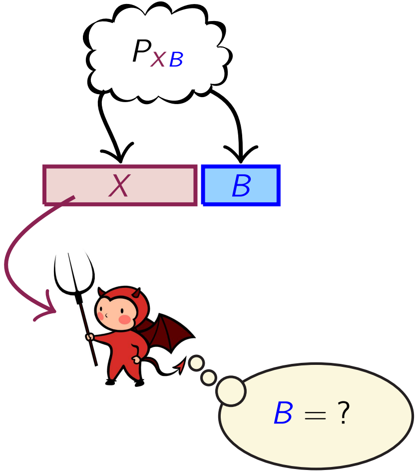

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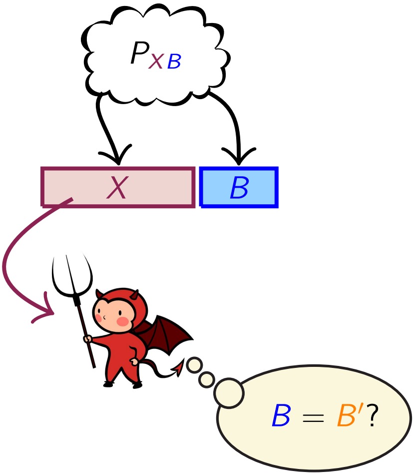

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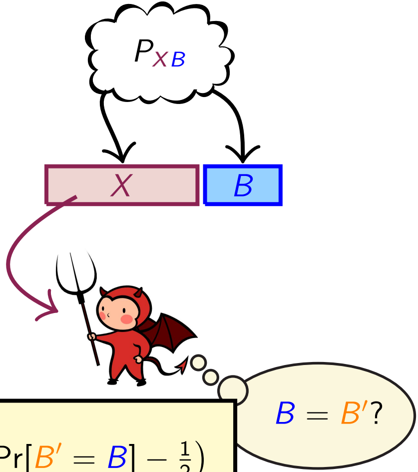

$$\mathbf{Guess}^{\mathbf{A}}(B \mid X) := 2 \cdot \left(\Pr[B' = B] - \tfrac{1}{2}\right)$$

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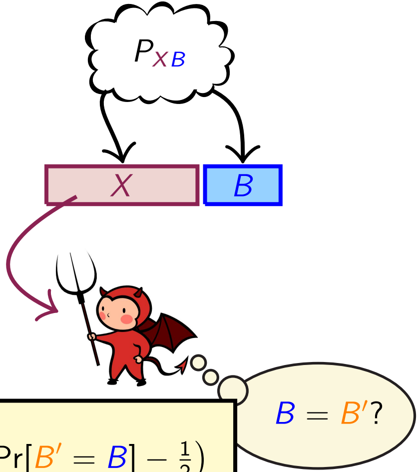

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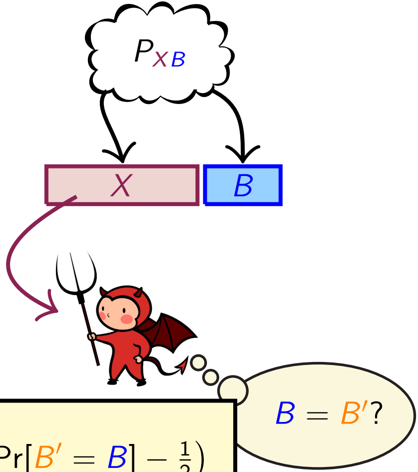

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Example. $f : \{0,1\}^n \to \{0,1\}^n$, $P : \{0,1\}^n \to \{0,1\}$

$$U \xleftarrow{\$} \{0,1\}^n,\ X := f(U),\ B := P(U)$$

$B = B'$?

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$\mathbf{Guess}_{\mathrm{poly}}(B \mid X) = \mathrm{negl} \iff P$ is hardcore predicate for f

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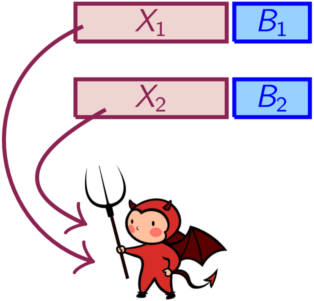

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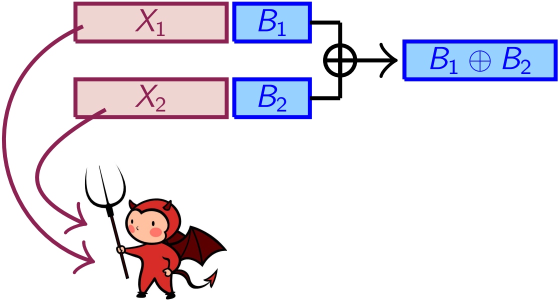

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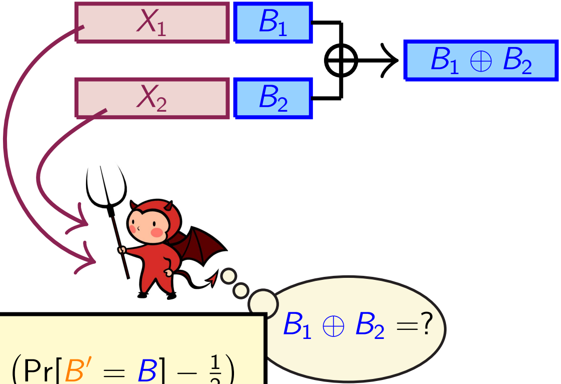

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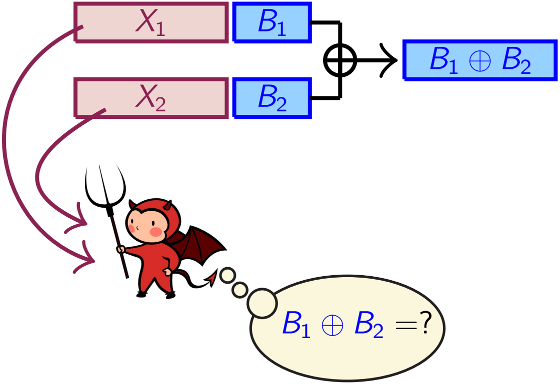

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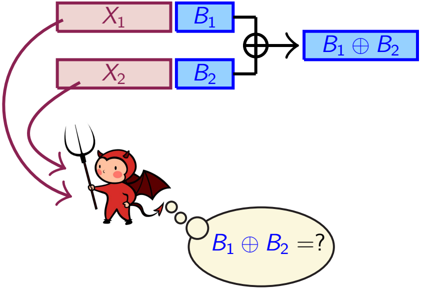

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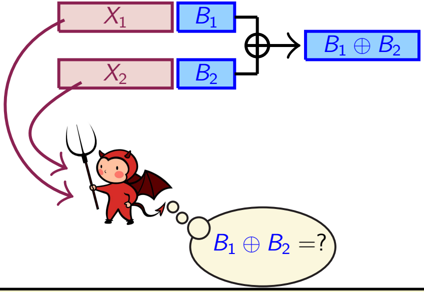

Theorem [Y82]. $\forall\, (X_1, B_1), \ldots, (X_m, B_m)$,

$$\mathbf{Guess}_t(B_1 \oplus \cdots \oplus B_m \mid X_1, \ldots, X_m)$$

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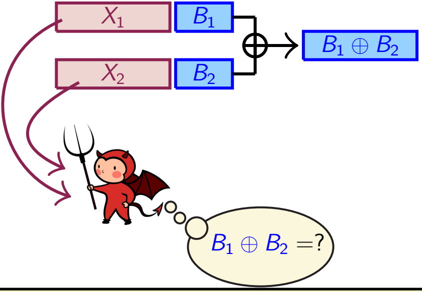

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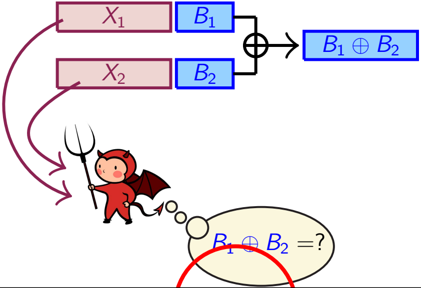

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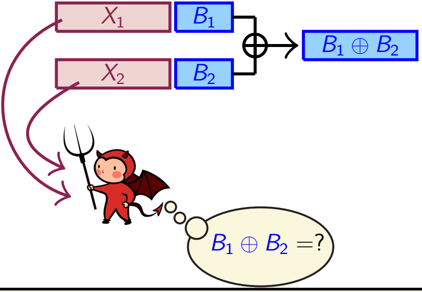

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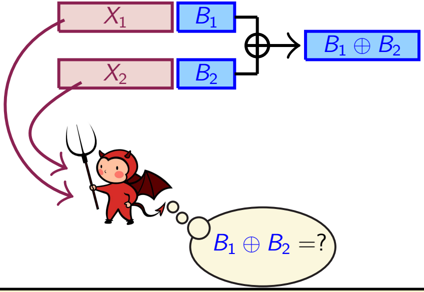

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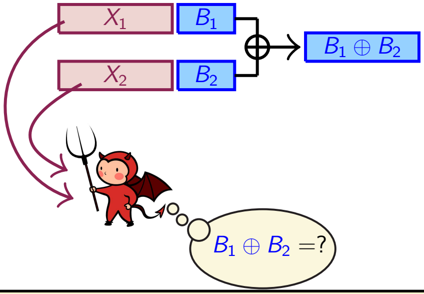

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TRADE OFF

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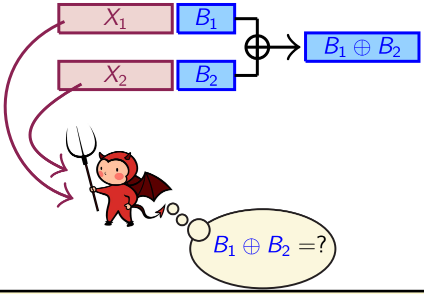

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TRADE OFF

Several proofs [L87,I95,GNW95,...]

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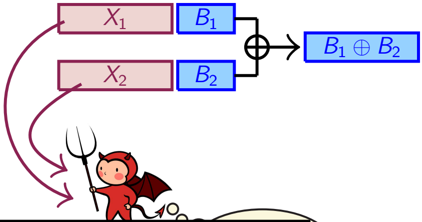

Asymptotically: $\mathbf{Guess}_{\text{poly}}(B_i \mid X_i) \leq \varepsilon \implies$

$$\mathbf{Guess}_{\text{poly}}(B_1 \oplus \cdots \oplus B_m \mid X_1, \ldots, X_m) \leq \varepsilon^m + \text{negl}$$

$$\mathbf{Guess}_t(B_1 \oplus \cdots \oplus B_m \mid X_1, \ldots, X_m) \leq \prod_{i=1}^{m} \mathbf{Guess}_{t'}(B_i \mid X_i) + \gamma$$

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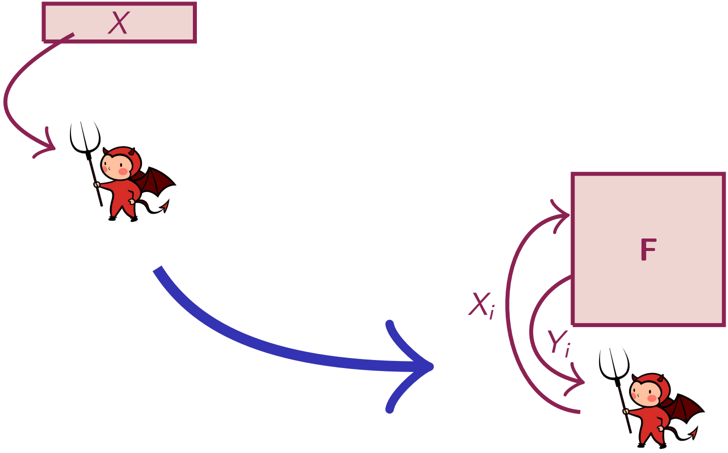
This Paper: Generalization

random variables



interactive systems

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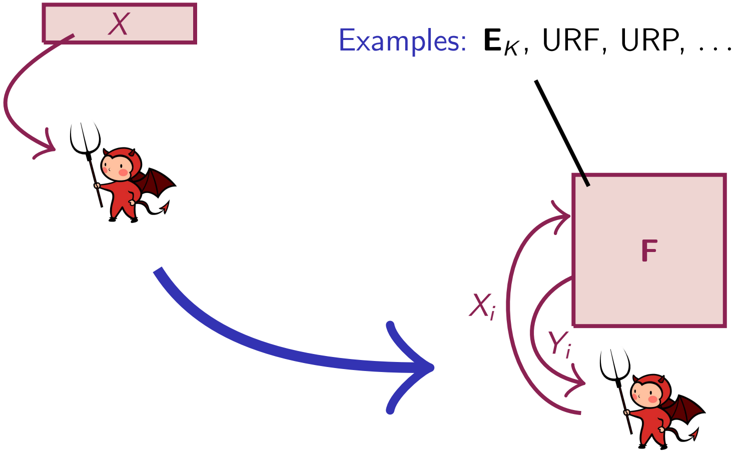

interactive systems

System-Bit Pairs

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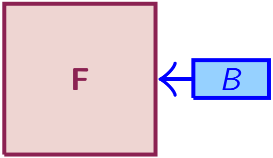

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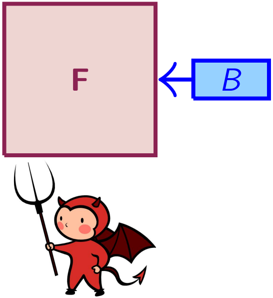

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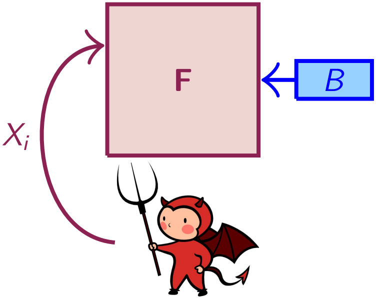

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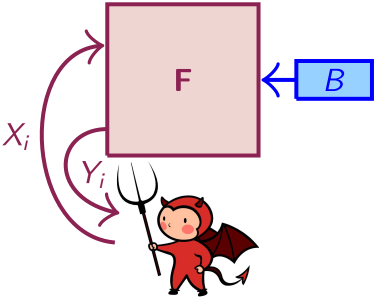

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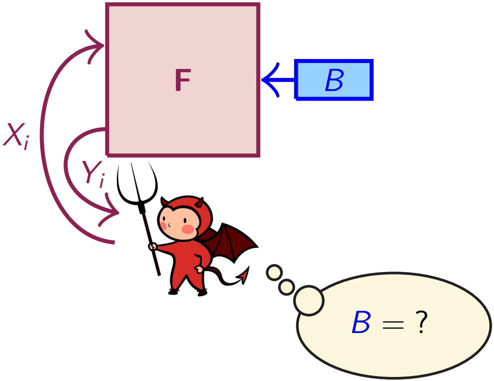

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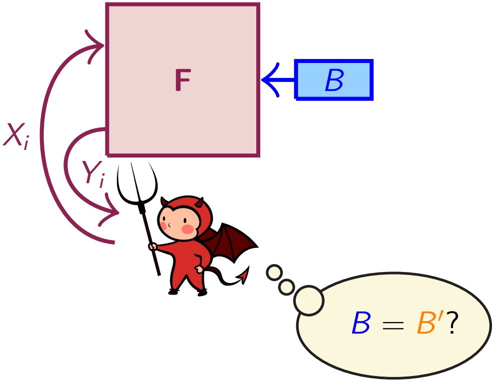

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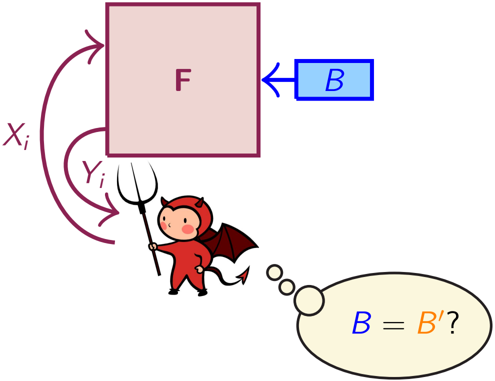

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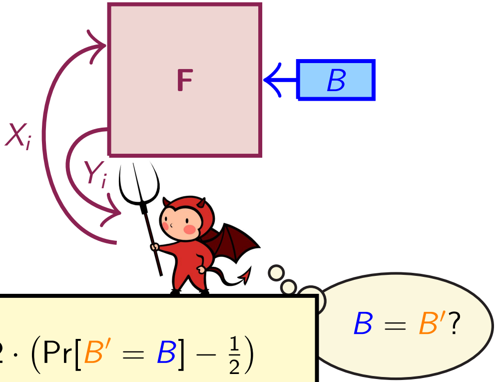

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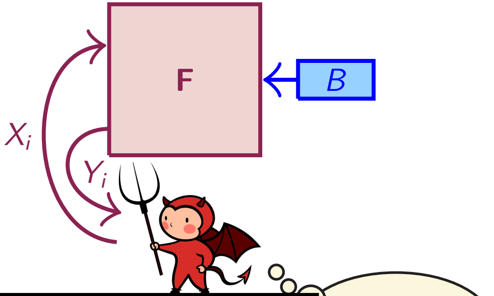
System-Bit Pairs



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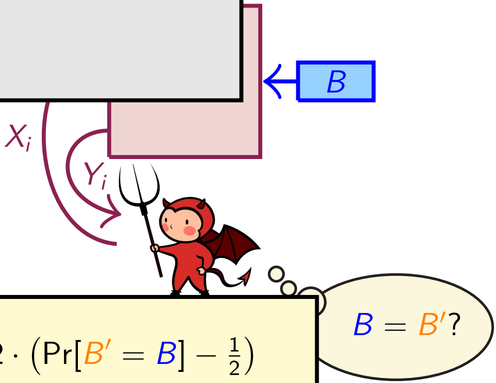
System-Bit Pairs

Example. B unbiased random bit

- $B = 0 \Longrightarrow \mathbf{F} := \mathbf{E}_K$
- $B = 1 \Longrightarrow \mathbf{F} := \mathbf{R}$ URF



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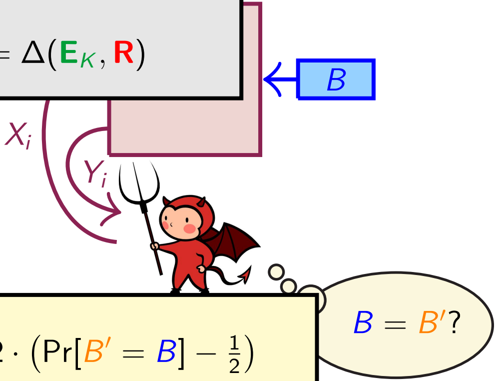
- $B = 1 \implies \mathbf{F} := \mathbf{R}$ URF

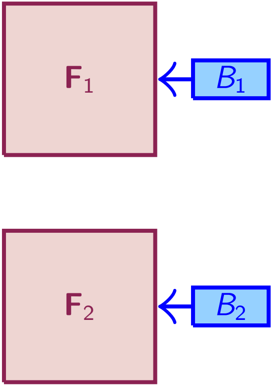
$$\mathbf{Guess}(B \mid \mathbf{F}) = \Delta(\mathbf{E}_K, \mathbf{R})$$



$$\mathbf{Guess}^{\mathbf{A}}(B \mid \mathbf{F}) := 2 \cdot \left(\Pr[B' = B] - \tfrac{1}{2}\right)$$

$$\mathbf{Guess}_{t,q}(B \mid \mathbf{F}) := \max_{\mathbf{A}: t_{\mathbf{A}} \leq t, q_{\mathbf{A}} \leq q} \mathbf{Guess}^{\mathbf{A}}(B \mid \mathbf{F})$$

Generalized XOR Lemma



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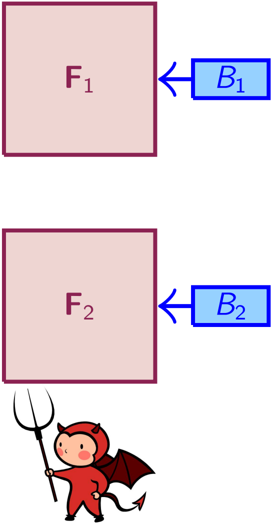

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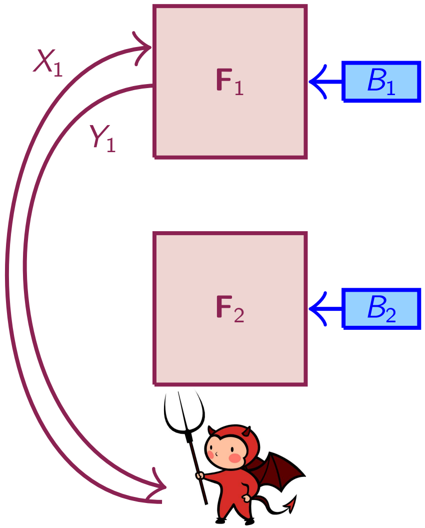

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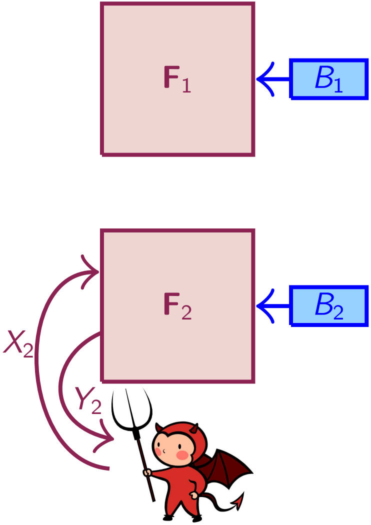

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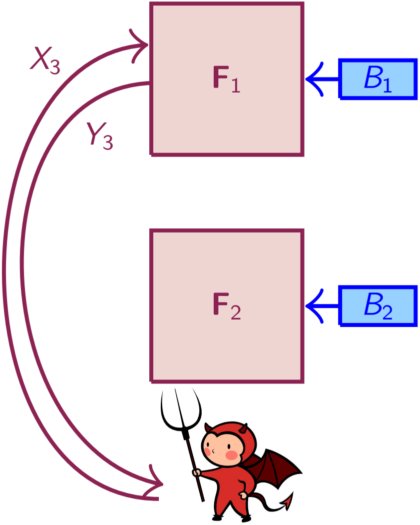

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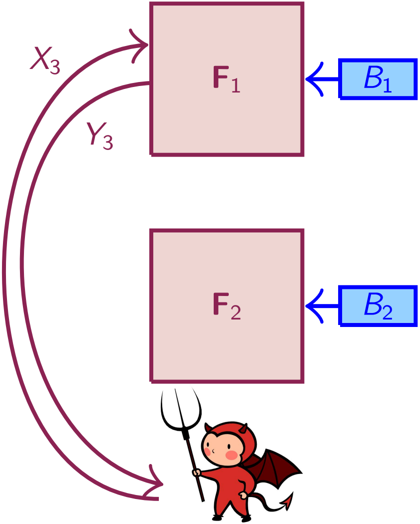

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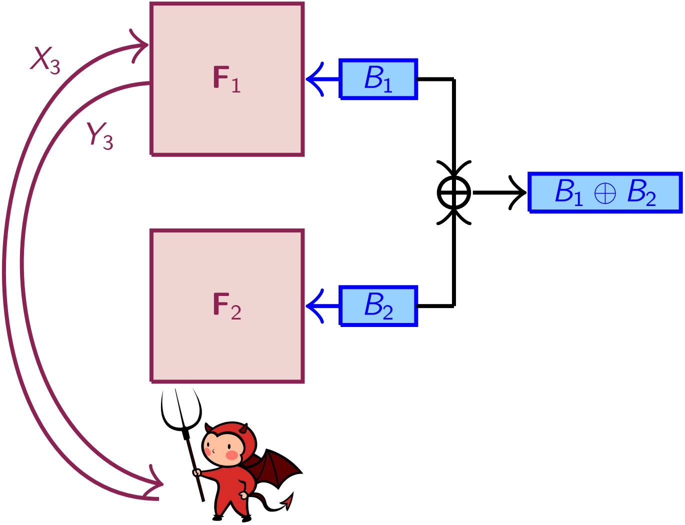

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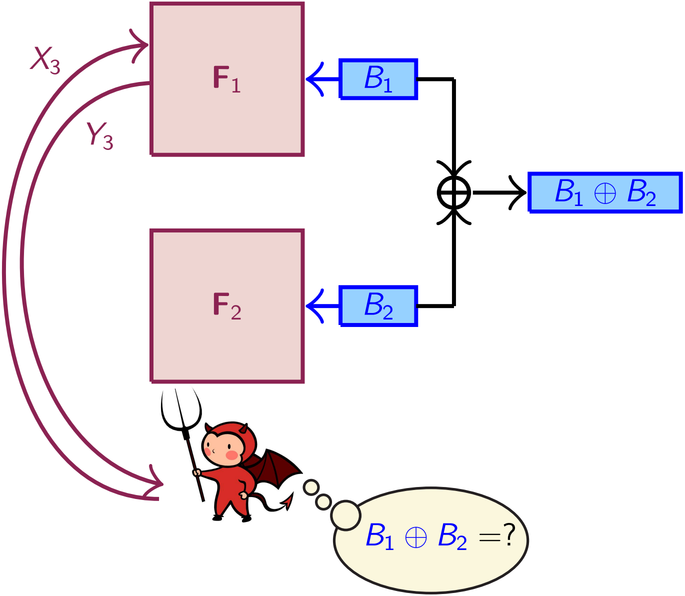

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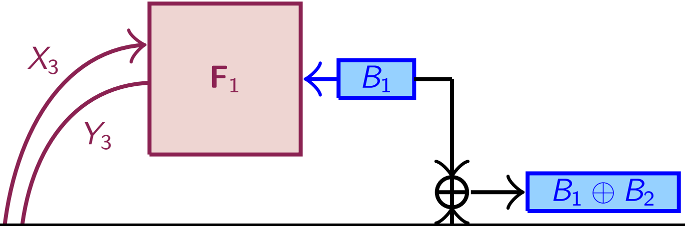

Theorem. $\forall$ cc-stateless $(\mathbf{F}_1, B_1), \ldots, (\mathbf{F}_m, B_m)$, $\forall\ \gamma > 0$

$$\mathbf{Guess}_{t,q}(B_1 \oplus \cdots \oplus B_m \mid \mathbf{F}_1 \| \ldots \| \mathbf{F}_m) \leq \prod_{i=1}^{m} \mathbf{Guess}_{t',q'}(B_i \mid \mathbf{F}_i) + \gamma,$$

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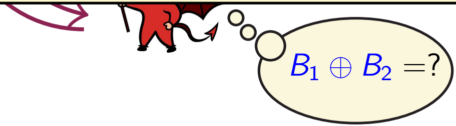

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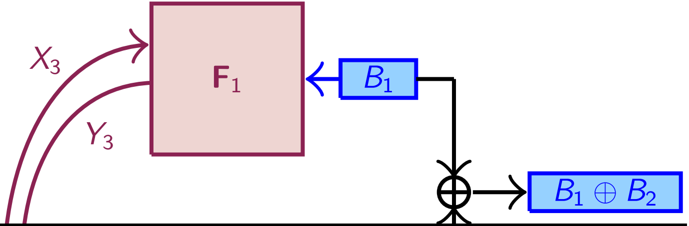

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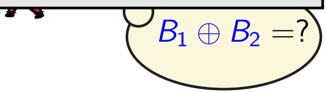
[HR08]: sequential access (not sufficient here)



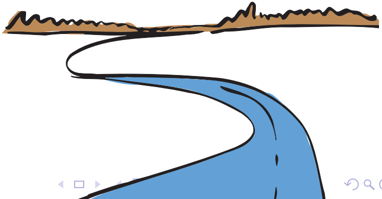
Outline

1. Generalizing Yao's XOR Lemma

2. Neutralizing Constructions

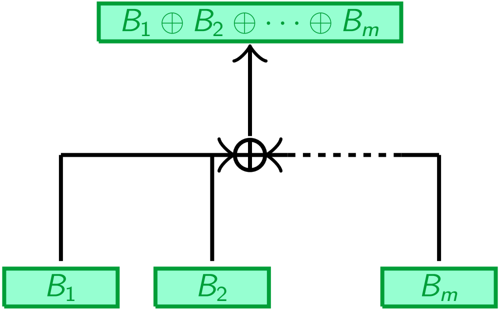
3. Strong Indistinguishability Amplification

4. Concluding Remarks



XOR of Random Bits

$B_1, \ldots, B_m$: independent (biased) random bits



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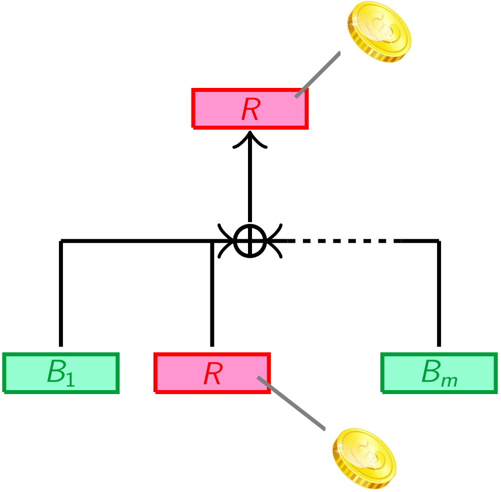
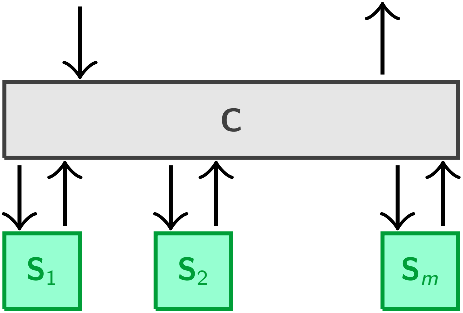


Neutralizing Constructions [MPR07]

$\mathbf{C}(\cdot)$ **neutralizing** for $\mathcal{F}$ and ideal $\mathbf{I}_1, \ldots, \mathbf{I}_m \in \mathcal{F}$

$$\forall \mathbf{S}_1, \ldots, \mathbf{S}_m \in \mathcal{F} : (\exists i : \mathbf{S}_i \equiv \mathbf{I}_i) \implies \mathbf{C}(\mathbf{S}_1, \ldots, \mathbf{S}_m) \equiv \mathbf{C}(\mathbf{I}_1, \ldots, \mathbf{I}_m)$$



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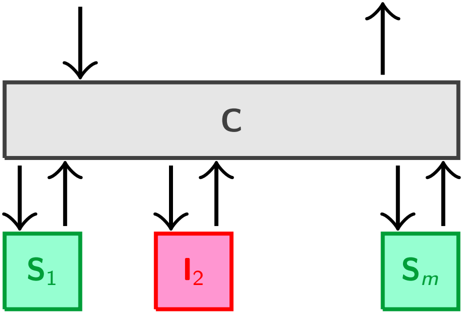

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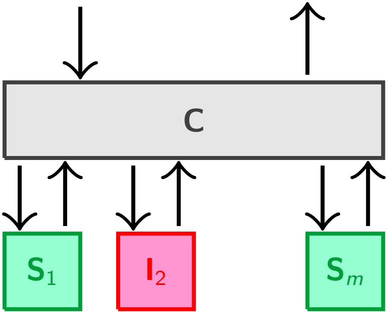

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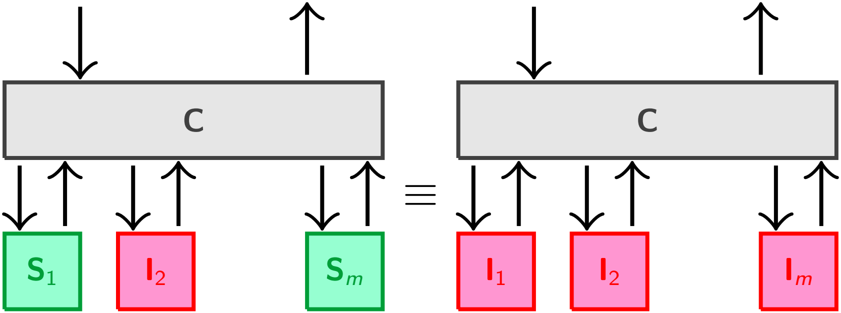

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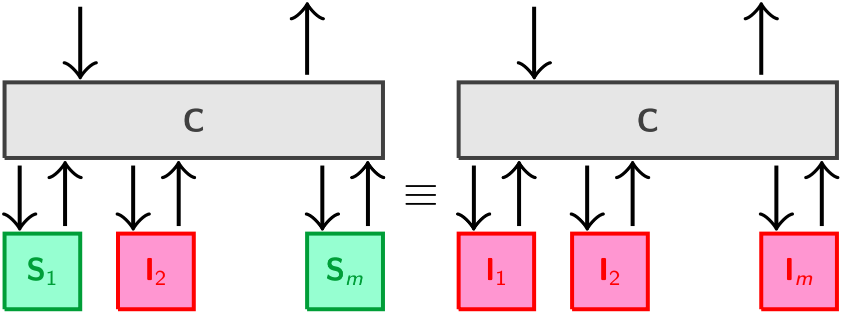

Security Amplification for All Neutralizing Constructions

First Product Theorem

Given: $\mathbf{C}(\cdot)$ neutralizing for $\mathcal{F}$ and cc-stateless $\mathbf{I}_1, \ldots, \mathbf{I}_m$

Then: $\forall$ cc-stateless $\mathbf{F}_1, \ldots, \mathbf{F}_m \in \mathcal{F}$ and $\forall\ \gamma > 0$

$$\Delta_{t,q}(\mathbf{C}(\mathbf{F}_1, \ldots, \mathbf{F}_m), \mathbf{C}(\mathbf{I}_1, \ldots, \mathbf{I}_m))$$

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Remarks

- Security amplification for all combiners!
- Matches tight IT-bounds [MPR07]

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$< \frac{1}{2}$

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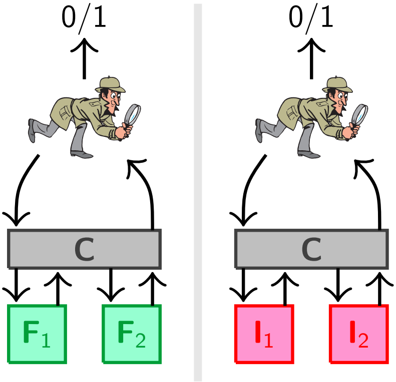
Proof Idea: Reduction to the XOR Lemma



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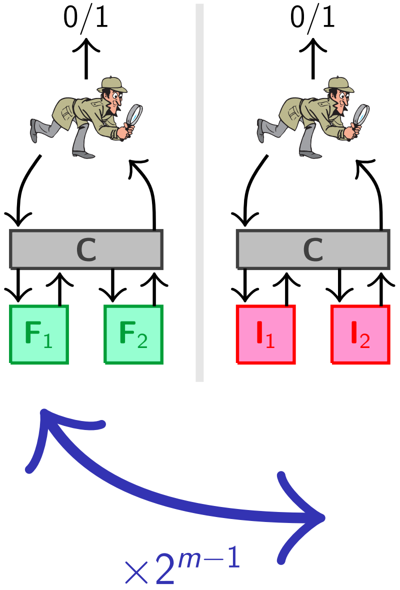

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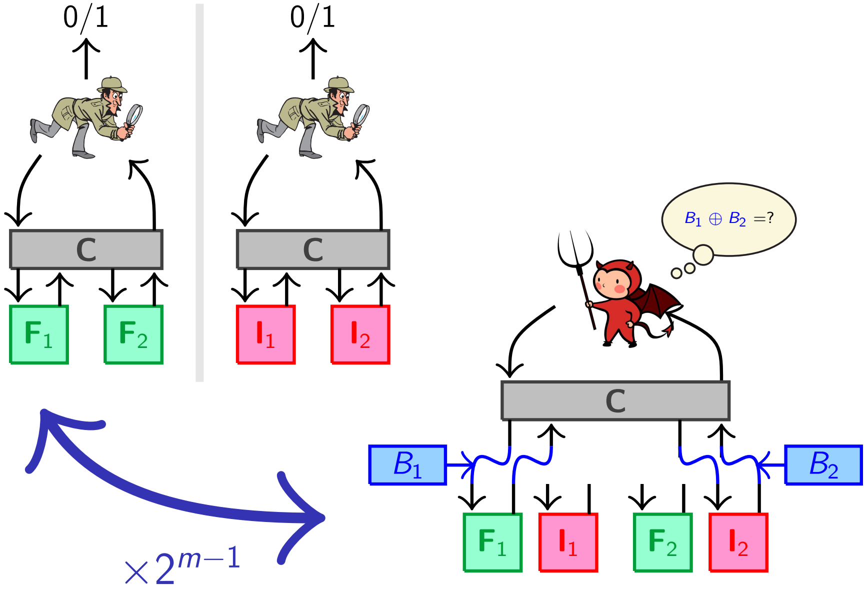

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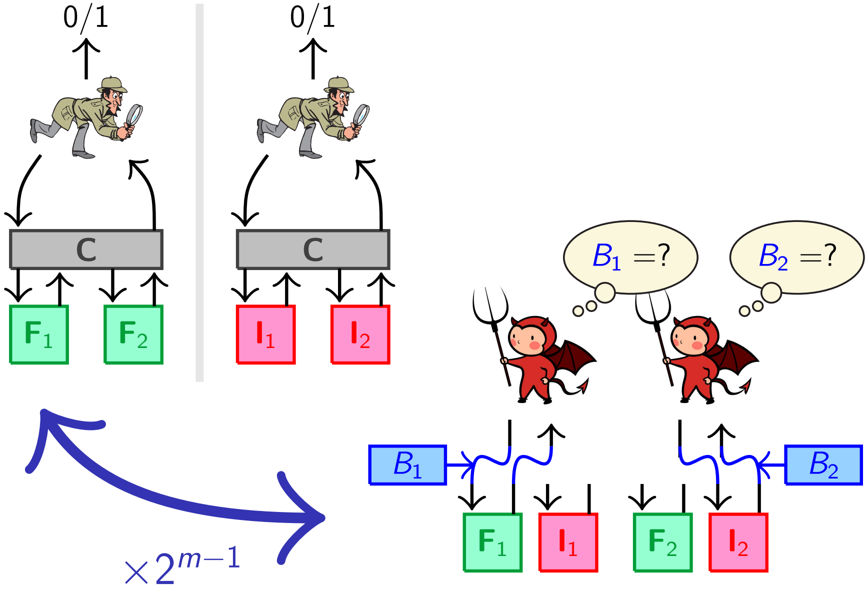
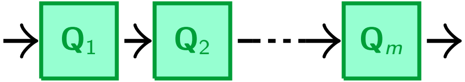


Example – Cascade of PRPs

$\mathbf{Q}_1, \ldots, \mathbf{Q}_m$: permutations $D \longrightarrow D$ (e.g. $\mathbf{E}_K$)



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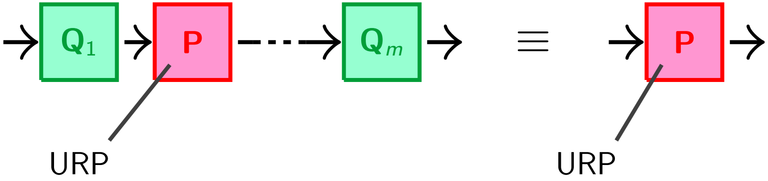

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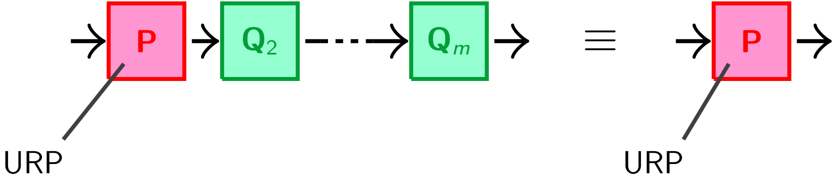

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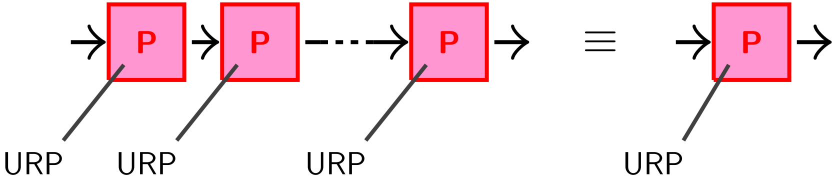

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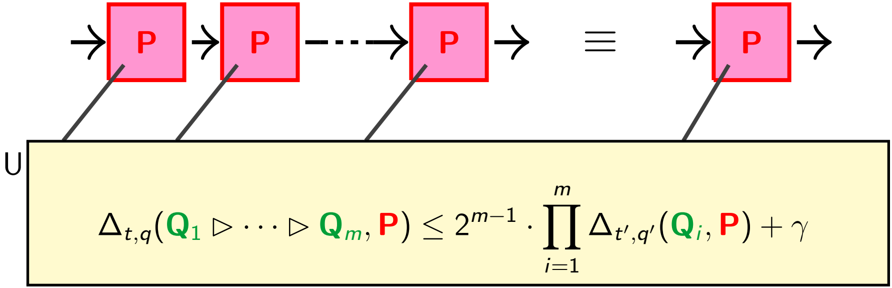

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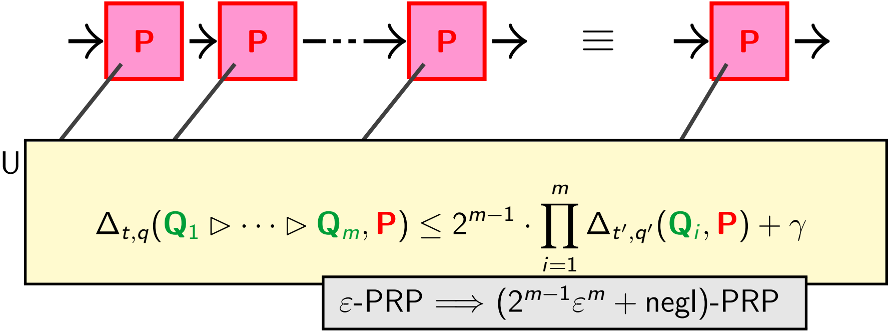

Example – Casca

two-sided

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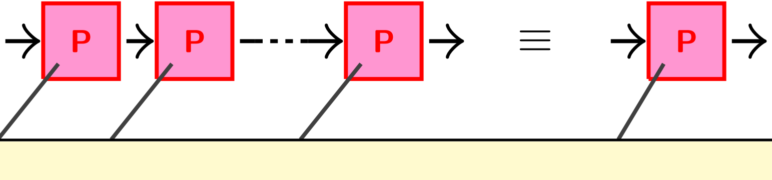

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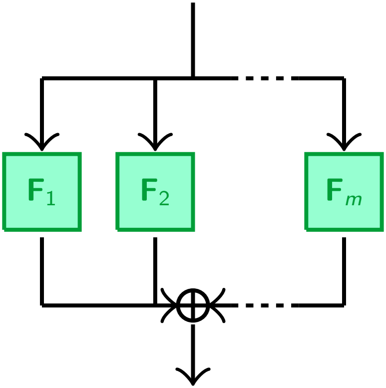

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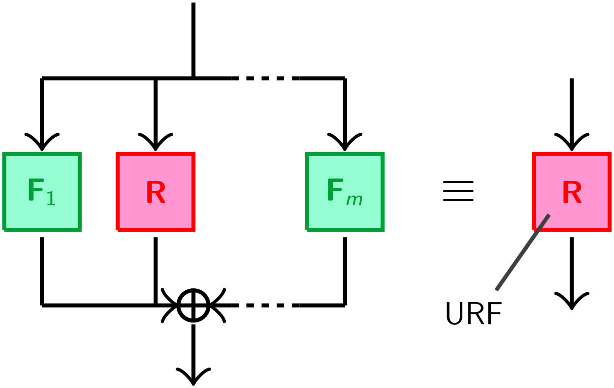

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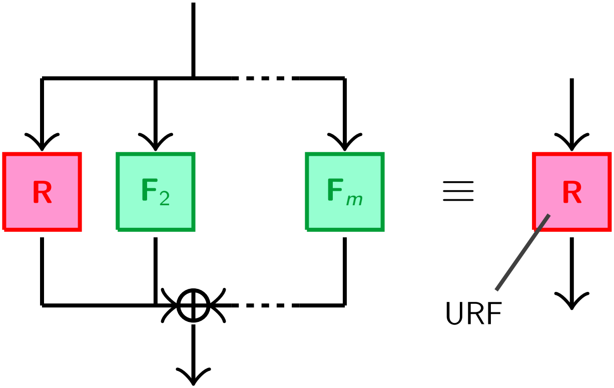

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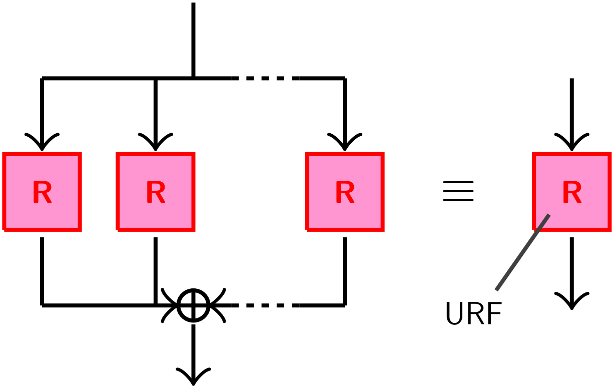

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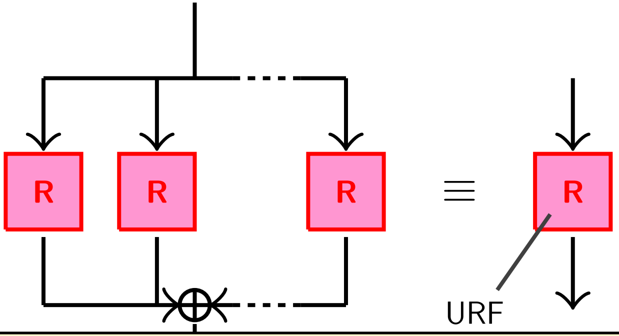

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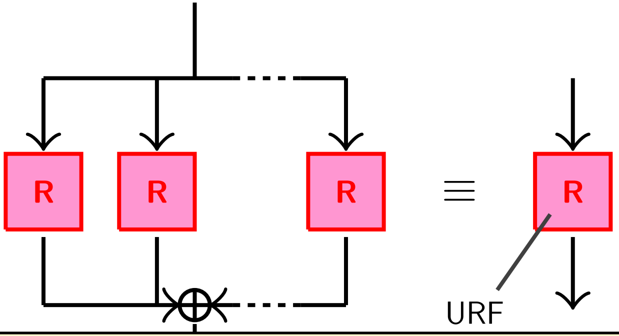

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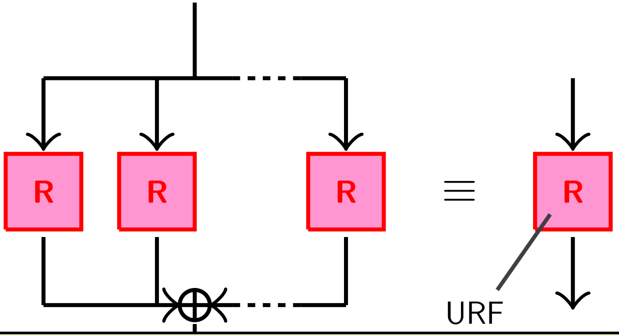

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Improves bounds of [DIJK09]

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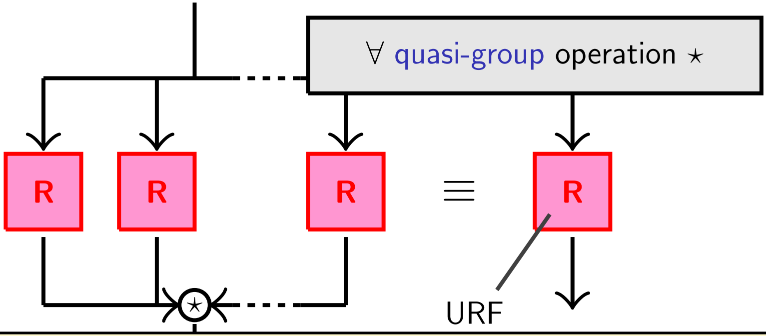

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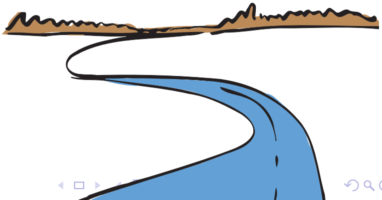

Security Amplification for Neutralizing Constructions

First Product Theorem

Given: $\mathbf{C}(\cdot)$ neutralizing for $\mathcal{F}$ and cc-stateless $\mathbf{I}_1, \ldots, \mathbf{I}_m$

Then: $\forall$ cc-stateless $\mathbf{F}_1, \ldots, \mathbf{F}_m \in \mathcal{F}$ and $\forall\ \gamma > 0$

$$\Delta_{t,q}(\mathbf{C}(\mathbf{F}_1, \ldots, \mathbf{F}_m), \mathbf{C}(\mathbf{I}_1, \ldots, \mathbf{I}_m)) \leq 2^{m-1} \prod_{i=1}^{m} \Delta_{t',q'}(\mathbf{F}_i, \mathbf{I}_i) + \gamma,$$

with $t' = \mathcal{O}(\frac{t}{\gamma^2})$ and $q' = \mathcal{O}(\frac{q}{\gamma^2})$.

$< \frac{1}{2}$

+ η-self independence

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with $t' = \mathcal{O}(\frac{t}{\gamma^2})$ and $q' = \mathcal{O}(\frac{q}{\gamma^2})$.

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Second

~~First~~ Product Theorem

Given: $\mathbf{C}(\cdot)$ neutralizing for $\mathcal{F}$ and cc-stateless $\mathbf{I}_1, \ldots, \mathbf{I}_m$

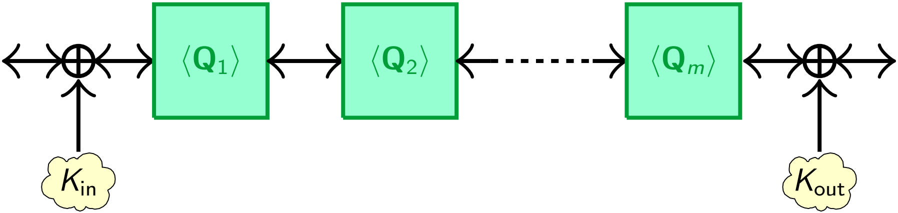
Then: $\forall$ cc-stateless $\mathbf{F}_1, \ldots, \mathbf{F}_m \in \mathcal{F}$ and $\forall\ \gamma > 0$

$$\Delta_{t,q}(\mathbf{C}(\mathbf{F}_1, \ldots, \mathbf{F}_m), \mathbf{C}(\mathbf{I}_1, \ldots, \mathbf{I}_m)) \leq \cancel{2^{m-1}} \prod_{i=1}^{m} \Delta_{t',q'}(\mathbf{F}_i, \mathbf{I}_i) + m \cdot \eta\left(q, \frac{1}{\gamma^2}\right) + \gamma,$$

with $t' = \mathcal{O}(\frac{t}{\gamma^2})$ and $q' = \mathcal{O}(\frac{q}{\gamma^2})$.

Example — Randomized Cascade of ↔PRPs

$\langle \mathbf{Q}_1 \rangle, \ldots, \langle \mathbf{Q}_m \rangle$: two-sided permutations $D \rightarrow D$ (e.g. $\langle \mathbf{E}_K \rangle$)



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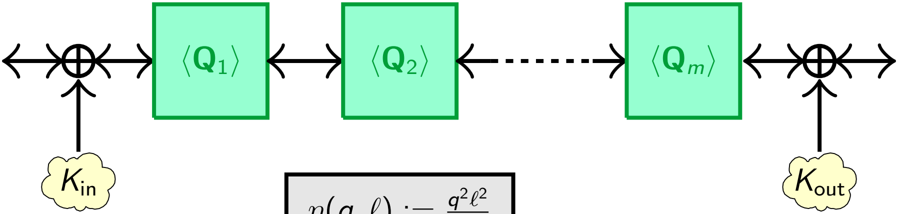

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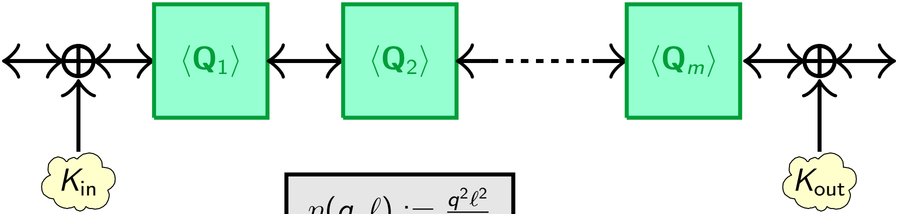

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$$\Delta_{t,q}(\langle \mathbf{Q}_1 \rangle \triangleright \cdots \triangleright \langle \mathbf{Q}_m \rangle, \langle \mathbf{P} \rangle) \leq \prod_{i=1}^{m} \Delta_{t',q'}(\langle \mathbf{Q}_i \rangle, \langle \mathbf{P} \rangle) + \frac{mq^2}{|D|\gamma^4} + \gamma$$

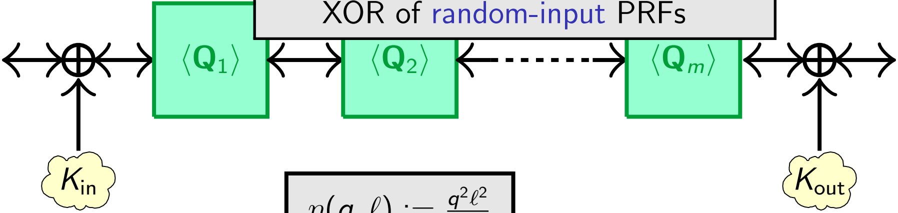
ε-↔PRP $\Longrightarrow$ (ε^m + negl)-↔PRP

Example — Randomized Cascade of ↔PRPs

Further Applications:

- Strong security amplification of PRFs [M03]
- Strong security amplification for XOR of random-input PRFs

$\langle \mathbf{Q}_1 \rangle, \ldots, \langle \mathbf{Q}_m \rangle$: … $\mathbf{E}_K \rangle)$



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ε-↔PRP $\Longrightarrow$ $(\varepsilon^m + \text{negl})$-↔PRP

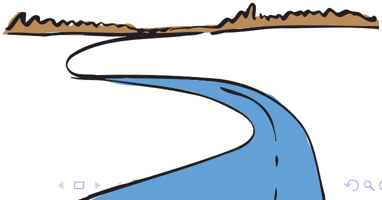
Outline

1. Generalizing Yao's XOR Lemma

2. Neutralizing Constructions

3. Strong Indistinguishability Amplification

4. Concluding Remarks



General Framework

- Improves all existing computational indistinguishability amplification results
- First standard-model analysis of cascaded encryption
- Strong security amplification for PRPs

Open Problems

- Further applications
- Specialized product theorems

Thank you!

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