Constructing MACs using blockciphers that are only secure as MACs

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- Must be resistant to chosen message attack

$$f_K(x_1) \quad f_K(x_2) \quad \cdots \quad f_K(x_q)$$

$$A$$

$$\longrightarrow (x_{q+1}, y) \text{ s.t. } f_K(x_{q+1}) = y$$

Blockcipher as a MAC

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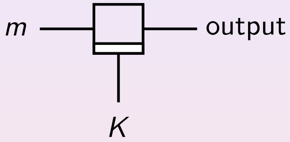

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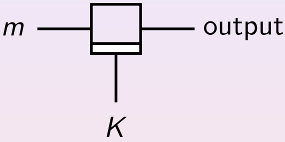

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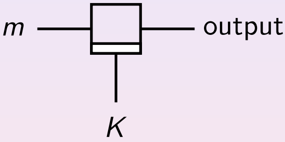

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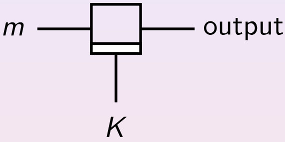

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- Blockciphers typically modeled as PRP's, much stronger than being a MAC
- MACs only need to be unpredictable and not pseudorandom

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Result: A rate 3 variable input length MAC function whose security is at most $q^2 \log^2(q)$ worse than the MAC security of the underlying blockcipher.

Previous Constructions

- Many constructions secure under pseudorandomness of f fail (in general) assuming only unpredictability.
  - Includes CBC-MAC [AB99], HMAC, hash-then-mac using universal hash functions, contant-round Feistel Network [AB99,DP07],...

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- Previous Best: The rate 2 "enciphered CBC MAC" of Dodis, Pietrzak and Puniya (Eurocrypt 08) whose security is q^4 worse than the MAC security of the underlying blockcipher (q is number of queries).

Additional Features

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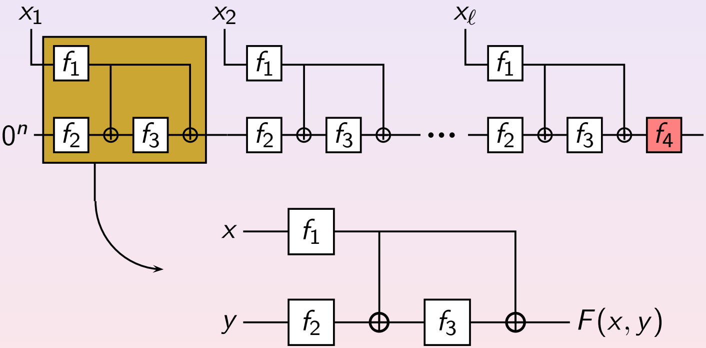
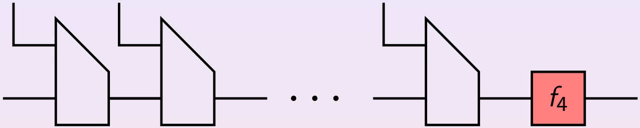
(2) Still indistinguishable from PRF even when the adversary is allowed to make "transcript queries" showing all blockcipher query data $\Longrightarrow$ the hash function can have a completely leaky implementation as long as the blockcipher keys aren't leaked

Our Construction



Shrimpton-Stam '07

Proof Framework (An and Bellare 99)



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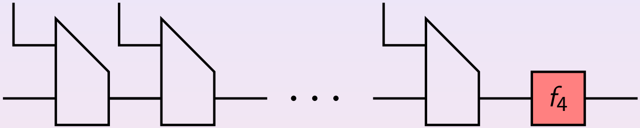

- To forge, the adversary must either find an internal collision or forge f_4.
- Main task is therefore to bound the collision resistance of the compression function using only the MAC security of the underlying blockcipher

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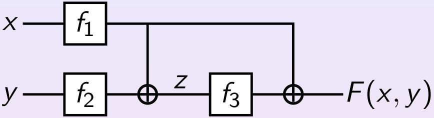
- Comparison to SS'09: information-theoretic argument assuming perfectly random f_i's versus a computational reduction from one type of adversary to another.

Proof of Main Theorem



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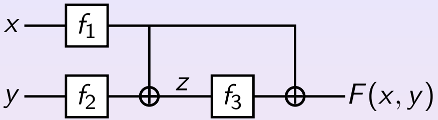

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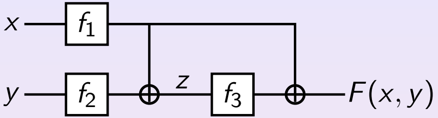

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Definition: For each $z \in \{0,1\}^n$ let $\mathrm{Pairs}(z) = \{(x, y)$ s.t. $f_1(x) \oplus f_2(y) = z$, A has made the queries $f_1(x), f_2(y)\}$.

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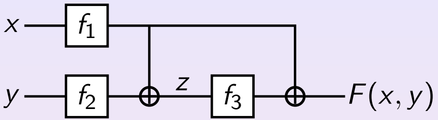

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Observation: If f_1, f_2 are behaving randomly then (i) $C := \max_z |\text{Pairs}(z)|$ is small, (ii) with each query $f_3(z)$, A learns at most $|\text{Pairs}(z)| \leq C$ new values $F(x, y)$, (iii) A learns at most Cq values $F(x, y)$ total.

Proof of Main Theorem

Strategy 1: If f_1, f_2 are behaving randomly then B can guess the answer to a query $f_3(z)$ by guessing that $F(x, y) = F(x', y')$ for some $(x, y) \in \mathrm{Pairs}(z)$ and some (x', y') for which $F(x', y')$ is already known. More precisely, since $F(x, y) = f_1(x) \oplus f_3(z)$, guess $f_3(z) = f_1(x) \oplus f_1(x') \oplus f_3(z')$.

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Strategy 2: If f_1, f_2 are not behaving randomly and $|\text{Pairs}(z)| > \log(q)$ for some z, then use the non-randomness of f_1, f_2 to forge either f_1 or f_2.

Exploiting the non-randomness of f_1, f_2

- Will display a strategy for B that forges f_1 or f_2 with probability $1/4q^2$ whenever $|\mathrm{Pairs}(z)| > \log(q)$ for some z.

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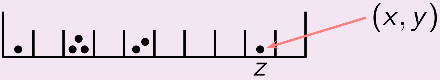

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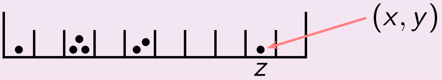

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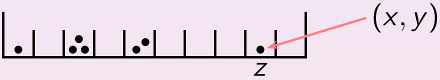

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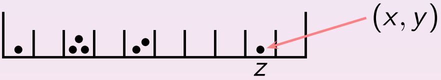

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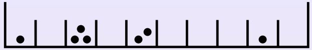
- Can reduce to the case where the balls are thrown one by one.

Balls-in-Bins Game



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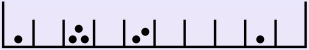

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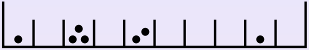

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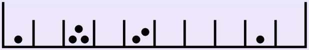

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- Let c_j = total number of balls thrown into bins with $\geq j$ balls in them already. Then for fixed t, B chance's of winning is $\geq \frac{c_t}{q^2} \frac{1}{c_{t-1}} = \frac{1}{q^2} \frac{c_t}{c_{t-1}}$.

Cute Computation

B's chance of winning is

$$
\begin{aligned}
\sum_{t=1}^{\log(q)} \frac{1}{\log(q)} \frac{1}{q^2} \frac{c_t}{c_{t-1}} &= \frac{1}{q^2} \text{ ArithmeticMean} \left(\frac{c_1}{c_0}, \ldots, \frac{c_{\log(q)}}{c_{\log(q)-1}} \right) \\
&\geq \frac{1}{q^2} \text{ GeometricMean} \left(\frac{c_1}{c_0}, \ldots, \frac{c_{\log(q)}}{c_{\log(q)-1}} \right) \\
&= \frac{1}{q^2} \left(\frac{c_{\log(q)}}{c_0} \right)^{\frac{1}{\log(q)}} \geq \frac{1}{q^2} \left(\frac{1}{q^2} \right)^{\frac{1}{\log(q)}} \\
&= \frac{1}{q^2} \frac{1}{2^{\log(q^2)\frac{1}{\log(q)}}} \\
&= \frac{1}{4q^2}
\end{aligned}
$$

QED.

Theorem (Take-Away Fact)

If Q objects are sequentially placed into infinitely many slots such that some slot accumulates more than $\log(Q)$ objects by the end of the process, it is possible to forecast the position of one of the objects with probability at least $1/Q$.

Open Questions/Remarks

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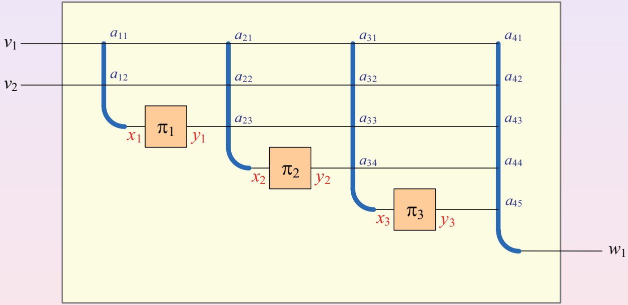

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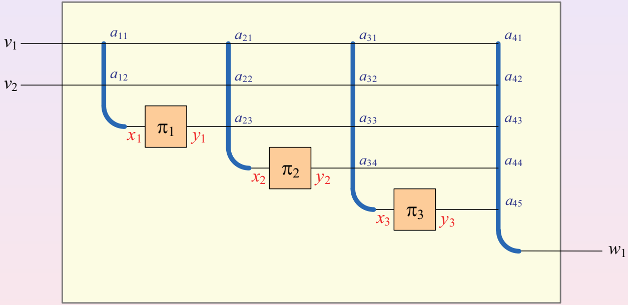

- Open question: going beyond birthday security