New Birthday Attacks on Some MACs Based on Block Ciphers

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Joint work with Zheng Yuan, Keting Jia, Guangwu Xu, and Xiaoyun Wang

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Outline

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Main Results

Inner near-collision with some specific differences

- Part I – Distinguishing and forgery attack on ALRED and its AES-based instance ALPHA-MAC
  - Joint work with Zheng Yuan, Keting Jia, and Xiaoyun Wang
  - Distinguishing and forgery attack on ALRED construction
  - Internal state recovery attack on ALPHA-MAC
- Part II – Impossible differential cryptanalysis of PELICAN, MT-MAC-AES and PC-MAC-AES
  - Joint work with Xiaoyun Wang and Guangwu Xu
  - The first impossible differential attack on MACs
  - Recover the internal state of PELICAN, a subkey of MT-MAC-AES, and two 128-bit key of PC-MAC-AES

Outline

1. Main Results
2. Brief Introduction of MAC Algorithms
3. Related Works on Cryptanalysis of MACs
   - A General Forgery Attack on Iterated MACs
   - Distinguishing Attack on HMAC/NMAC-MD5
4. Our Works
   - Distinguishing and Forgery Attacks on ALRED and Its AES-based Instance ALPHA-MAC
   - Impossible Differential Cryptanalysis of PELICAN, MT-MAC-AES and PC-MAC-AES

Message Authentication Code

Secret Key $+$ Message $\Rightarrow$ $\boxed{\text{MAC Algorithm}}$ $\Rightarrow$ Tag (MAC)

Applications:

- Guarantee data integrity and data origin authentication
- Internet security: IPsec, SSL, SSH, SNMP, etc
- Finance: Banking, electronic purses, etc

Constructions:

- Based on hash function with secret key, e.g., HMAC
- Based on block cipher, e.g., CBC-MAC
  Block cipher and reduced block cipher, e.g., PELICAN
- Based on universal hash function, e.g., Wegman-Carter MAC

MAC Security

Three kinds of attacks:

- Distinguishing Attack
  - Distinguishing-R Attack
  - Distinguishing-H Attack
- Forgery Attack
  - Existential Forgery:
    For a new message M, compute a valid MAC
  - Selective Forgery: The adversary can select a message M, and compute $M' \neq M$ with $MAC_K(M') = MAC_K(M)$
  - Universal Forgery:
    For any given message M, compute a valid MAC
- Key Recovery Attack

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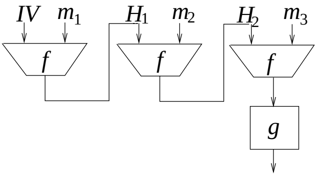
A General Forgery Attack on Iterated MACs

- Preneel and van Oorschot, Crypto'95
- Applicable to all iterated MACs based on both hash functions and block ciphers



f: compression function

g: output transformation

$\boxed{\text{Detect the } \textit{internal collision}}$

$2^{(n+1)/2}$ randomly chosen M_i

$\xrightarrow{\text{birthday paradox}} \exists(M_j, M_k)$ collide

$\xrightarrow{\text{query with}(M_j||P, M_k||P)}$ if still collide

$\rightarrow$ internal collision

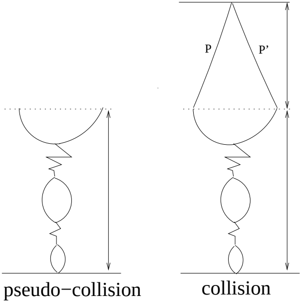
$\rightarrow MAC_K(M_j||Q) = MAC_K(M_k||Q)$

Outline

Distinguishing Attack on HMAC/NMAC-MD5

- Wang et al., EuroCrypt'09
- The first attack on HMAC/NMAC-MD5 without related-key

Detect the *inner near-collision with some specific difference*



- A pseudo-collision differential path of MD5 with prob. 2^{-46} (den Boer and Bosselaers, EuroCrypt'93)
- 2^{66} randomly chosen P $\xrightarrow{\text{birthday paradox}} \exists(P, P')$ s. t. conditions on IV
- $\xrightarrow{\text{Query with } 2^{47} (P||M, P'||M)}$ internal collision/ dBB collision/ others
- Partial key recovery attack on MD5-MAC

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ALRED Construction

- Daemen and Rijmen, FSE 2005
- For a message $M = (x_1, x_2, \cdots, x_t)$,
  1. Apply the block cipher to the state of all-zero block

     $$y_0 = \mathrm{Enc}_k(0)$$

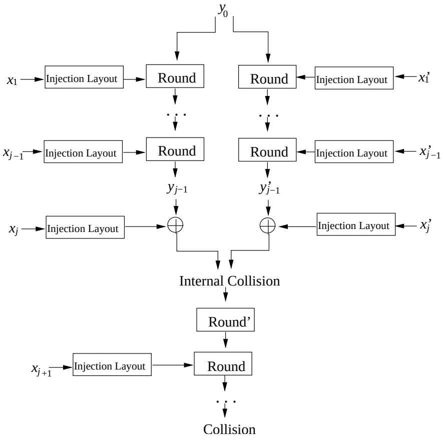
  2. Perform an iteration for each message word x_i

     $$y_i = \mathrm{ReducedEnc}_{x_i}(y_{i-1}), i = 1, 2, \cdots, t$$

  3. Apply the block cipher to the state again, and truncate the first l_m bits of the state as the output

     $$C = Trunc(\mathrm{Enc}_k(y_t))$$

Distinguishing Attack on ALRED Construction



- $2^{(n+1)/2}$ randomly chosen M^i
- $\xrightarrow{\text{birthday paradox}}$

  $\exists(M^a, M^b)$ collide, where

  $$M^a = (x_1^a, \ldots, x_j^a, x_{j+1}, \ldots, x_t)$$
  $$M^b = (x_1^b, \ldots, x_j^b, x_{j+1}, \ldots, x_t)$$
- Query the MAC with

  $$\overline{M^a} = (x_1^a, \ldots, x_{j-1}^a, \overline{x_j^a}),$$
  $$\overline{M^b} = (x_1^b, \ldots, x_{j-1}^b, \overline{x_j^b}),$$

  where $\overline{x_j^a} \oplus \overline{x_j^b} = x_j^a \oplus x_j^b$

  – Collide $\Rightarrow$ ALRED

  – Else $\Rightarrow$ a random function

$\boxed{\text{Collision} \Rightarrow \text{Internal collision} \Rightarrow \textit{Inner near-collision with } \Delta y_{j-1} = \Delta InLayout(x_j)}$

Forgery Attack on ALRED Construction

Obtain a colliding pair (M^a, M^b), where $M^a = (x_1^a, \ldots, x_{j-1}^a, x_j^a)$, $M^b = (x_1^b, \ldots, x_{j-1}^b, x_j^b)$, and $\Delta y_{j-1} = \Delta x_j$

1. Query the MAC oracle with $\widetilde{M^a} = (x_1^a, \ldots, x_{j-1}^a, \widetilde{x_j^a}, s)$, where s is an arbitrary message string
2. Get the forgery of $\widetilde{M^b} = (x_1^b, \ldots, x_{j-1}^b, \widetilde{x_j^a} \oplus \Delta x_j, s)$

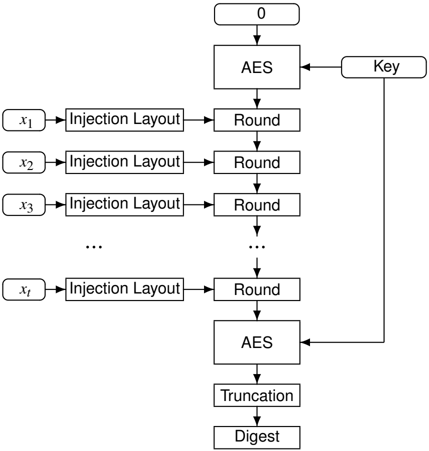
Work for:
MACs based on block ciphers, e.g., CBC-MAC, OMAC, TMAC

$$H_i = f(H_{i-1}, x_i) = E_k(H_{i-1} \oplus x_i)$$

MACs based on CFB mode

$$H_i = f(H_{i-1}, x_i) = E_k(H_{i-1}) \oplus x_i$$

AES Based Instance: ALPHA-MAC



- Round: 1-round AES
- Round: AK, SB, SR, MC
- Message $M = (x_1, x_2, \cdots, x_t)$
- 32-bit word $x_i = (x_{i,0}, x_{i,1}, x_{i,2}, x_{i,3})$
- Injection layout $(x_i) =$

$$\begin{pmatrix} x_{i,0} & 0 & x_{i,1} & 0 \\ 0 & 0 & 0 & 0 \\ x_{i,2} & 0 & x_{i,3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Other Attacks on ALPHA-MAC

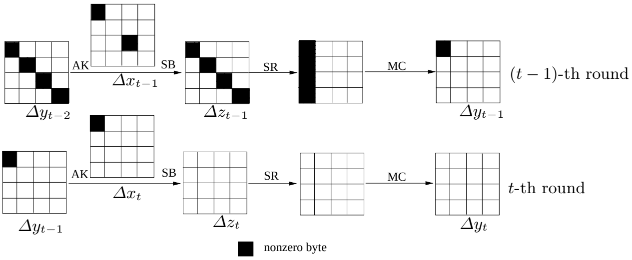
- Huang et al. exploited the algebraic properties of the AES, and applied a selective forgery attack on ALPHA-MAC, on the assumption that a key or an internal state is known
- Biryukov et al. proposed a side-channel collision attack on ALPHA-MAC recovering its internal state, and mounted a selective forgery attack

All forgery attacks are based on the recovery of the internal state

Distinguishing Attack on ALPHA-MAC I

Fact

Given two messages $M = (x_1, \ldots, x_{t-1}, x_t)$ and $M' = (x'_1, \ldots, x'_{t-1}, x'_t)$ following the 2-round collision differential path, there exists an algorithm to find another message pair $\overline{M} = (x_1, \ldots, \overline{x_{t-1}}, x_t)$ and $\overline{M'} = (x'_1, \ldots, \overline{x'_{t-1}}, x'_t)$ satisfying the 2-round collision differential path with 2^9 queries and 2^9 chosen messages.



Distinguishing Attack on ALPHA-MAC II

1. Construct two structures, each with $2^{65.5}$ texts, where $\Delta x_{t-1}, \Delta x_t$ as shown in the above collision path
   $T_1 = \{M^a = (x_1^a, x_2^a, \ldots, x_{t-1}^a, x_t)\}$
   $T_2 = \{M^b = (x_1^b, x_2^b, \ldots, x_{t-1}^b, x_t \oplus (\eta, 0, 0, 0))\}$
2. Search for (M^a, M^b) s.t. $C^a = C^b$ by the birthday attack. Query the MAC with the new message pair $(\overline{M^a}, \overline{M^b})$, where $\overline{M^a} = (x_1^a, \ldots, x_{t-1}^a, \overline{x_t^a})$, $\overline{M^b} = (x_1^b, \ldots, x_{t-1}^b, \overline{x_t^b})$, $\Delta \overline{x_t} = \Delta x_t$
   - If collide $\Rightarrow$ ALRED-MAC, and goto step 3
   - Otherwise $\Rightarrow$ a random function
3. Randomly choose 2^8 different $(\overline{x_{t-1,0}^a}, \overline{x_{t-1,0}^b})$ to replace $(x_{t-1,0}^a, x_{t-1,0}^b)$. Query the MACs of the new messages.
   - If a collision appears $\Rightarrow$ ALPHA-MAC
   - Otherwise $\Rightarrow$ a random function

Internal State Recovery of ALPHA-MAC I

$$y_0 \longleftarrow \cdots \longleftarrow \Delta y_{t-3} = \begin{pmatrix} * & ? & * & ? \\ ? & * & ? & * \\ * & ? & * & ? \\ ? & * & ? & * \end{pmatrix}$$

$$\xleftarrow{AK^{-1}\ SB^{-1}} \Delta z_{t-2} = \begin{pmatrix} * & ? & * & ? \\ ? & * & ? & * \\ * & ? & * & ? \\ ? & * & ? & * \end{pmatrix} \xleftarrow{SR^{-1}} \begin{pmatrix} * & ? & * & ? \\ * & ? & * & ? \\ * & ? & * & ? \\ * & ? & * & ? \end{pmatrix} \xleftarrow{MC^{-1}} \Delta y_{t-2} = \begin{pmatrix} * & 0 & 0 & 0 \\ 0 & ? & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & ? \end{pmatrix}$$

$$\xleftarrow{AK^{-1}\ SB^{-1}} \Delta z_{t-1} = \begin{pmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{pmatrix} \xleftarrow{SR^{-1}} \begin{pmatrix} * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & 0 & 0 & 0 \end{pmatrix} \xleftarrow{MC^{-1}} \Delta y_{t-1} = \begin{pmatrix} \Delta x_{t,0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Internal State Recovery of ALPHA-MAC II

1. Recover $(y^a_{t-2,0}, y^b_{t-2,0}, y^a_{t-2,10}, y^b_{t-2,10})$

$$\begin{cases} S(y^a_{t-2,0} \oplus x^a_{t-1,0}) \oplus S(y^b_{t-2,0} \oplus \underline{x^b_{t-1,0}}) = \Delta z_{t-1,0} \\ S(y^a_{t-2,0} \oplus \overline{x^a_{t-1,0}}) \oplus S(y^b_{t-2,0} \oplus x^b_{t-1,0}) = \Delta z_{t-1,0} \end{cases}$$

$$\Delta y_{t-2} = \begin{pmatrix} * & 0 & 0 & 0 \\ 0 & ? & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & ? \end{pmatrix} \xleftarrow{AK^{-1}\ SB^{-1}}$$

$$\Delta z_{t-1} = \begin{pmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{pmatrix} \xleftarrow{SR^{-1}\ MC^{-1}} \Delta y_{t-1} = \begin{pmatrix} \Delta x_{t,0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Internal State Recovery of ALPHA-MAC III

2. Recover $(y^a_{t-3,0}, y^b_{t-3,0}, y^a_{t-3,2}, y^b_{t-3,2}, y^a_{t-3,8}, y^b_{t-3,8}, y^a_{t-3,10}, y^b_{t-3,10})$
   Making use of the property of the S-box and MixColumn
   $S(y_{t-3,0} \oplus x^a_{t-2,0}) \oplus S(y'_{t-3,0} \oplus x^b_{t-2,0}) = \Delta z_{t-2,0} \Rightarrow 2^8$ candidates
   –Right candidate can lead to a collision with prob. 2^{-8}
   –Wrong candidates produce a collision with prob. $\leq 2^{-16}$

$$\Delta y_{t-3} = \begin{pmatrix} \boxed{*} & ? & * & ? \\ ? & * & ? & * \\ * & ? & * & ? \\ ? & * & ? & * \end{pmatrix}$$

$$\xleftarrow{AK^{-1}\ SB^{-1}} \Delta z_{t-2} = \begin{pmatrix} \boxed{*} & ? & * & ? \\ ? & * & ? & * \\ * & ? & * & ? \\ ? & * & ? & * \end{pmatrix} \xleftarrow{SR^{-1}\ MC^{-1}} \Delta y_{t-2} = \begin{pmatrix} \boxed{*} & 0 & 0 & 0 \\ \boxed{0} & ? & 0 & 0 \\ \boxed{0} & 0 & * & 0 \\ \boxed{0} & 0 & 0 & ? \end{pmatrix}$$

$$\xleftarrow{AK^{-1}\ SB^{-1}} \Delta z_{t-1} = \begin{pmatrix} \boxed{*} & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{pmatrix} \xleftarrow{SR^{-1}\ MC^{-1}} \Delta y_{t-1} = \begin{pmatrix} \Delta x_{t,0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Internal State Recovery of ALPHA-MAC IV

3. Recover $(y^a_{t-3,5}, y^b_{t-3,5}, y^a_{t-3,7}, y^b_{t-3,7}, y^a_{t-3,13}, y^b_{t-3,13}, y^a_{t-3,15}, y^b_{t-3,15})$
Making use of some equations derived from the difference and recovered bytes, such as

$$\begin{cases} \Delta z_{t-2,5} & = & S(y^a_{t-3,5}) \oplus S(y^b_{t-3,5}) \\ \Delta z_{t-2,15} & = & S(y^a_{t-3,15}) \oplus S(y^b_{t-3,15}) \\ y^a_{t-2,0} & = & 3S(y^a_{t-3,0} \oplus x^a_{t-2,0}) \oplus 2S(y^a_{t-3,5}) \oplus S(y^a_{t-3,10} \oplus x^a_{t-2,3}) \oplus S(y^a_{t-3,15}) \\ y^b_{t-2,0} & = & 3S(y^b_{t-3,0} \oplus x^b_{t-2,0}) \oplus 2S(y^b_{t-3,5}) \oplus S(y^b_{t-3,10} \oplus x^b_{t-2,3}) \oplus S(y^b_{t-3,15}) \end{cases}$$

$$\Delta y_{t-3} = \begin{pmatrix} * & ? & * & ? \\ ? & \boxed{*} & ? & * \\ * & ? & * & ? \\ ? & * & ? & \boxed{*} \end{pmatrix} \xleftarrow{AK^{-1}\ SB^{-1}} \Delta z_{t-2} = \begin{pmatrix} * & ? & * & ? \\ ? & \boxed{*} & ? & * \\ * & ? & * & ? \\ ? & * & ? & \boxed{*} \end{pmatrix}$$

Internal State Recovery of ALPHA-MAC V

4. Recover the internal state y_0
Guess all the 2^{64} possibilities of the rest 8 bytes of y^a_{t-3}

$$y_{t-3} \xrightarrow[decryption]{(x_{t-3},\cdots,x_1)} y_0 \xrightarrow[encryption]{(x'_1,\cdots,x'_{t-3})} \overline{y_{t-3}} \begin{cases} \overline{y_{t-3}} = y'_{t-3} & \text{right} \\ \text{Else} & \text{wrong} \end{cases}$$

Second Preimages for ALPHA-MAC

Once the internal state y_0 is recovered, the selective forgery attacks can be performed by Huang et al. or Biryukov et al.'s attacks

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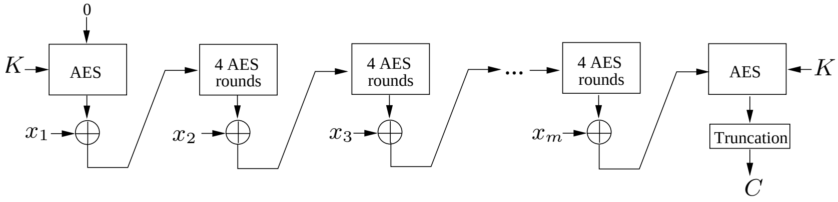
PELICAN Algorithm I

- Daemen and Rijmen, 2005
- An optimized version of ALPHA-MAC
- x_i: 128-bit message word
- Round function: 4-round AES with round subkeys set to 0



PELICAN Algorithm II

For a message $M = (x_1, x_2, \ldots, x_b)$

1. Initialization: $y_0 = E_K(0)$
   where E is the AES, and K is the secret key

2. Chaining:
   1. $y_1 = y_0 \oplus x_1$
   2. For each message word x_i $(i = 2, \ldots, b)$, perform an iteration operation:
      $y_i = f(y_{i-1}) \oplus x_i$
      where f consists of 4-round AES with the round subkeys set to 0

3. Finalization: $C = Trunc(E_K(y_m))$

Main Idea of the Impossible Differential Cryptanalysis

1. Find an impossible differential path
   - For AES, several 4-round impossible differential paths have been found in literature
   - For PELICAN, a 3-round impossible differential path is OK
2. Collect and sieve the message pairs with the required differences (obstacle)
   - For block ciphers, sieve directly
   - For PELICAN, need new techniques

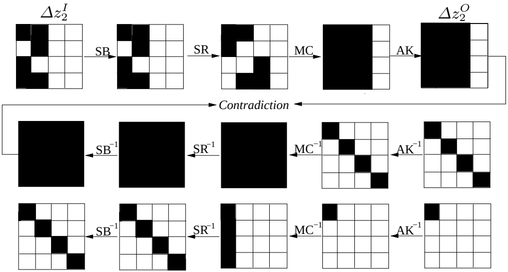
$$\boxed{\text{AES}} \rightarrow \boxed{4\text{-}r\ \text{AES}} \rightarrow \cdots \rightarrow \boxed{4\text{-}r\ \text{AES}} \rightarrow \boxed{\text{AES}}$$

3. For each sieved pair, discard the wrong subkeys (or internal state), and only the correct one is left

Three-Round Impossible Differential of AES

Proposition

For 3-round AES, given an input pair $(z_2^I, z_2^{I\prime})$ *whose bytes equal in all except six indexed by* $(0,1,5,8,12,13)$ *(or* $(0,1,4,5,9,12)$*,* $(0,4,5,8,9,13)$*,* $(1,4,8,9,12,13)$*), the difference of the output pair* $(z_4^O, z_4^{O\prime})$ *can not have exactly one nonzero byte.*

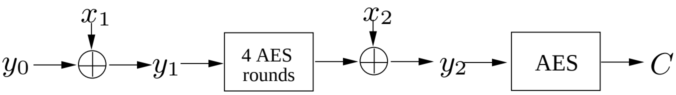


Sieve Useful Message Pairs

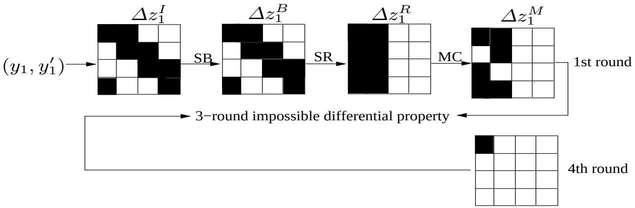
$\boxed{\text{Detect the inner near-collision with specific difference}}$

PELICAN algorithm with two message words:

$$
\begin{aligned}
\text{External Collision} \quad &\Rightarrow \quad \text{Internal Collision} \\
& \qquad AES^{4r}(y_1) \oplus x_2 = AES^{4r}(y'_1) \oplus x'_2 \\
&\Rightarrow \quad \text{Output difference of 4-round AES} \\
& \qquad AES^{4r}(y_1) \oplus AES^{4r}(y'_1) = {\color{red} x_2 \oplus x'_2}
\end{aligned}
$$



Impossible Differential Cryptanalysis of PELICAN I



Message Pairs Collection Phase

1. Construct two structures, each with 2^{64} messages $S_1 = \{(x_1, x_2)\},\ S_2 = \{(x_1', x_2')\}$ where Δx_1 is zero at bytes $(2, 3, 4, 7, 8, 9, 13, 14)$, and Δx_2 has only one nonzero byte

Impossible Differential Cryptanalysis of PELICAN II

2. Query MAC and search collisions between the two structures by the birthday attack
   - If there is no truncation at the final output, this means an inner collision at y_2
   - Else, for all colliding pairs $(x_1\|x_2, x_1'\|x_2')$, query the MAC on $(x_1\|x_2', x_1'\|x_2)$. If still collide, $(x_1\|x_2, x_1'\|x_2')$ must collide at y_2

Internal State Recovery Phase

- Recover 8 bytes of y_0 at position $(0, 1, 5, 6, 10, 11, 12, 15)$ by exhaustive search directly
- Recover the other 8 bytes in a similar manner

Selective Forgery Attack

$\boxed{\text{Recover } y_0 \Rightarrow \text{Control of the internal states}}$

Select $M = (x_1, x_2, \cdots, x_b)$, and obtain the MAC value C
Compute $M' = (x'_1, x'_2, \cdots, x'_b)$ with the same C:

1. Randomly choose x'_1, where $x'_1 \neq x_1$
2. Compute $y_1 = x_1 \oplus y_0, y'_1 = x'_1 \oplus y_0, AES^{4r}(y_1)$ and $AES^{4r}(y'_1)$
3. Set $x'_2 = AES^{4r}(y_1) \oplus AES^{4r}(y'_1) \oplus x_2$, then

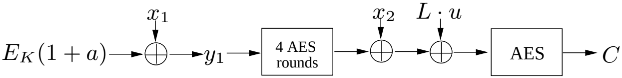
$$y'_2 = AES^{4r}(y'_1) \oplus x'_2 = AES^{4r}(y_1) \oplus x_2 = y_2$$

Set $x'_3 = x_3, \cdots, x'_b = x_b$

Obviously, $MAC_K(M') = C = MAC_K(M)$

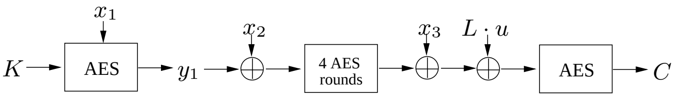
Impossible Differential Cryptanalysis of MT-MAC-AES

- MT-MAC: designed by Minematsu and Tsunoom, FSE'06
- MT-MAC-AES: MT-MAC instantiated with AES and simplified 4-round AES
- Adopt the above attack on PELICAN directly
- Recover the subkey $E_K(1+a)$
- Complexity: $2^{85.5}$ chosen messages and $2^{85.5}$ queries



Impossible Differential Cryptanalysis of PC-MAC-AES

- PC-MAC: designed by Minematsu and Tsunoom, FSE'06
- PC-MAC-AES: PC-MAC instantiated with AES and simplified 4-round AES
- Recover the internal state y_1
- Recover the two 128-bit secret key (K, L) by exhaustive search, respectively
- Complexity: $2^{85.5}$ chosen messages and 2^{128} queries



Thank you very much!