# Reset Indifferentiability and its Consequences

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# Introduction

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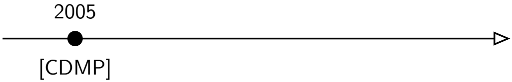


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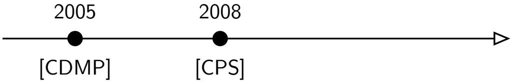


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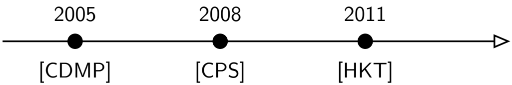


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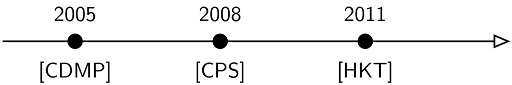


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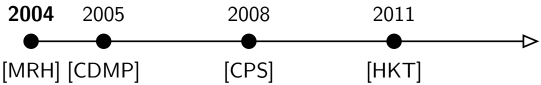
but what is "$\equiv$"?

# Equivalence Through Indifferentiability

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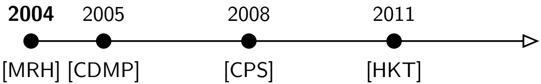


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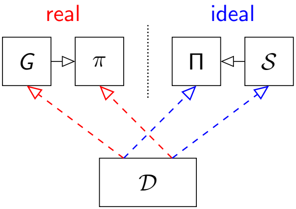
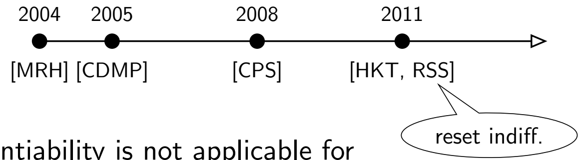



- e.g., $G$: constructed "random oracle"; $\pi$: ideal cipher; $\Pi$: real random oracle
- ask for simulator $\mathcal{S}$ such that $(G^{\pi}, \pi) \stackrel{c}{\approx} (\Pi, \mathcal{S}^{\Pi})$

# Limitations of Indifferentiability

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- $\ldots, x \leftarrow \mathcal{A}_1, \ldots, y \leftarrow \mathcal{A}_2, \ldots$

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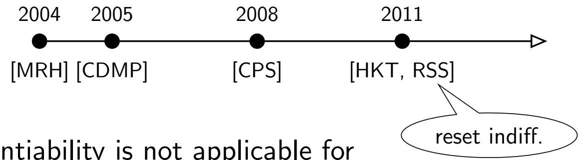


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no shared state

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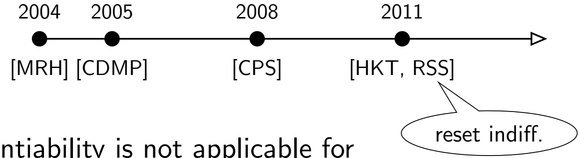


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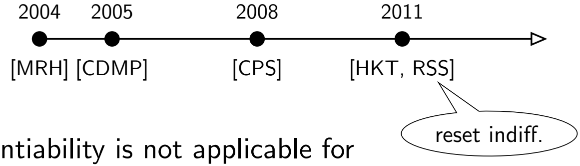


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- problem (roughly): distinct stages result in distinct simulators, distinct simulators are inconsistent
- allow the distinguisher to reset the simulator, reset indifferentiability [RSS11]

$$\text{ROM} \stackrel{?}{\equiv} \text{ICM}$$

# ROM $\stackrel{?}{\equiv}$ ICM, Revisited

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- reset-indifferentiable constructions cannot be domain extending [LAMP12, DGHM13]
    - assuming that ROs have infinite domain, ICM $\not\Rightarrow$ ROM

# In This Work

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1. under reset indifferentiability, ROM $\not\equiv$ ICM
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2. "Duality Lemma": two primitives are either equivalent or incomparable
3. $n$-reset indifferentiability $\equiv$ 1-reset indifferentiability

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   - (no result for length-preserving constructions)
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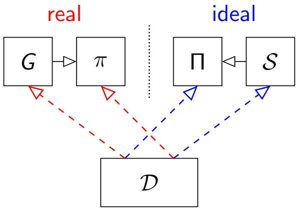
# Multi-Stage Indifferentiability




- instead of resettable simulators, consider stateless ones
- think "reset after each query"
- equivalent to reset indifferentiability
- simulators are *pseudo deterministic*—why?

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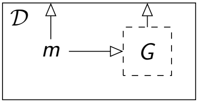

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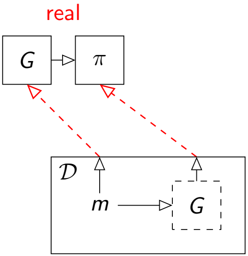


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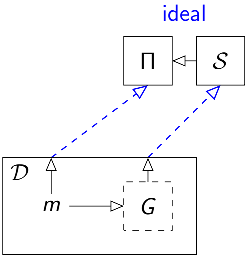


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- ideal world
  - $\mathcal{S}$ needs to query $\Pi$ on $m$
  - gets $k$ inputs of size $\frac{\ell}{2} < \ell = |\Pi(m)|$
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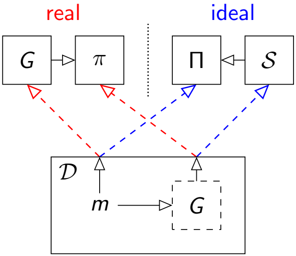


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- note: choice of primitives arbitrary

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- can switch roles!

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- positive (resp. negative) result in one direction translates to other direction
- no domain-extending constructions $\Rightarrow$ no domain-shrinking constructions; ROM and ICM are incomparable

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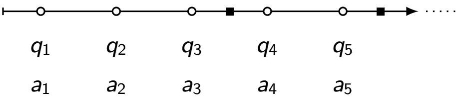
- turns out $n$-reset $=$ $n'$-reset $=$ 1-reset
- idea: at least one reset must be "critical", find it

# Eliminating Resets

consider the distinguisher $\mathcal{D}_n$ on randomness $r$ (max. $n$ resets)

$\mathcal{D}_n(r)$



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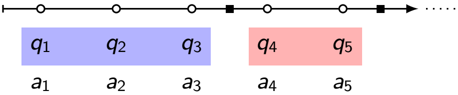

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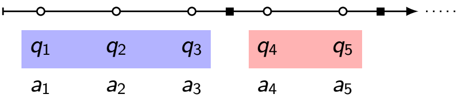


construct $\mathcal{D}_1$

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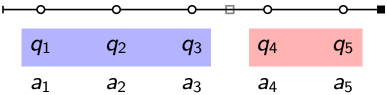


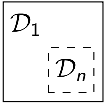

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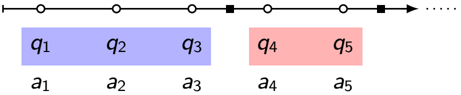


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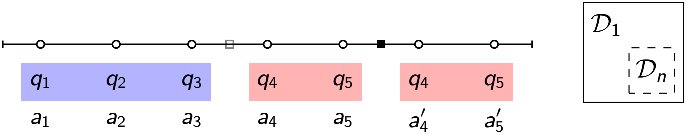


let $\mathcal{D}_1$'s output be $(a_4, a_5) \stackrel{?}{=} (a'_4, a'_5)$

next, consider $\mathcal{D}_{n-1}$

# Summary

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- is the ROM equivalent to the ICM?
- answer—depends on "equivalent"
  - for composing single-stage games: ✓
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  - multi stage / length preserving:

**open question**

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# The End

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