Review of the book
"Mathematics in Games, Sports, and Gambling"
by Ronald J. Gould, CRC Press, 2010

ISBN: 978-1-4398-0163-5

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September 16, 2015

Summary of the review

This is a review of Gould's book on the mathematics used in games, sports, and gambling. The text covers elementary discrete probability and statistics plus some combinatorics. As the title announces, motivating illustrations and examples are taken from sports (mostly in the North American context) and games. The book grew out of lecture notes he used to teach the topics to first year university students. Due to the audience in mind, the prerequisite is high school algebra.

The author's stated purposes are to demonstrate some of the principles underlying many of the games people play, to show what statistics is meant to study and how statistical insights can help us, as well as to show how mathematics can be used to analyze games and sports. Lastly, and very importantly, though, the purpose is to have fun playing.

Summary of the book

The book has 9 chapters, the last being an appendix. It can be roughly divided into two parts: the first one covers the first six chapters dealing mostly with introductory statistics while Chapters 7 and 8, which form the second part, focus on the combinatorial aspects.

Chapter 1: Basic Probability

Chapter 1 builds the laws of probability and show some of their fundamental consequences. Brief history of how probability started as a rigorous discipline pioneered by Cardano, Pascal, and Fermat is outlined. The *Problem of Points* on how to fairly divide the prize based on the present score in a partially completed game figures prominently. Here, one finds concepts like *sample space* and *independent events* leading to the *multiplication principle*, and learns how to use *choice trees*. Next comes *probability* as it is formally defined. Five rules governing probability are then stated explicitly. They can be considered as explication of Kolmogorov's axiomatization of probability.

A treatment on *conditional probability* follows. The discussion begins with the notion of *dependent events* and ends with *Bayes' Formula* with a connection to the total probability formula used as a bridge. Discussion on measures of *fairness* and the St. Petersburg Paradox shed a light on the law of large numbers. One needs to take care when using the computed *expected value* as fairness indicator since expectation is meaningful only for long-term play. Some of the most common measures in statistics: *mean*, *variance*, and *standard deviation* are defined and explained. Measures in the case of conditional probability are also treated.

Chapter 2: The Game's Afoot

Chapter 2 covers the applications of counting techniques, basic and conditional probabilities, and expectancy in games such as *roulette*, *craps*, *poker* and *backgammon*. The chapter explains various poker terminologies and variants. It counts the probability values of various hands in poker and provides an easy way to approximate the probability of hitting a desired card. The analysis on (American) roulette is interesting to me who was ignorant of this game. The house always holds a better-than-5%-advantage over the players, making the game fairly unattractive to those in the know. The overall discussion reminds us that, in the long run, the house never loses.

Some variations of the *Monty Hall problem* where either Monty Hall does not know where the car is or when we use 4 doors instead of 3 are also entertaining to try. The author completes the chapter with interesting historical notes on the development of backgammon from the Mesopotamian time to the introduction of the doubling cube in 1925 that altered the game's dynamics considerably.

Chapter 3: Repeated Play

This chapter centers on repeated play in fixed probability games, allowing us a statistical look at the games in which the outcomes of the games become statistical sample population.

Binomial coefficients are derived by proving the binomial theorem and their presentation as *Pascal triangle*. Here one can learn how to approximate the probability distributions of two-outcome events as well as their *cumulative probability*. The discussion culminates in the analysis of *normal distribution* and the central limit theorem (CLT). The *Poisson distribution* as a way to estimate the number of events occurring in a specified time period or range is explained.

The chapter examines the idea of *streaks* (either winning or losing) in games of cards and uses the Poisson distribution to explain why streaks are a natural part of random processes. How to connect streaks to winning strategies, *martingale*, anti-martingale and the cancellation system are analysed and both their strength and weaknesses discussed.

The author argues that, in gambling, the secret is to know when to quit and take your winnings. There are no systems for betting fixed probability games with working guarantee. The *gambler's ruin problem* is dissected to convince the reader of the futility of trying to break the house in real cases.

Chapter 4: Card Tricks and More

Under consideration in this chapter are several math-based card tricks and games to motivate learning counting techniques. The second, which is the longest, section presents the five-card game and its generalization. The *Pigeon Hole Principle* is used in the analysis. Recasting the problem as a bipartite graph before applying the *Hall's Marriage Theorem* highlights the delightful exposition.

Next, using the two-deck matching game, the notion of *derangement* is introduced and examined in details. In the process, the *Principle of Inclusion and Exclusion* is proved before being used in determining the probability of derangement. Here, the uninitiated may be pleasantly surprised of the appearance of e in the formula.

Some nice but not-too-mathematically-rich arithmetic games are treated as well. They can be entertaining, yet those well-versed in the art of counting would see the tricks right away. The 3-player paint ball war illustrates that, if you're the worst shooter but has the first turn in shooting, sometimes it pays to deliberately miss your shot.

Chapter 5: Dealing with Data

Statistics can be defined simply as collection of data. It can be more properly understood as the mathematics of collection, organization, and interpretation of numerical data. This chapter introduces statistics in the latter sense by using sport statistics as the data, using statistical models to answer questions on strategy and performance. The author emphasizes the difference between samples and populations and highlights that statistical inferences use sample to make predictions and gain information about the corresponding population.

Methods to view, interpret, and present data such as stem plots, time plots, and regression lines are discussed with progressive details. The roles and limitations of regression lines to make prediction and comparison are treated by using concepts such as the *sum of squares* and *linear correlation coefficients*. To explain that generally more data leads to more information leading to higher confidence in inference making, this chapter also deals with *confidence intervals*. In the last section, using illustrations from data points indicating players' performances in several sports, the notion of *coefficients of variations* is used to derive a player's relative performance.

Chapter 6: Testing and Relationships

The chapter focuses on typical statistical tests to verify claims and to test for relationships between certain values. Which players in the best one? Is Player A better than Player B? The methods introduced in Chapter 5 can be used to answer such questions. Yet, often, statistics can not help in deciding which is the better player. For instance, their confidence intervals may be greatly intersect and, hence, no clear decision can be made.

To decide whether to accept or reject hypotheses, the importance of the test's *significance level* is put forward. The interpretative nature of statistical inference is highlighted since it is the tester who determines how low the acceptable probability should be at a given instance to accept or reject a hypothesis. The possibility of committing different types of errors are also mentioned. Adjusting the previously discussed techniques for small data size, say below 30 data points, leads to the explanation on the Student t-statistics and its *degree of freedom*.

Statistics can be used to test if the old adages such as the home field advantage and left handed vs. right handed advantage in sports hold true. Comparing a set of players to another can be done by using hypothesis testing both in the case of large sample size where normal distribution can be safely assumed and when one needs to use the t-statistics due to small sample size.

This chapter brings the first part of the book to conclusion. The next two chapters veers away from statistics into more combinatorial approaches.

Chapter 7: Games and Puzzles

Opening up the second part, this chapter considers a number of games and puzzles with some interesting inherent mathematics. It explores the underlying mathematics and uses it to gain advantage in playing the games.

Three kinds of games form most of the coverage: *magic squares*, variations of magic squares, and *sudoku*. Definitions and constructions of magic squares and their variations are provided. An interesting open problem to determine the number of **distinct** magic $n \times n$ squares (excluding rotations and reflections), except for small values of n, is mentioned. Variations of magic squares come from relaxing some of the original conditions. Heuristics and computer programs that can solve sudoku problems are briefly touched on.

Prototypical problems in combinatorics such as the Tower of Hanoi problem is then used to pave the way to discussing other games and puzzles.

Chapter 8: Combinatorial Games

Games are said to be combinatorial if they are two-player finite-step games with complete information and clearly-defined allowable moves. The chapter starts with an introduction to *subtraction games*, which look deceptively simple but possess far ranging generalization and analogues, say in bioinformatics algorithms. The focus of discussion, however, is in the *game of Nim* and its variants. Other games being mentioned are the *Northcott's* and the *Silver Dollar games*, the *Wythoff's game*, and the *game of Kayles*.

A powerful analysis tool is to model games as *directed graphs* or *digraphs* by associating each possible position of the game with a vertex. A directed edge is inserted from vertex A to B if there is a move taking position A to B. The digraphs are then used to derive winning strategies. Two games G_1 and G_2 can be combined to produce a bigger game $G = G_1 + G_2$ and the digraph of G can be easily constructed from the digraphs of the component games. The winning strategy for G, however, does not follow immediately from those of the G_is. One needs to use the recursive *Sprague-Grundy function* for further analysis.

The main technical result in this chapter is the Grundy's theorem which says that any impartial game G with only finitely many possible positions is equivalent in play to some Nim heap. Many games are in fact Nims in disguise. The last two sections conclude the analysis of combinatorial games by classifying them into four game types from which winning strategies follows.

Chapter 9: Appendix

This short chapter contains five parts: review of elementary set theory, standard normal distribution table, student t-distribution, and solution to problems presented in the text, as well as hints to selected even-numbered exercises.

What is the book like?

The book's exposition is fluid and engaging. Being an introductory textbook, it is rather light in mathematical contents. It comes replete with descriptions, illustrations, and examples to bring points across.

The exposition is suitable for motivated high school students, beginning undergraduates, and sport and games enthusiasts who are curious enough about the mathematics behind them. There is no direct connection to modern cryptography since the statistics and combinatorics presented are introductory.

The context is too North American at times and this may discourage the adoption of the book in other parts of the world where the favorite sports are football (soccer) instead of American football, badminton or table tennis rather than basketball.

Would you recommend this book?

This is not a book for researchers, except when such persons want to enjoy some light entertainment in their leisure times. So, who might benefits from this text? What makes this book stand out is the focus on sports and games as motivating background and source of worked-out examples and exercises.

I would heartily recommend this book to the following audiences:

- Secondary school's students or beginning undergraduates who like sports and games and are curious to learn about the underlying mathematics.
- Teachers and professors who teach introductory course in descriptive statistics and combinatorics. The examples can be used as templates for projects designed to let students explore useful concepts further, either in groups or individually. They can pick some favorite sports or games, search for available data, and proceed to perform analysis and derive inferences.
- Sport enthusiasts who want to gain better understanding and make predictions about their teams' or favorite players' performance. Doing so may make them more successful bettors and winners of pub debates on who's the best player or which is the best team. Mind, however, that one should not state that F.C. Barcelona is the best when having a pint in Madrid.

Miscellaneous Notes

The images used in the book are of uneven quality, not surprising since they are mostly taken from online sources. Some are really bad and tend to disturb rather than to help. The publisher could have done a better job at this.

For future edition(s), better figure and picture presentations should be a priority. Most of them can be typeset better, see, for instance, the equations on page 95. Sharper choices are available for illustrative photos and the likes *e.g.* Figure 2.3.

Minor mistakes that lead to confusion should be corrected. One such occasion is the fact that there are three different figures with the label Figure 9.3: one each on pages 324, 326, and 327.

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