Non-Interactive Keyed-Verification Anonymous Credentials

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Abstract. Anonymous credential (AC) schemes are protocols which allow for authentication of authorized users without compromising their privacy. Of particular interest are non-interactive anonymous credential (NIAC) schemes, where the authentication process only requires the user to send a single message that still conceals its identity. Unfortunately, all known NIAC schemes in the standard model require pairing based cryptography, which limits them to a restricted set of specific assumptions and requires expensive pairing computations. The notion of keyed-verification anonymous credential (KVAC) was introduced in (Chase et al., CCS'14) as an alternative to standard anonymous credential schemes allowing for more efficient instantiations; yet, making existing KVAC non-interactive either requires pairing-based cryptography, or the Fiat-Shamir heuristic.

In this work, we construct the first non-interactive keyed-verification anonymous credential (NIKVAC) system in the standard model, without pairings. Our scheme is efficient, attribute-based, supports multi-show unlinkability, and anonymity revocation. We achieve this by building upon a combination of algebraic MAC with the recent designated-verifier non-interactive zero-knowledge (DVNIZK) proof of knowledge of (Couteau and Chaidos, Eurocrypt'18). Toward our goal of building NIKVAC, we revisit the security analysis of a MAC scheme introduced in (Chase et al., CCS'14), strengthening its guarantees, and we introduce the notion of *oblivious* non-interactive zero-knowledge proof system, where the prover can generate non-interactive proofs for statements that he *cannot check by himself*, having only a part of the corresponding witness, and where the proof can be checked efficiently given the missing part of the witness. We provide an efficient construction of an oblivious DVNIZK, building upon the specific properties of the DVNIZK proof system of (Couteau and Chaidos, Eurocrypt'18).

Keywords. Anonymous credentials, keyed-verification anonymous credentials, non-interactive anonymous credentials, designated-verifier non-interactive zero-knowledge proofs.

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1   Introduction

1.1   Anonymous Credentials

Anonymous credentials, introduced in the seminal work of Chaum [14], allow users to authenticate in an anonymous way to a variety of services. Each user can receive credentials from authorities, and register pseudonyms with authorities and verifiers. These pseudonyms are associated to the identity of the user, but should be *unlinkable* to its exact identity. That is, another entity should not be able to check whether two pseudonyms are associated with the same identity. Authorities can issue credentials to users which can be shown to verifiers, and the presentation of a credential should only leak the information that the user knows the identity associated to the pseudonym, and owns a credential from the authority for this identity. This guarantees the *anonymity* of users. In order for credentials to make sense, they must be *unforgeable*: a user should not be able to present a credential without having received one from the authority first. Due to their wide range of real-world applications, anonymous credentials have received a constant attention from the cryptographic community [1,2,4,6–9,13,17,21,23,24,26,29].

Non-Interactive Anonymous Credentials. Non-interactive anonymous credentials (NIAC) are anonymous credentials where the process of showing possession of a valid credential to a verifier requires sending a single message from the user to the verifier. Non-interactivity in anonymous credentials is considered to be a highly desirable security property, and was the focus on an important research effort [3,4,24]. However, a downside of existing NIAC scheme is that all known constructions in the standard model require the use of pairing based cryptography, which limits their efficiency (since pairing are a relatively expensive cryptographic operation) and restricts the set of assumptions their security can be based on. While some interactive anonymous credential schemes can be made non-interactive in the random oracle model under the Fiat-Shamir transform, this is known to provide only heuristic security arguments in the standard model [10,20,22].

Keyed-Verification Anonymous Credential. Most commonly, anonymous credential schemes allow for a single credential to be shown more than once to multiple verifiers. The notion of *keyed-verification anonymous credentials* (KVAC) was introduced in [13]; it restricts credential to only be valid with respect to one verifier and requires the authority and verifier to share a secret key. The key observation of [13] is that such restricted anonymous credentials can be instantiated very efficiently, using algebraic message authentication codes. Therefore, in numerous applications where the restriction to keyed-verification is not an issue, they can be used to allow for more efficient instantiations. Think for example of a bus company issuing monthly pass, where the pass must be shown each time a user boards a bus; here, it is reasonable to assume that the bus device can share a secret-key with the bus company (since both belong to the same organisation).

A downside of the KVAC scheme of [13], however, is that the process of showing possession of a credential requires an interactive protocol between the user and the verifier. This protocol can be made non-interactive, but this either requires the Fiat-Shamir transform (leading to a protocol secure in the random oracle model only), or the use of pairing-based cryptography, nullifying the efficiency advantages of KVAC with respect to their publicly verifiable counterpart.

1.2   Our Contribution

In this work, we construct the first non-interactive keyed-verification anonymous credential scheme (NIKVAC) in the standard model, without relying on pairing-based cryptography. Our NIKVAC is very expressive: it natively supports multi-show unlinkability (*i.e.*, when showing possession of a credential multiple time to a verifier, the latter cannot tell whether these correspond to the same user) or pseudonyms (the verifier knows a pseudonym that he can link a credential to, but that he cannot link to the actual identity of the user), without any additional cost (*i.e.*, we do not require to generate an additional commitment to the identity and prove knowledge of its content to obtain pseudonyms, as in most alternative approaches; rather, such commitments are natively and implicitly defined by our scheme). Our NIKVAC is also attribute-based (it supports vectors of attributes as opposed to identities, and can handle a variety of relations on the attributes), and supports anonymity revocation (there exists a global trapdoor which a trusted authority can use to revoke the anonymity of a misbehaving user, efficiently extracting his identity from any accepting credential).

While our scheme is the first NIKVAC in the standard model without pairings, we observe (this is in fact the starting point of our work) that there is a relatively natural construction of a NIKVAC which is obtained by starting with the (interactive) scheme of [13], and replacing the underlying zero-knowledge proof system by the designated-verifier non-interactive zero-knowledge proof system of [11]. While this observation is interesting in itself, the security analysis of the resulting construction does not present major technical difficulties (although it is not entirely straightforward). In this work, we refine this approach, adopting a different strategy to better exploit the structural properties of the proof system of [11]. Our optimized approach provides strong efficiency improvements (which we detail in Section 1.6) over the previous alternative.

1.3   Our Approach

Our starting point is the interactive KVAC scheme of [13]. In this scheme, a credential is an algebraic MAC signature on the identity of the user. Anonymous presentation of a credential is done (informally) by masking the credential, and providing some zero-knowledge proofs of knowledge of the identity together with the masking informations satisfying the appropriate relation, which allows the verifier (who knows the secret MAC key) to check that the masked credential does indeed verify correctly with respect to the (hidden) identity of the user.

To make this scheme non-interactive, the basic observation is that it suffices to rely on a *designated-verifier* non-interactive zero-knowledge (DVNIZK) proof of knowledge of the appropriate values. Unlike their publicly-verifiable counterpart, there exists efficient constructions of DVNIZK proof systems which does not rely on pairings. However, until recently, all known constructions of DVNIZK proof systems [12,16,18] suffered from two important downside, each of them preventing their use in a NIKVAC scheme:

– they can only deal with *existential* statements, while anonymous credentials crucially rely on proving *knowledge* of the signed identity, and
– they only satisfy a *bounded* notion of security, where the soundness of the proof is only guaranteed to hold if the prover is restricted to query a verification oracle an a priori bounded number of times. In an anonymous credential system, however, the users can interact freely with a verifier and receive feedback on whether proofs of credential possession was accepted or not; hence, for all of these scheme, a malicious user could forge a credential which is accepted by the verifier even though it was not issued by the authority, by interacting a sufficient (polynomial) number of times with the verifier.

This situation recently changed with the introduction in [11] of the first DVNIZK proof system which allows to provide proofs of *knowledge* of a witness, for a wide variety of algebraic statements, where soundness is *unbounded* (it holds even if the prover is given arbitrary access to a verification oracle). Furthermore, the framework of [11] allows for efficient DVNIZK proofs, directly proportional to the size of the algebraic statement to be proven.

A natural approach toward building a NIKVAC scheme is therefore to rely on the KVAC scheme of [13], and to replace the underlying zero-knowledge proofs by appropriate DVNIZK, using the framework of [11]. However, while this approach should lead to a secure NIKVAC, it misses the opportunity to exploit the specific structure of the scheme of [11] to get improved efficiency guarantees. Therefore, we choose instead to tackle the problem directly and construct an optimized NIKVAC system, heavily building upon the specific structure of the DVNIZK of [11].

1.4   Our Techniques

To describe our strategy, it is helpful to start from a natural but insecure approach. As in [13], a credential will simply be a signature on the identity of the user using an algebraic MAC. To show possession of a credential, the user can simply send this credential (but not his identity) and prove with a DVNIZK that he knows an identity such that the MAC verification algorithm returns 1 when given as input this identity and the credential. A first observation is that this approach allows for a straightforward optimization: in the most common setting, the verifier must know a pseudonym associated to the user (which cannot be linked to his identity), which will usually take the form of a commitment to the identity of the user. We observe, however, that a DVNIZK proof of knowledge within the framework of [11]

does already include an encryption of the witness, and the proof of knowledge property does in particular guarantee that the witness whose knowledge is proven is indeed the one encrypted in the proof. Therefore, it is not necessary to add a commitment to the identity and prove that the committed value is the one for which the user knows a credential; rather, the user can simply compute this encryption ahead of time (this does not require knowing the credential) and send it to the verifier, which will store it as being the user pseudonym. Then, each time the user wants to authenticate, he only have to generate the "missing part" of the proof with respect to this encryption. This strongly reduces the size of the proof (since the proof does not need to include an explicit proof regarding a commitment anymore), and allows to reuse a significant portion of the proof across many authentications.

However, the natural approach of disclosing a credential σ and proving knowledge of an identity that verifies correctly with respect to σ fails, for two reasons:

- First, the above approach does not guarantee anonymity, because the verifier (who knows the secret MAC key) could find out the identity of the user simply by colluding with the authority, and evaluating the MAC verification algorithms on all identities previously submitted to the authority, to find out which one verifies correctly with respect to this credential.
- Second, and more importantly, the MAC verification requires knowledge of the secret MAC key, which the user does not know; hence, he cannot possibly issue a proof that his credential verifies correctly, since checking this statement does already require knowing the secret MAC key.

We first explain how we address the second concern. Our idea is to build upon the specific malleability property of the DVNIZK proof system of [11] to build an *oblivious* DVNIZK proof system, which allows the prover to issue a proof for a statement *even if he does not know himself whether the statement does hold.* This does not contradict the security of the MAC scheme, since the proof system is not publicly verifiable: hence, even after he builds the proof, the prover cannot check by himself whether this proof verifies correctly. Intuitively, the prover will construct a "partial non-interactive proof" which is malleable in the following sense: given this proof and the secret MAC key, the verifier can reconstruct himself the complete proof that the credential verifies correctly. If the prover does not know the appropriate witness, the reconstructed proof will not verify correctly. The partial proof should not leak any more information about the witness held by the prover than what is leaked by the reconstructed proof; hence, by the zero-knowledge property of the DVNIZK proof system, this proof will only reveal whether the statement (which depends on both the prover witness and the secret key known to the verifier) is true. We believe that the concept of non-interactive oblivious proofs, which allows to prove that a statement is true while knowing only a part of the witness to a verifier knowing the "missing part" of the witness, might be of independent interest (we briefly elaborate on this in Section 1.5); in particular, it formalizes the approach taken (in the interactive setting) in previous works on keyed-verification anonymous credentials [13].

To tackle the first concern, the prover will randomize his credential and mask it with appropriate random values, and issue a partial proof that the *unmasked* credential does verify with respect to the secret key. We formalize both properties at once by introducing a new primitive, oblivious designated-verifier non-interactive zero-knowledge proofs of knowledge, which can be used to prove statements non-interactively even when the prover only knows a part of the witness, and can be simulated by a simulator that does not know neither the witness nor the word for which the proof is constructed, guaranteeing that the verifier will not only be unable to recover the witness, but also that he cannot possibly recover the credential, which would allow him to break anonymity by colluding with the authority.

Next, we provide an optimized construction of an oblivious DVNIZK proof system for the language of valid credentials, building upon the DVNIZK proof system of [11]. Proving security of the resulting proof system, however, runs into a subtle issue: when considering the more general setting of *attribute-based* anonymous credentials, where the user will have a secret *vector of attributes* instead of a secret identity, the unforgeability property of the underlying MAC scheme does not suffice to prove the soundness of the oblivious proof system. We provide two alternatives to overcome this issue:

– When the vector of attributes is of length one (*i.e.*, when we restrict our attention to non-attribute-based anonymous credentials, where the secret of the user is only his identity), we show that the public parameters of the MAC scheme suffice to reduce the security directly to the unforgeability of the MAC scheme. This setting already captures many possible applications.
– In the general setting, where the vector of attributes can be longer than 1, we show that the security can be proven if the MAC scheme satisfies a stronger notion of unforgeability, which we call *extended unforgeability*. Then, we revisit the security analysis of one of the two MAC schemes constructed in [13], which is secure in the generic group model, and prove that this scheme does in fact already satisfy extended unforgeability. While the second MAC scheme constructed in [13] (which is based on the decisional Diffie-Hellman assumption) does plausibly satisfy extended unforgeability, we leave it as an interesting open problem to prove it under a standard assumption, or to construct a MAC scheme with extended unforgeability under the DDH assumption. We note that considering algebraic MACs with stronger unforgeability guarantees is a relatively natural approach in the setting of anonymous credentials (see e.g. [3,4]), but the specific strengthening we require in our construction was not, to our knowledge, considered in previous works.

There is an additional requirement which we must take care of: the MAC schemes of [13] are only proven secure in groups of prime order, while the most natural instantiation of the DVNIZK proof system of [11] typically requires composite-order groups. While the security of their DDH-based MAC easily extends to the composite order setting by assuming in addition that it is infeasible for any polytime adversary to find a generator of a strict subgroup (which is a

standard and well-studied assumption), the proof of their generic-group-model-based (GGM-based) MAC is unconditional, hence it assumes that the adversary is unbounded, in which case there is an explicit attack on the composite-order variant of the scheme where the unbounded adversary constructs an invalid MAC signature in a strict subgroup of the group. We therefore revisit the security proof of the GGM-based MAC, and show that it holds in the generic group model assuming in addition that the adversary is polynomially bounded, and that the computational subgroup assumption holds. Altogether, we show that this gives rise to a highly optimized NIKVAC. In the next section, we discuss in more details the efficiency of our scheme.

1.5 Applications of Oblivious DVNIZK

Given the intermediate abstraction of oblivious designated non-interactive zero-knowledge proofs, the construction of NIKVAC follows very naturally. In fact, we could have provided a direct construction of NIKVAC from this approach, without formalizing the intermediate primitive. However, we believe that oblivious DVNIZKs can be interesting in their own right. We elaborate below.

Secure computation protocols allow a group of parties to securely evaluate a public function on their joint private input. We focus in this discussion on the case of two parties for simplicity. A common approach to secure two-party computation is to first design a scheme secure against passive adversaries, which do not deviate from the specifications of the protocol, and then to use zero-knowledge proofs to let all adversaries prove their honest behavior throughout the protocol. This transformation makes the protocol secure against malicious adversaries, which can deviate arbitrarily from the specifications of the protocol. To obtain round-efficient compilation of passively secure computation protocols into maliciously secure protocols, the most natural strategy is to rely on (designated-verifier) non-interactive zero-knowledge proofs (an alternative is to use implicit zero-knowledge proofs [5], but this adds two more rounds to the protocol) to prove honest behavior of each user after each round.

Oblivious DVNIZK allow for an alternative compilation strategy, which starts from a protocol with stronger security guarantees, but is in general more efficient. Let us call (informally) *half-maliciously secure* a secure computation protocol which is passively secure, and such that no malicious adversary can compromise the *privacy* of the inputs (but can possibly compromise the *correctness* of the computation). Let Π be a half-maliciously secure protocol, securely computing a function f. Let (x_1, x_2) denote the inputs of the parties. To convert Π into a fully secure protocol, we first modify Π to include commitments (c_1, c_2) to the inputs (if Π does not already include them). Then, to guarantee full security, one of the parties, which we call the prover, must send a single oblivious DVNIZK to the other party (the receiver) at the very end of the protocol, which is a proof that $y = f(x_1, x_2)$, where y is the output of the protocol, and (x_1, x_2) is committed in (c_1, c_2). Note that the prover does not have the full witness for this statement (since it depends, in particular, on the private input of the verifier), but the prover and the verifier jointly have the full witness, allowing the verifier to check

the proof without further interaction. This is in contrast with DVNIZK-based compilation, which requires proving honest behavior of all users at each round (here, we only prove *correctness* of the computation in the last round). We leave the formal proof of this observation to future work.

1.6 Efficiency

There is, to our knowledge, no existing previous construction of NIKVAC in the standard model. However, as we pointed out previously, there is a relatively natural construction which is obtained by starting from the scheme of [13], and replacing the underlying zero-knowledge proofs by DVNIZKs instantiated with [11]. Let us call this construction the CMZ+CC construction. We use CMZ+CC as a basis for comparison with our improved construction. We focus on the communication cost of showing possession of a credential, since the computation is directly proportional to the communication (hence, an improvement factor with respect to communication translates to a comparable improvement factor with respect to computation), and since the cost of issuing a credential depends on the specific secure computation scheme used to implement it, which is not the focus of our work (we require the same blind issuance of an algebraic MAC as in previous works on KVAC).

For simplicity, we focus on the cost obtained when implementing the MAC with the more efficient GGM-based MAC scheme of [13]; when using the other, DDH-based MAC scheme, all costs must be roughly scaled up by 50% (up to constants), and the improvement factor of our method compared to the naive approach will be essentially identical. Let β denote the length of the vector of attributes. In the minimal setting where the verifier knows a pseudonym, implemented as a commitment to the user's vector of attributes, instantiating the zero-knowledge proofs in [13] using the DVNIZK proof system of [11] leads to a proof size of $3\beta + 3$ group elements, and $6\beta + 2$ ciphertexts (in a typical instantiation of the DVNIZK of [11], the group will be a composite order abelian group, and the encryption scheme will be the Paillier encryption scheme).

In comparison, our proof of credential possession requires sending $\beta + 2$ group elements, and $2\beta + 2$ ciphertexts. Furthermore, all the ciphertexts can be sent once for all to the verifier (they form the pseudonym of the prover); each new credential presentation then requires only generating and sending $\beta + 2$ group elements (in comparison, the pseudonym in [13] is a tuple of β commitments, hence sending the pseudonym ahead of time saves only β group elements). For the important case of $\beta = 1$ attribute, and instantiating the DVNIZK with Paillier and a 2048-bit modulus, this corresponds to a factor of improvement of more than 7 in the proof size compared to [13]. In addition, using an optimization which we describe in the full version [15], the number of ciphertexts can be further reduced, from $2\beta + 2$ to 2β. We summarize the comparison between our scheme and CMZ+CC in the table 1.

Eventually, we sketch a straightforward computational optimization (assuming an instantiation with the Paillier scheme and a 2048-bit modulus for concreteness): the exponents manipulated when constructing and verifying the proof are either

Table 1. Comparison of our optimized NIKVAC to a direct construction from [13] with the DVNIZK of [11].

NIKVAC [1]	CMZ+CC	This Work	This Work (full version [15])
β attributes, group element length n, ciphertext size m			
Pseudonym Size	βn	$(2\beta+2)m$	$2\beta m$
Proof Size	$(2\beta+3)n+(6\beta+2)m$	$(\beta+2)n$	$(\beta+2)n$
Prover Computation [2]	$(5\beta+2)A+(3\beta+1)(B+C)$	$(2\beta+3)A$	$(2\beta+3)A$
Assumption	GGM+IND-CPA	GGM+IND-CPA	GGM+IND-CPA + short-exp dlog
(with Paillier) 1 attribute, group element length 2048, ciphertext size 4096			
Pseudonym Size	256 Byte	2048 Byte	1024 Byte
Proof Size	5,38 kB	756 Byte	756 Byte
Prover Computation	$7A+4(B+C)$	$5A$	$5A$
Assumption	GGM+Paillier	GGM+Paillier	GGM+Paillier + short-exp dlog

[1] We consider a minimal setting where the prover shows possession of a valid credential with respect to an identity committed in a *pseudonym* known to the verifier. We use the GGM-based scheme of [13] as the underlying algebraic MAC (the efficiency gain of our approach is essentially the same if one uses the DDH-based MAC of [13]).

[2] A denotes the cost of an exponentiation in the group $\mathbb{G}$, B denotes the cost of an encryption, C denotes the cost of an homomorphic scalar multiplication. We note that, under the short-exponent discrete logarithm assumptions, all exponentiations in $\mathbb{G}$ (resp. all homomorphic scalar multiplications) can be performed with exponents (resp. scalars) of length at most 256 bits.

attributes, random coins, or masks. If attributes are, say, up to 128-bit long, then under the short-exponent discrete logarithm assumption (which states that it is hard to find x from g^x even if x is random but *short*, e.g. 128-bit long), all exponents can be taken either 128-bit long (for the attributes and the random coins) or 256-bit long (for the masks, since they must statistically mask the attributes over the integers). This makes computing exponentiations and scalar multiplications considerably more efficient than with full-size (i.e., 2048-bit) values.

Comparison with Plain [13]. We briefly comment on the comparison with the plain scheme of [13] (which is either interactive, or non-interactive in the random oracle model). Our main efficiency bottleneck is the fact that we use the DVNIZK of [11], which requires to use a large order group.[1] Therefore, using natural parameters, we manipulate group elements of size 2048 bits, and ciphertexts of size 4096 bits. In constrast, [13] can work exclusively with group elements and exponents over any DDH-hard group, e.g. of size 256 bits. However, the proof size of [13] (not counting the size of the pseudonym) is $\beta+2$ group elements and $3\beta+2$ 256-bit exponents, for a total of 256 Byte. Our proof system achieves a proof size 756 Byte, less than three times larger in spite of our use of an 8-time larger group - and unlike [13], it is secure in the standard model (the ratio remains essentially the same if we instantiate instead the underlying MAC scheme with the DDH-based scheme of [13]).

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[1] In [11], the size of the group must be equal to the size of the plaintext space of a DVNIZK-friendly encryption scheme, such as Paillier.

1.7   Organization

In Section 2, we recall necessary preliminaries (further preliminaries are given in the full version [15]). In Section 3, we recall the definition of MAC schemes, introduce a general algebraic MAC scheme, and define the stronger notion of extended unforgeability. In Section 4, we formally define non-interactive keyed-verification anonymous credentials and their security properties. In Section 5, we introduce the concept of oblivious DVNIZK and their security properties, provide an explicit instantiation tailored to our application, and formally prove its security. In Section 6, we show how to construct a non-interactive keyed-verification anonymous credential from a MAC scheme and an oblivious DVNIZK proof system. Eventually, in the full version [15], we prove that the first MAC scheme of [13] satisfies extended unforgeability in the generic group model (with composite order groups), and we describe further improvements to our NIKVAC construction relying on the short-exponent discrete logarithm assumption.

2   Preliminaries

Throughout this paper, λ denotes the security parameter. A probabilistic polynomial time algorithm (PPT, also denoted *efficient* algorithm) runs in time polynomial in the (implicit) security parameter λ. A positive function f is *negligible* if for any polynomial p there exists a bound $B > 0$ such that, for any integer $k \geq B$, $f(k) \leq 1/|p(k)|$. An event depending on λ occurs with *overwhelming probability* when its probability is at least $1 - \mathrm{negl}(\lambda)$ for a negligible function negl. Given a finite set S, the notation $x \xleftarrow{\$} S$ means a uniformly random assignment of an element of S to the variable x. We represent adversaries as interactive probabilistic Turing machines; the notation $\mathscr{A}^{\mathcal{O}}$ indicates that the machine $\mathscr{A}$ is given oracle access to $\mathcal{O}$. Adversaries will sometime output an arbitrary state st to capture stateful interactions.

Abelian Groups and Modules. We use additive notation for groups for convenience, and write $(\mathbb{G}, +)$ for an abelian group of order k. When it is clear from the context, we denote 0 its neutral element (otherwise, we denote it $0_{\mathbb{G}}$). We denote by $ord(\mathbb{G})$ the order of $\mathbb{G}$. We denote by $\bullet$ the scalar-multiplication algorithm (*i.e.* for any $(x, G) \in \mathbb{Z}_k \times \mathbb{G}$, $x \bullet G = G + G + \ldots + G$, where the sum contains x terms). Observe that we can naturally view $\mathbb{G}$ as a $\mathbb{Z}_k$-module $(\mathbb{G}, +, \bullet)$, for the ring $(\mathbb{Z}_k, +, \cdot)$. For simplicity, we write $-G$ for $(-1) \bullet G$. We use lower case to denote elements of $\mathbb{Z}_k$, upper case to denote elements of $\mathbb{G}$, and bold notations to denote vectors. We extend the notations $(+, -)$ to vectors and matrices in the natural way, and write $\boldsymbol{x} \bullet \boldsymbol{G}$ to denote the scalar product $x_1 \bullet G_1 + \ldots + x_t \bullet G_t$ (where $\boldsymbol{x}, \boldsymbol{G}$ are vectors of the same length t). For a vector $\boldsymbol{v}$, we denote by $\boldsymbol{v}^{\intercal}$ its transpose. By $\mathsf{GGen}(1^{\lambda})$, we denote a probabilistic efficient algorithm that, given the security parameter λ, generates an abelian group $\mathbb{G}$ in which the CSG and DLSE assumption defined below holds in respect to λ. Note that this implies that the normal discrete log problem is hard in this group, as

well. In the following, we write $(\mathbb{G}, k) \xleftarrow{\$} \mathsf{GGen}(1^\lambda)$. Additionally, we denote by $\mathsf{GGen}(1^\lambda, k)$ a group generation algorithm that allows us to select the order k beforehand.

RSA Groups. A *strong prime* is a prime $p = 2p' + 1$ such that p' is also a prime. We call *RSA modulus* a product $n = pq$ of two strong primes. We denote by φ Euler's totient function; it holds that $\varphi(n) = (p-1)(q-1)$. We denote by $\mathbb{J}_n$ the cyclic subgroup of $\mathbb{Z}_n^*$ of elements with Jacobi symbol 1 (the order of this group is $\varphi(n)/2$), and by QR_n the cyclic subroup of squares of $\mathbb{Z}_n^*$ (which is also a subgroup of $\mathbb{J}_n$ and has order $\varphi(n)/4$). By $\mathsf{Gen}(1^\lambda)$, we denote a probabilistic efficient algorithm that, given the security parameter λ, generates a strong RSA modulus n and secret parameters (p, q) where $n = pq$, such that the best known algorithm for factoring n takes time 2^λ. In the following, we write $(n, (p, q)) \xleftarrow{\$} \mathsf{Gen}(1^\lambda)$ and call abelian groups with order n composite order groups, if n is a RSA modulus.

Generic Group Model. The generic group model (GGM) was introduced in [31] and is an idealized model of groups. It captures groups with no additional structure apart from being a group. In such generic groups, the only possibility of attacking a cryptographic primitive is utilizing generic algorithms which only make use of group operations.

In proofs, the generic group model is captured by giving an adversary access to the group through random encodings of the group elements as bitstrings and a group operation oracle. Note that if a cryptographic primitive is proven secure in the GGM, it only ensures that it can not be broken with generic algorithms. In order to simulate the oracle in this work, we will require the following lemma, based on [30].

Lemma 1 (Generalised Schwartz-Zippel). *Let* $(n, (p, q)) \xleftarrow{\$} \mathsf{Gen}(1^\lambda), \mathbb{G} \xleftarrow{\$} \mathsf{GGen}(1^\lambda, n)$ *and* $F \in \mathbb{Z}_n[\overline{x_1}, \overline{x_2}, .., \overline{x_l}]$ *with* $F \neq 0 \wedge deg(F) = d \geq 0$. *Let* $p' \in \{p, q\}$ *and* $\mathbb{P}$ *a subgroup of* $\mathbb{G}$ *of order* p'. *It holds that*

$$\Pr\left[x = (x_1, x_2, .., x_l) \xleftarrow{\$} \mathbb{P}^l : F(x) = 0\right] \leq \frac{d}{p'}$$

2.1 Assumptions

Computational Subgroup Assumption (CSG). The computational subgroup assumption states that no bounded adversary can output a generator for a non-trivial subgroup. Or more formally, for all PPT adversaries $\mathscr{A}$, it holds that

$$\Pr\left[\begin{array}{l}(n, (p, q)) \xleftarrow{\$} \mathsf{Gen}(1^\lambda), \\ \mathbb{G} \xleftarrow{\$} \mathsf{GGen}(1^\lambda, n), \quad : G \neq 0_\mathbb{G} \wedge (p \bullet G = 0_\mathbb{G} \vee q \bullet G = 0_\mathbb{G}) \\ G \leftarrow \mathscr{A}(\mathbb{G}, n),\end{array}\right] \leq \mu(\lambda)$$

where $\mu(\lambda) = \mathrm{negl}(\lambda)$.

Decisional-Diffie-Hellman (DDH) Assumption. Let $\mathbb{G}$ be a group with order n. For all PPT adversaries $\mathscr{A}$ it holds that

$$\left| \Pr \begin{bmatrix} a, b, c \overset{\$}{\leftarrow} \mathbb{Z}_{\mathsf{n}} & : \mathscr{A}(G, A, B, C) = 1 \\ A \leftarrow a \bullet G, B \leftarrow b \bullet G, C \leftarrow ab \bullet G \end{bmatrix} - \Pr \begin{bmatrix} a, b, c \overset{\$}{\leftarrow} \mathbb{Z}_{\mathsf{n}} & : \mathscr{A}(G, A, B, C) = 1 \\ A \leftarrow a \bullet G, B \leftarrow b \bullet G, C \leftarrow c \bullet G \end{bmatrix} \right| \leq \mu(\lambda)$$

2.2 Encryption Schemes

A public-key encryption scheme S is a triple of PPT algorithms ($S.\mathsf{KeyGen}$, $S.\mathsf{Enc}$, $S.\mathsf{Dec}$), such that $S.\mathsf{KeyGen}$ generates encryption and decryption keys $(\mathsf{ek}, \mathsf{dk})$, $S.\mathsf{Enc}_{\mathsf{ek}}$, given a plaintext, outputs a (randomized) ciphertext, and $S.\mathsf{Dec}_{\mathsf{dk}}$, given a ciphertext, outputs a plaintext. An encryption scheme must be correct ($S.\mathsf{Enc}_{\mathsf{dk}}(S.\mathsf{Enc}_{\mathsf{ek}}(m)) = m$ for every message m) and IND-CPA secure (no adversary can distinguish between the encryptions of two messages of its choice). Because of space constraints, we defer to the full version [15] the formal definition of encryption schemes and their security properties.

In this work, we will focus on additively homomorphic encryption schemes, which are homomorphic for both the message and the random coin. More formally, we require that the message space $\mathcal{M}$ and the random source $\mathcal{R}$ are integer sets $(\mathbb{Z}_M, \mathbb{Z}_R)$ for some integers (M, R), and that there exists an efficient operation $\oplus$ such that for any $(\mathsf{ek}, \mathsf{sk}) \overset{\$}{\leftarrow} \mathsf{KeyGen}(1^\lambda)$, any $(m_1, m_2) \in \mathbb{Z}_M^2$ and $(r_1, r_2) \in \mathbb{Z}_R^2$, denoting $(C_i)_{i \leq 2} \leftarrow (S.\mathsf{Enc}_{\mathsf{ek}}(m_i; r_i))_{i \leq 2}$, it holds that $C_1 \oplus C_2 = S.\mathsf{Enc}_{\mathsf{ek}}(m_1 + m_2 \bmod M; r_1 + r_2 \bmod R)$. We say an encryption scheme is *strongly additive* if it satisfies these requirements. Note that the existence of $\oplus$ implies (via a standard square-and-multiply method) the existence of an algorithm that, on input a ciphertext $C = S.\mathsf{Enc}_{\mathsf{ek}}(m; r)$ and an integer $\rho \in \mathbb{Z}$, outputs a ciphertext $C' = S.\mathsf{Enc}_{\mathsf{ek}}(\rho m \bmod M; \rho r \bmod R)$. We denote by $\rho \odot C$ the external multiplication of a ciphertext C by an integer ρ, and by $\ominus$ the operation $C \oplus (-1) \odot C'$ for two ciphertexts (C, C'). We will sometimes slightly abuse these notations, and write $C \oplus m$ (resp. $C \ominus m$) for a plaintext m to denote $C \oplus S.\mathsf{Enc}_{\mathsf{ek}}(m; 0)$ (resp. $C \ominus S.\mathsf{Enc}_{\mathsf{ek}}(m; 0)$). We extend in a natural way the algorithm Enc over vectors: for vectors $\boldsymbol{m} = (m_i)_i \in \mathbb{Z}_M^*$ and $\boldsymbol{r} = (r_i)_i \in \mathbb{Z}_R^*$ of the same size, $S.\mathsf{Enc}_{\mathsf{ek}}(\boldsymbol{m}; r)$ denotes the vector $(S.\mathsf{Enc}_{\mathsf{ek}}(m_i, r_i))_i$. We extend the algorithm Dec to vectors of ciphertexts in a similar way.

The Paillier Encryption Scheme. The Paillier encryption scheme [27] is a well-known additively homomorphic encryption scheme over $\mathbb{Z}_n$ for an RSA modulus n. We describe here a standard variant [19,25], where the random coin is an exponent over $\mathbb{J}_n$ rather than a group element. Note that the exponent space of $\mathbb{J}_n$ is $\mathbb{Z}_{\varphi(n)/2}$, which is a group of unknown order; however, it suffices to draw exponents at random from $\mathbb{Z}_{n/2}$ to get a distribution statistically close from uniform over $\mathbb{Z}_{\varphi(n)/2}$. The IND-CPA security of the Paillier encryption scheme reduces to the DCR assumption, which states that it is computationally infeasible to distinguish random n'th powers over $\mathbb{Z}_{n^2}^*$ from random elements of $\mathbb{Z}_{n^2}^*$.

- KeyGen(1^λ): run $(n, (p, q)) \xleftarrow{\$} \mathsf{Gen}(1^\lambda)$, pick $g \xleftarrow{\$} \mathbb{J}_n$, set $h \leftarrow g^n \bmod n^2$, and compute $\delta \leftarrow n^{-1} \bmod \varphi(n)$ (n and $\varphi(n)$ are relatively prime). Return $\mathsf{ek} = (n, h)$ and $\mathsf{dk} = \delta$;
- Enc($\mathsf{ek}, m; r$): given $m \in \mathbb{Z}_n$, for a random $r \xleftarrow{\$} \mathbb{Z}_{n/2}$, compute and output $c \leftarrow (1 + n)^m \cdot h^r \bmod n^2$;
- Dec(dk, c): compute $x \leftarrow c^{\mathsf{dk}} \bmod n$ and $c_0 \leftarrow [c \cdot x^{-n} \bmod n^2]$. Return $m \leftarrow (c_0 - 1)/n$.

DVNIZK-Friendly Encryption Scheme. We say that a strongly additive encryption scheme is DVNIZK-*friendly*, when it satisfies the following additional properties:

- Coprimality Property: we require that the size M of the plaintext space and the size R of the random source are coprime, *i.e.*, $\gcd(M, R) = 1$;
- Decodable: for any $(\mathsf{ek}, \mathsf{sk}) \xleftarrow{\$} \mathsf{KeyGen}(1^\lambda)$, the function $f_{\mathsf{ek}} : m \mapsto \mathsf{Enc}_{\mathsf{ek}}(m; 0)$ must be efficiently invertible (*i.e.*, there is a PPT algorithm, which is given ek, computing f_{ek}^{-1} on any value from the image of f_{ek}).

Note that the Paillier cryptosystem is DVNIZK-friendly: $(\gcd(n, \varphi(n)) = 1$, and any message m can be efficiently recovered from $\mathsf{Enc}_{\mathsf{ek}}(m; 0) = (1 + n)^m \bmod n^2)$.

2.3   Non-Interactive Zero-Knowledge Proof of Knowledge Systems

A (designated-verifier) non-interactive zero-knowledge (DVNIZK) proof system for a language $\mathscr{L}$ is a quadruple $(\Pi.\mathsf{Setup}, \Pi.\mathsf{KeyGen}, \Pi.\mathsf{Prove}, \Pi.\mathsf{Verify})$, as follows: $\Pi.\mathsf{Setup}$ generates the setup parameters; $\Pi.\mathsf{KeyGen}$ generate the (public) proving key and the verification key (which is private in a designated-verifier scheme, and public in a publicly-verifiable one); $\Pi.\mathsf{Prove}$, given the proving key, a word x an a witness w for $x \in \mathscr{L}$, outputs a proof π; and $\Pi.\mathsf{Verify}$, given the verification key, x, and π, outputs either accept or reject.

A DVNIZK proof system must be complete (if $x \in \mathscr{L}$, the verifier accept), sound (if $x \notin \mathscr{L}$, no malicious prover can cause the verifier to accept; we usually want a stronger security notion, *unbounded extractability*, which states that a polytime extractor can extract a valid witness from any accepting proof, even if the proof was adversarially generated with arbitrary access to a verification oracle), and *zero-knowledge* (the proof can be simulated without knowledge of the witness). Because of space constraints, we defer to the full version [15] the formal definition of DVNIZKs and their security properties.

The DVNIZK of Chaidos and Couteau. This DVNIZK proof of knowledge system was introduced in [11] and satisfies composable zero-knowledge, and statistical adaptive unbounded knowledge-extractability. The proofs are generated for statements defined by a linear map over $\mathbb{G}$:
Let k be an integer, $(\mathbb{G}, \boldsymbol{+})$ be an abelian group of order k, and (α, β, γ) be three

integers. Let $\boldsymbol{G} \in \mathbb{G}^\alpha$ denote a vector of *public parameters*, and let $\boldsymbol{C} \in \mathbb{G}^\beta$ denote a public *word*. This system considers statements $\mathsf{St}_\Gamma(\boldsymbol{G}, \boldsymbol{C})$ defined by a linear map $\Gamma : (\mathbb{G}^\alpha, \mathbb{G}^\beta) \mapsto \mathbb{G}^{\gamma \times \beta}$ such that $\mathsf{St}_\Gamma(\boldsymbol{G}, \boldsymbol{C}$ coresponds to the statement "I know $\boldsymbol{x} \in \mathbb{Z}_k^\gamma$ such that $\boldsymbol{x} \bullet \Gamma(\boldsymbol{G}, \boldsymbol{C}) = \boldsymbol{C}$". Let $S = (S.\mathsf{KeyGen}, S.\mathsf{Enc}, S.\mathsf{Dec})$ denote a DVNIZK-friendly encryption scheme with plaintext space $\mathbb{Z}_k$. The algorithms $(\Pi_\mathsf{K}.\mathsf{Setup}, \Pi_\mathsf{K}.\mathsf{KeyGen}, \Pi_\mathsf{K}.\mathsf{Prove}, \Pi_\mathsf{K}.\mathsf{Verify})$ form a DVNIZK of knowledge Π_K for a statement $\mathsf{St}_\Gamma(\boldsymbol{G}, \boldsymbol{C})$ over a word $\boldsymbol{C} \in \mathbb{G}^\beta$, with public parameters $\boldsymbol{G} \in \mathbb{G}^\alpha$, defined by a linear map $\Gamma : (\mathbb{G}^\alpha, \mathbb{G}^\beta) \mapsto \mathbb{G}^{\gamma \times \beta}$:

- $\Pi_\mathsf{K}.\mathsf{Setup}(1^\lambda)$: compute $(\mathsf{ek}, \mathsf{dk}) \overset{\$}{\leftarrow} S.\mathsf{KeyGen}(1^\lambda)$. Output $\mathsf{crs} \leftarrow \mathsf{ek}$. Note that ek defines a plaintext space $\mathbb{Z}_k$ and a random source $\mathbb{Z}_R$. As the $\mathsf{IND\text{-}CPA}$ and strong additive properties of S require R to be unknown, we assume that a bound B on R is publicly available. We denote $\ell \leftarrow 2^\lambda k B$.
- $\Pi_\mathsf{K}.\mathsf{KeyGen}(1^\lambda)$: pick $e \leftarrow \mathbb{Z}_\ell$, set $\mathsf{pk} \leftarrow S.\mathsf{Enc}_\mathsf{ek}(0; e)$ and $\mathsf{vk} \leftarrow e$.
- $\Pi_\mathsf{K}.\mathsf{Prove}(\mathsf{pk}, \boldsymbol{C}, \boldsymbol{x})$: on a word $\boldsymbol{C} \in \mathbb{Z}_k^\beta$, with witness $\boldsymbol{x}$ for the statement $\mathsf{St}_\Gamma(\boldsymbol{G}, \boldsymbol{C})$, pick $\boldsymbol{x}' \overset{\$}{\leftarrow} \mathbb{Z}_k^\gamma$, $\boldsymbol{r} \overset{\$}{\leftarrow} \mathbb{Z}_{2^\lambda B}^\gamma$, compute

$$
\begin{aligned}
\boldsymbol{X} &\leftarrow S.\mathsf{Enc}_\mathsf{ek}(\boldsymbol{x}; \boldsymbol{r}), \\
\boldsymbol{X}' &\leftarrow S.\mathsf{Enc}_\mathsf{ek}(\boldsymbol{x}'; 0) \ominus (\boldsymbol{r} \odot \mathsf{pk}) = S.\mathsf{Enc}_\mathsf{ek}(\boldsymbol{x}'; -e \cdot \boldsymbol{r}), \\
\boldsymbol{C}' &\leftarrow \boldsymbol{x}' \bullet \Gamma(\boldsymbol{G}, \boldsymbol{C}),
\end{aligned}
$$

and output $\boldsymbol{\pi} \leftarrow (\boldsymbol{X}, \boldsymbol{X}', \boldsymbol{C}')$.
- $\Pi_\mathsf{K}.\mathsf{Verify}(\mathsf{pk}, \mathsf{vk}, \boldsymbol{C}, \boldsymbol{\pi})$: parse $\boldsymbol{\pi}$ as $(\boldsymbol{X}, \boldsymbol{X}', \boldsymbol{C}')$. Check that $e \odot \boldsymbol{X} \oplus \boldsymbol{X}'$ is decodable, and decode it to a vector $\boldsymbol{d} \in \mathbb{Z}_k^\gamma$. Check that

$$
\boldsymbol{d} \bullet \Gamma(\boldsymbol{G}, \boldsymbol{C}) = e \bullet \boldsymbol{C} + \boldsymbol{C}'.
$$

If all checks succeeded, accept. Otherwise, reject.

3   Message Authentication Codes

In this section, we recall the definition of message authentication codes (MAC), and outline a general MAC scheme (which we call "abstract MAC"), which unifies several existing MAC scheme with a natural algebraic structure. Then, we introduce a stronger unforgeability notion for this abstract MAC scheme. In the full version [15], we prove that one of the MAC schemes of [13] does satisfy this security notion in the generic group model.

3.1   Definition

Definition 1 (Message Authentication Code). We recall the definition of a message authentication code. A message authentication code (MAC) M is a quadruple of PPT algorithms $(M.\mathsf{Setup}, M.\mathsf{KeyGen}, M.\mathsf{Sign}, M.\mathsf{Verify})$, such that

- $M.\mathsf{Setup}(1^\lambda)$ generates the public parameters pp of the MAC. We assume that pp specifies the tag space $\mathcal{S}$ and the message space $\mathcal{M}$;

- $M.\mathsf{KeyGen}(\mathsf{pp})$ generates and outputs a key sk and optionally public issuer parameters ipp.
- $M.\mathsf{Sign}_{\mathsf{sk}}(m)$ given the message $m \in \mathcal{M}$, outputs a tag σ;
- $M.\mathsf{Verify}_{\mathsf{sk}}(m, \sigma)$ given the message $m \in \mathcal{M}$ and a tag $\sigma \in \mathcal{S}$, outputs a bit b whose value depends on the validity of the tag σ with respect to m.

We assume for simplicity that once generated, the public parameters pp are implicitly passed as an argument to the algorithms ($M.\mathsf{KeyGen}$, $M.\mathsf{Sign}$, $M.\mathsf{Verify}$).

Definition 2 (Correctness of a MAC). A Message Authentication Code M is *correct* if for any $\mathsf{pp} \xleftarrow{\$} M.\mathsf{Setup}(1^\lambda)$, any $\mathsf{sk} \xleftarrow{\$} M.\mathsf{KeyGen}(\mathsf{pp})$, any message $m \in \mathcal{M}$ and for $\sigma \xleftarrow{\$} M.\mathsf{Sign}_{\mathsf{sk}}(m)$, it holds that $M.\mathsf{Verify}_{\mathsf{sk}}(m, \sigma) = 1$.

Definition 3 (UF-CMVA Security of a MAC). A MAC M is UF-CMVA *secure* if for any PPT adversary $\mathscr{A}$, it holds that

$$\Pr \left[\begin{array}{l} Q \leftarrow \emptyset, \mathsf{pp} \xleftarrow{\$} M.\mathsf{Setup}(1^\lambda), \\ \mathsf{sk} \xleftarrow{\$} M.\mathsf{KeyGen}(\mathsf{pp}), \\ (m, \sigma) \xleftarrow{\$} \mathscr{A}^{\mathcal{O}_{\mathsf{sk}}[Q]}(\mathsf{pp}) \end{array} : M.\mathsf{Verify}_k(m, \sigma) = 1 \wedge m \notin Q \right] \leq \frac{1}{2} + \mu(\lambda)$$

for some function $\mu(\lambda) = \mathrm{negl}(\lambda)$. $\mathscr{A}$ has access to an oracle $\mathcal{O}_{\mathsf{sk}}[Q]$ which answers to verification and signing queries:

- $\mathcal{O}.\mathsf{Sign}(m)$ sets $Q \leftarrow Q \cup \{m\}$ and outputs $M.\mathsf{Sign}_{\mathsf{sk}}(m)$;
- $\mathcal{O}.\mathsf{Verify}(m, \sigma)$ outputs $M.\mathsf{Verify}_{\mathsf{sk}}(m, \sigma)$.

In this paper we will need algebraic MACs which means that the signing and verification algorithms require only group operations to be performed.

3.2   An Abstract MAC Scheme

Let $\mathbb{G}$ be an abelian group of order n. Given a vector $\boldsymbol{x} = (x_0, \cdots, x_\beta)$ for some integer β, we denote by $H_{\boldsymbol{x}} : \mathbb{Z}_{\mathsf{n}}^\beta \mapsto \mathbb{Z}_{\mathsf{n}}$ the affine function which, on input $(m_1, \cdots, m_\beta)$, outputs $x_0 + \sum_{i=1}^{\beta} x_i \cdot m_i$. Consider now the following generic MAC scheme, parametrized with integers (α, β):

- $M.\mathsf{Setup}(1^\lambda)$: pick a generator G of $\mathbb{G}$ and output $\mathsf{pp} \leftarrow (\mathbb{G}, \mathsf{n}, G, \alpha, \beta)$;
- $M.\mathsf{KeyGen}(\mathsf{pp})$: pick α vectors $(\mathbf{k}_i)_{i \leq \alpha} \in (\mathbb{Z}_{\mathsf{n}}^{\beta+1})^\alpha$ (which can be either random or fixed) of length $\beta+1$, and α random group elements $(G_i)_{i \leq \alpha} \xleftarrow{\$} \mathbb{G}^\alpha$. Set $H_{i,j} \leftarrow \mathsf{k}_{i,j}^{-1} \bullet G$ for $i \in [1..\alpha], j \in [1..\beta]$, $G_i' \leftarrow \mathsf{k}_{i,0} \bullet G_i$ for $i \in [1..\alpha]$, and $\mathsf{ipp} \leftarrow ((H_{i,j})_{1 \leq j \leq \beta})_{i \leq \alpha}, (G_i, G_i')_{i \leq \alpha})$. Output $\mathsf{sk} = (\mathbf{k}_i)_{i \leq \alpha}$ and ipp.
- $M.\mathsf{Sign}_{\mathsf{sk}}(\boldsymbol{m})$: given a message $\boldsymbol{m} = (m_1, .., m_\beta) \in \mathbb{Z}_{\mathsf{n}}^\beta$, pick a random group element $U \xleftarrow{\$} \mathbb{G}$ and output

$$\sigma \leftarrow (U, (H_{\mathbf{k}_i}(\boldsymbol{m}) \bullet U)_{i \leq \alpha}).$$

- $M.\mathsf{Verify}_{\mathsf{sk}}(\boldsymbol{m}, \sigma)$: parse σ as $(U, (V_i)_{i \leq \alpha})$ and check that for $i = 1$ to α, $V_i = H_{\mathbf{k}_i}(\boldsymbol{m}) \bullet U$.

Example 1. The scheme $\mathsf{MAC_{GGM}}$ from [13] is obtained by setting $\alpha = 1$, and sampling the key $\mathbf{k}$ uniformly at random. This scheme is $\mathsf{UF\text{-}CMVA}$-secure in the generic group model. Similarly, we recover the scheme $\mathsf{MAC_{DDH}}$ from [13] by setting $\alpha = 3$, sampling $\mathbf{k}_1, \mathbf{k}_2$ at random, and setting $\mathbf{k}_3 \leftarrow (\mathsf{k}_{3,0}, 0, \cdots, 0)$ for a uniformly random $\mathsf{k}_{3,0}$. This scheme is $\mathsf{UF\text{-}CMVA}$-secure under the decisional Diffie-Hellman assumption.

Note that for our construction of an anonymous credential scheme, we will require the security of the underlying MAC scheme to hold in a group of composite order. In the full version [15], we slightly modify $\mathsf{MAC_{GGM}}$ and prove that the modified version is secure in non-prime order groups in the generic group model.

3.3 Extended Unforgeability

The $\mathsf{UF\text{-}CMVA}$ security property states that no PPT adversary should be able to forge a MAC on a message, even given access to signing and verification oracles, as long as this message was never queried to the signing oracle. One can consider stronger notions of unforgeability, where the adversary is given access to an additional oracle. In particular, it will be useful in our setting to consider the following *extended unforgeability* property for the abstract MAC scheme defined above:

Definition 4 (Extended Unforgeability). An abstract MAC M is $\mathsf{XUF\text{-}CMVA}$ *secure* if for any PPT adversary $\mathscr{A}$, it holds that

$$\Pr \left[\begin{array}{l} Q \leftarrow \emptyset, \mathsf{pp} \xleftarrow{\$} M.\mathsf{Setup}(1^\lambda), \\ \mathsf{sk} \xleftarrow{\$} M.\mathsf{KeyGen}(\mathsf{pp}), \\ (m, \sigma) \xleftarrow{\$} \mathscr{A}^{\mathcal{O}_{\mathsf{sk}}[Q]}(\mathsf{pp}) \end{array} : M.\mathsf{Verify}_{\mathsf{sk}}(m, \sigma) = 1 \wedge m \notin Q \right] \leq \frac{1}{2} + \mu(\lambda)$$

for some function $\mu(\lambda) = \mathrm{negl}(\lambda)$. $\mathscr{A}$ has access to an oracle $\mathcal{O}_{\mathsf{sk}}[Q]$ which answers to verification and signing queries, as well as another specific type of query:

- $\mathcal{O}.\mathsf{Sign}(m)$ sets $Q \leftarrow Q \cup \{m\}$ and outputs $M.\mathsf{Sign}_{\mathsf{sk}}(m)$;
- $\mathcal{O}.\mathsf{Verify}(m, \sigma)$ outputs $M.\mathsf{Verify}_{\mathsf{sk}}(m, \sigma)$;
- $\mathcal{O}.\mathsf{Check}((A_{i,j})_{i \leq \alpha, j \leq \beta}, (B_{i,j})_{i \leq \alpha, j \leq \beta})$ checks $\sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet A_{i,j} = \sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet B_{i,j}$ for all $i \leq \alpha$, and outputs 1 iff all checks succeed (note: the $\mathcal{O}.\mathsf{Check}$ oracle could equivalently check whether $\sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet A_{i,j} = 0$).

In the full version [15], we will prove that the scheme $\mathsf{MAC_{GGM}}$ from [13], which was proven $\mathsf{UF\text{-}CMVA}$-secure over prime order groups in the generic group model in [13], is in fact $\mathsf{XUF\text{-}CMVA}$-*secure* in the generic group model over *composite order groups* (the use of groups of composite order is required for compatibility of the MAC scheme with the DVNIZK scheme of [11]), under the computational subgroup assumption. Note that, while it is uncommon to prove security in the GGM under an additional assumption, it is necessary here: there exists an explicit attack against the security of the MAC if the adversary is able to compute a generator of a strict subgroup of $\mathbb{G}$. However, in the usual formulation of the

GGM, the adversary is unbounded and receives as input the order of the group, hence he can trivially factor this order and efficiently compute generators of strict subgroups of $\mathbb{G}$, showing that $\mathsf{MAC}_{\mathsf{GGM}}$ is in fact *not* unconditionally secure in the GGM over composite order groups.

4   Non-Interactive Keyed-Verification Anonymous Credentials

In this section, we formally introduce non-interactive keyed-verification anonymous credentials and their security properties. Our definition mostly follows the blueprint of [13].

Definition 5 (Non-Interactive Keyed-Verification Anonymous Credentials). An non-interactive keyed-verification anonymous credentials (NIKVAC) scheme Θ is a set of algorithms ($\Theta.\mathsf{Setup}, \Theta.\mathsf{CredKeyGen}, \Theta.\mathsf{BlindIssue}, \Theta.\mathsf{BlindObtain}, \Theta.\mathsf{Show}, \Theta.\mathsf{ShowVerify}$) such that

- $\Theta.\mathsf{Setup}(1^\lambda)$, outputs the public parameters pp of the AC and a trapdoor td, the public parameters fix the set of supported statements $\mathbf{\Phi}$, the universe of attributes $\mathcal{U}$ and are passed to the following algorithms implicitly, the trapdoor can be used to revoke anonymity;
- $\Theta.\mathsf{CredKeyGen}(\mathsf{pp})$, generates a secret key sk and public issuer parameters ipp for an issuing organization;
- $\Theta.\mathsf{BlindIssue}(\mathsf{sk}, S) \leftrightarrow \Theta.\mathsf{BlindObtain}(\mathsf{ipp}, (m_1, ..., m_l))$, interactively generates a credential cred for the attributes $(m_1, .., m_l) \in \mathcal{U}$, where $S \subset \{m_1, .., m_l\}$ (here, S refers to the subset of attributes that the user wants to keep private; it allows to flexibly choose which attributes should be revealed, and which should not);
- $\Theta.\mathsf{Show}(\mathsf{ipp}, \mathsf{cred}, (m_1, ..., m_l), \Phi)$, outputs a proof of possession π of a credential cred for organization with issuer parameters ipp in respect to the attributes $(m_1, .., m_l) \in \mathcal{U}$ with associated statements $\Phi \in \mathbf{\Phi}$;
- $\Theta.\mathsf{ShowVerify}(\mathsf{sk}, \pi, \Phi)$, checks the proof π with sk with respect to the statements $\Phi \in \mathbf{\Phi}$;

which satisfies the *correctness*, *anonymity*, *unforgeability*, *blind issuance* and *key-parameter consistency* properties defined below.

We define two extra algorithms to simplify the security definitions:

- $\mathsf{Issue}(\mathsf{sk}, (m_1, .., m_l))$: generates a credential for the attributes $(m_1, .., m_l)$ using sk;
- $\mathsf{CredVerify}(\mathsf{sk}, (m_1, .., m_l), \mathsf{cred})$: verifies the credential cred using sk.

Here we define correctness, which guarantees that Issue always outputs proper credentials and that a proof of possession for a valid credential verifies correctly.

Definition 6 (Correctness). A NIKVAC scheme Θ is *correct* if it holds that

$$\Pr\left[\begin{array}{l}(\mathsf{pp},\mathsf{td}) \xleftarrow{\$} \Theta.\mathsf{Setup}(1^\lambda), (m_1,..,m_l) \xleftarrow{\$} \mathcal{U}, \\ (\mathsf{sk},\mathsf{ipp}) \xleftarrow{\$} \Theta.\mathsf{CredKeyGen}(1^\lambda), \\ \mathsf{cred} \xleftarrow{\$} \mathsf{Issue}(\mathsf{sk},(m_1,..,m_l)), \\ b \xleftarrow{\$} \mathsf{CredVerify}(\mathsf{sk},(m_1,..,m_l),\mathsf{cred})\end{array} : b = 1 \right] = 1$$

and

$$\Pr\left[\begin{array}{l}(\mathsf{pp},\mathsf{td}) \xleftarrow{\$} \Theta.\mathsf{Setup}(1^\lambda), \\ \Phi \in \mathbf{\Phi}, (m_1,..,m_l) \xleftarrow{\$} \mathcal{U} \text{ with } \Phi(m_1,..,m_l) = 1, \\ (\mathsf{sk},\mathsf{ipp}) \xleftarrow{\$} \Theta.\mathsf{CredKeyGen}(1^\lambda), \\ \mathsf{cred} \xleftarrow{\$} \mathsf{Issue}(\mathsf{sk},(m_1,..,m_l)), \\ \pi \xleftarrow{\$} \Theta.\mathsf{Show}(\mathsf{ipp},\mathsf{cred},(m_1,...,m_l),\Phi) \\ b \xleftarrow{\$} \Theta.\mathsf{ShowVerify}(\mathsf{sk},\pi,\Phi),\end{array} : b = 1 \right] = 1$$

Unforgeability ensures that users cannot successfully show credentials without having received one from the authority.

Definition 7 (Unforgeability). A NIKVAC scheme Θ is *unforgeable* if for any PPT adversary $\mathscr{A}$ it holds that

$$\Pr\left[\begin{array}{l}\mathsf{pp} \xleftarrow{\$} \mathsf{Setup}(1^\lambda), Q \leftarrow \emptyset, \\ (\mathsf{sk},\mathsf{ipp}) \xleftarrow{\$} \Theta.\mathsf{CredKeyGen}(1^\lambda), \\ (\Phi,\pi) \xleftarrow{\$} \mathscr{A}^{\mathcal{O}_{\mathsf{sk}}[Q]}(\mathsf{pp},\mathsf{ipp}), \\ b \xleftarrow{\$} \Theta.\mathsf{ShowVerify}(\mathsf{sk},\pi,\Phi),\end{array} : \begin{array}{l} b = 1 \land \forall (m_1,..,m_l) \in Q : \\ \Phi(m_1,..,m_l) = 0 \end{array} \right] \le \mu(\lambda)$$

for some function $\mu(\lambda) = \mathsf{negl}(\lambda)$. $\mathscr{A}$ has access to an oracle $\mathcal{O}_{\mathsf{sk}}[Q]$ which answers to issuing and verification queries:

- $\mathcal{O}.\mathsf{Issue}(m_1,..,m_l)$ sets $Q \leftarrow Q \cup \{m_1,..,m_l\}$ and outputs $\mathsf{Issue}(\mathsf{sk},\mathsf{ipp},(m_1,..,m_l))$;
- $\mathcal{O}.\mathsf{Verify}(\Phi,\pi)$ outputs $\Theta.\mathsf{ShowVerify}(\mathsf{sk},\pi,\Phi)$.

Anonymity ensures that a user that shows a credential stays anonymous. Note that, as observed in [13], this simulation-style notion of anonymity implies in particular the standard notion of multi-show unlinkability, which states that anonymity is preserved throughout multiple presentations of the credential (a property which is not satisfied by e.g. U-Prove [28]).

Definition 8 (Anonymity). A NIKVAC scheme Θ is *anonymous* if for any PPT adversary $\mathscr{A}$, there exists a PPT simulator Sim such that it holds that

$$\left| \Pr\left[\begin{array}{l}(\mathsf{pp},\mathsf{td}) \xleftarrow{\$} \Theta.\mathsf{Setup}(1^\lambda), \\ (\mathsf{sk},\mathsf{ipp}) \xleftarrow{\$} \Theta.\mathsf{CredKeyGen}(1^\lambda), \\ (\Phi,\mathsf{cred},(m_1,..,m_l),\mathsf{st}) \xleftarrow{\$} \mathscr{A}(\mathsf{pp},\mathsf{ipp},\mathsf{sk}), \\ \pi \xleftarrow{\$} \Theta.\mathsf{Show}(\mathsf{ipp},\mathsf{cred},(m_1,...,m_l),\Phi)\end{array} : \begin{array}{l}\mathsf{CredVerify}(\mathsf{sk},(m_1,..,m_l),\mathsf{cred}) \\ := 1 \land \Phi(m_1,..,m_l) = 1 \land \\ \mathscr{A}(\mathsf{st},\pi) = 1\end{array} \right] - \right.$$
$$\left. \Pr\left[\begin{array}{l}(\mathsf{pp},\mathsf{td}) \xleftarrow{\$} \Theta.\mathsf{Setup}(1^\lambda), \\ (\mathsf{sk},\mathsf{ipp}) \xleftarrow{\$} \Theta.\mathsf{CredKeyGen}(1^\lambda), \\ (\Phi,\mathsf{cred},(m_1,..,m_l),\mathsf{st}) \xleftarrow{\$} \mathscr{A}(\mathsf{pp},\mathsf{ipp},\mathsf{sk}), \\ \pi \xleftarrow{\$} \mathsf{Sim}(\mathsf{ipp},\mathsf{sk},\Phi)\end{array} : \begin{array}{l}\mathsf{CredVerify}(\mathsf{sk},(m_1,..,m_l),\mathsf{cred}) \\ := 1 \land \Phi(m_1,..,m_l) = 1 \land \\ \mathscr{A}(\mathsf{st},\pi) = 1\end{array} \right] \right| \le \mu(\lambda)$$

for some function $\mu(\lambda) = \text{negl}(\lambda)$.

Blind Issuance. The protocol BlindIssue $\leftrightarrow$ BlindObtain defines a secure two-party protocol for the function $f((S, \text{pp}, \text{ipp}), (\text{sk}, r), (m_1, .., m_l))$ for shared input $(S, \text{pp}, \text{ipp})$, issuer input (sk, r) and user input $(m_1, .., m_l)$ which returns cred $\leftarrow$ Issue$(\text{sk}, (m_1, .., m_l); r)$ to the user, if the input is correct. Since we will not cover this property explicitly in this paper, refer to [13] for more details.

Definition 9 (Key-Parameter Consistency). A NIKVAC scheme Θ fulfills the *key-parameter consistency* property if for any PPT adversary $\mathscr{A}$, it holds that

$$\Pr\left[\begin{array}{l} (\text{pp}, \text{td}) \overset{\$}{\leftarrow} \Theta.\text{Setup}(1^\lambda), \quad \text{for } i \in \{0, 1\}, \\ (\text{ipp}, \text{sk}_0, \text{sk}_1) \overset{\$}{\leftarrow} \mathscr{A}(\text{pp}) \ : (\text{ipp}, \text{sk}_i) \in \{x \mid x \overset{\$}{\leftarrow} \Theta.\text{CredKeyGen}(1^\lambda)\} \end{array}\right] \le \mu(\lambda)$$

for some function $\mu(\lambda) = \text{negl}(\lambda)$.

4.1 Additional Properties

Anonymity Revocation. The following property would allow a trusted third party to revoke anonymity if desired.

Definition 10 (Extractability). A NIKVAC scheme Θ is *extractable* if there exists an efficient extractor Ext such that

$$\Pr\left[\begin{array}{l} (\text{pp}, \text{td}) \overset{\$}{\leftarrow} \Theta.\text{Setup}(1^\lambda), \\ \Phi \in \mathbf{\Phi}, (m_1, .., m_l) \overset{\$}{\leftarrow} \mathcal{U}, \\ (\text{sk}, \text{ipp}) \overset{\$}{\leftarrow} \Theta.\text{CredKeyGen}(1^\lambda), \qquad : (m_1, .., m_l) \leftarrow \text{Ext}(\text{td}, \pi) \\ \text{cred} \overset{\$}{\leftarrow} \text{Issue}(\text{sk}, (m_1, .., m_l)), \\ \pi \overset{\$}{\leftarrow} \Theta.\text{Show}(\text{ipp}, \text{cred}, (m_1, ..., m_l), \Phi) \end{array}\right] = 1$$

5 Oblivious Designated-Verifier Non-Interactive Zero-Knowledge

In this section, we introduce oblivious (designated-verifier, non-interactive) zero-knowledge proof system. Intuitively, an oblivious DVNIZK enhances the security and the functionality of a DVNIZK with two properties:

– First, the oblivious DVNIZK on a word x can be used to show knowledge of a witness w such that $R_{\text{sk}}(x, w) = 1$, where R_{sk} is a *secret* witness relation, which depends on a secret information which is not known to the prover. The knowledge of sk is not required to generate a proof – but it is, obviously, necessary to verify the proof.

– Second, we consider words x which can be divided in subwords (x_0, x_1), such that x_0 is a *public subword*, while x_1 is a *private subword*. The privacy of x_1 is ensured by requiring, for the zero-knowledge property, the existence of a simulator which can simulate a proof without knowing the witness w and/or x_1. Note that this formalism is mainly chosen for notational convenience: the word x_1 could always be thought of as being part of the witness. However, defining it as a part of the word allows us to set the secret relation R_{sk} to be exactly the MAC verification, where the word is the signature and the witness is the message, in our concrete instantiation.

5.1   Definition

Definition 11 (Oblivious DVNIZK). An oblivious designated-verifier non-interactive zero-knowledge proof of knowledge Π for a family of *secret* witness relations $\{R_{\mathsf{crs}}(\cdot, \cdot, \cdot)\}_{\mathsf{crs}}$ (which take as input triples (sk, x, w) where sk is a *secret relation key*, x is a word, and w is a witness for the relation $R_{\mathsf{crs}}(\mathsf{sk}, \cdot, \cdot)$) is a five-tuple $(\Pi.\mathsf{Setup}, \Pi.\mathsf{RelSetup}, \Pi.\mathsf{KeyGen}, \Pi.\mathsf{Prove}, \Pi.\mathsf{Verify})$ of efficient algorithms such that

– $\Pi.\mathsf{Setup}(1^\lambda)$, on input the security parameter in unary, outputs a pair $(\mathsf{crs}, \mathsf{td})$ where crs is a *common reference string* and td is a *trapdoor*;
– $\Pi.\mathsf{RelSetup}(\mathsf{crs})$, on input crs, outputs a pair $(\mathsf{sk}, \mathsf{ipp})$, where sk is a secret *relation key*, and ipp are public issuer parameters;
– $\Pi.\mathsf{KeyGen}(\mathsf{crs})$, on input crs, outputs a pair $(\mathsf{pk}, \mathsf{vk})$ where pk is a public *proving key*, and vk is a secret *verification key*;
– $\Pi.\mathsf{Prove}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, (x_0, x_1), w)$, on input crs, the public key pk, the issuer parameters ipp, a word (x_0, x_1), where x_0 is a public subword and x_1 is a secret subword, and a witness w such that $R_{\mathsf{crs}}(\mathsf{sk}, (x_0, x_1), w) = 1$, outputs a proof π;
– $\Pi.\mathsf{Verify}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, x_0, \mathsf{vk}, \mathsf{sk}, \pi)$, on input crs, the public key pk, the issuer parameters ipp, the public subword x_0, the verification key vk, the secret relation key sk, and a proof π, outputs a bit $b \in \{0, 1\}$;

which satisfies the completeness, oblivious zero-knowledge, and oblivious knowledge-extractability properties defined below.

Definition 12 (Completeness). An oblivious DVNIZK proof system $\Pi = (\Pi.\mathsf{Setup}, \Pi.\mathsf{RelSetup}, \Pi.\mathsf{KeyGen}, \Pi.\mathsf{Prove}, \Pi.\mathsf{Verify})$ for a family of secret witness relations $\{R_{\mathsf{crs}}\}_{\mathsf{crs}}$ satisfies *completeness* if for every $(\mathsf{crs}, \mathsf{td})$ in the image of $\Pi.\mathsf{Setup}(1^\lambda)$, every $(\mathsf{sk}, \mathsf{ipp})$ in the image of $\Pi.\mathsf{RelSetup}(\mathsf{crs})$, every $(\mathsf{pk}, \mathsf{vk}, \mathsf{sk})$ in the image of $\Pi.\mathsf{KeyGen}(\mathsf{crs})$, every $((x_0, x_1), w)$ such that $R_{\mathsf{crs}}(\mathsf{sk}, (x_0, x_1), w) = 1$, and every π in the image of $\Pi.\mathsf{Prove}(\mathsf{pk}, \mathsf{ipp}, (x_0, x_1), w)$, it holds that $\Pi.\mathsf{Verify}(\mathsf{pk}, \mathsf{ipp}, x_0, \mathsf{vk}, \mathsf{sk}, \pi) = 1$.

Definition 13 (Oblivious Zero-Knowledge). An oblivious DVNIZK proof system $\Pi = (\Pi.\mathsf{Setup}, \Pi.\mathsf{RelSetup}, \Pi.\mathsf{KeyGen}, \Pi.\mathsf{Prove}, \Pi.\mathsf{Verify})$ for a family of

witness relations $\{R_{\mathsf{crs}}\}_{\mathsf{crs}}$ satisfies *oblivious zero-knowledge* if for any stateful PPT Adv, there exists a probabilistic polynomial-time simulator Sim such that

$$
\left| \Pr \begin{bmatrix} (\mathsf{crs}, \mathsf{td}) \xleftarrow{\$} \Pi.\mathsf{Setup}(1^\lambda), \\ (\mathsf{pk}, \mathsf{vk}) \xleftarrow{\$} \Pi.\mathsf{KeyGen}(\mathsf{crs}), \\ ((x_0, x_1), w, \mathsf{ipp}, \mathsf{sk}) \leftarrow \mathscr{A}(\mathsf{crs}, \mathsf{pk}, \mathsf{vk}), \\ \pi \leftarrow \Pi.\mathsf{Prove}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, (x_0, x_1), w), \end{bmatrix} : \begin{matrix} (R_{\mathsf{crs}}(\mathsf{sk}, (x_0, x_1), w) = 1) \\ \wedge (\mathscr{A}(\pi) = 1) \end{matrix} \right.
$$

$$
\left. - \Pr \begin{bmatrix} (\mathsf{crs}, \mathsf{td}) \xleftarrow{\$} \Pi.\mathsf{Setup}(1^\lambda), \\ (\mathsf{pk}, \mathsf{vk}, \mathsf{sk}) \xleftarrow{\$} \Pi.\mathsf{KeyGen}(\mathsf{crs}), \\ ((x_0, x_1), w, \mathsf{sk}, \mathsf{ipp}) \leftarrow \mathscr{A}(\mathsf{crs}, \mathsf{pk}, \mathsf{vk}), \\ \pi \leftarrow \mathsf{Sim}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, x_0, \mathsf{vk}, \mathsf{sk}), \end{bmatrix} : \begin{matrix} (R_{\mathsf{crs}}(\mathsf{sk}, (x_0, x_1), w) = 1) \\ \wedge (\mathscr{A}(\pi) = 1) \end{matrix} \right| \le \mu(\lambda)
$$

where $\mu(\lambda) = \mathrm{negl}(\lambda)$.

Definition 14 $((\mathcal{O}_0, \mathcal{O}_1)$-Knowledge-Extractability). An oblivious DVNIZK proof system $\Pi = (\Pi.\mathsf{Setup}, \Pi.\mathsf{RelSetup}, \Pi.\mathsf{KeyGen}, \Pi.\mathsf{Prove}, \Pi.\mathsf{Verify})$ for a family of secret witness relations $\{R_{\mathsf{crs}}\}_{\mathsf{crs}}$ satisfies $(\mathcal{O}_0, \mathcal{O}_1)$-*knowledge-extractability* if the following two conditions hold:

- for every PPT adversary $\mathscr{A}$, there is an efficient extractor Ext such that

$$
\Pr \begin{bmatrix} (\mathsf{crs}, \mathsf{td}) \xleftarrow{\$} \mathsf{Setup}(1^\lambda), \\ (\mathsf{sk}, \mathsf{ipp}) \xleftarrow{\$} \mathsf{RelSetup}(\mathsf{crs}), \\ (\mathsf{pk}, \mathsf{vk}) \xleftarrow{\$} \Pi.\mathsf{KeyGen}(\mathsf{crs}), \\ (\pi, x_0) \leftarrow \mathscr{A}^{\mathcal{V}, \mathcal{O}_0[\mathsf{sk}]}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}), \\ (x_1, w) \leftarrow \mathsf{Ext}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, x_0, \mathsf{td}, \pi), \end{bmatrix} : \begin{matrix} R_{\mathsf{crs}}(\mathsf{sk}, (x_0, x_1), w) = 0 \wedge \\ \Pi.\mathsf{Verify}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, x_0, \mathsf{vk}, \\ \mathsf{sk}, \pi) = 1 \end{matrix} \approx 0,
$$

  where $\mathcal{V}$ denotes $\Pi.\mathsf{Verify}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, \cdot, \mathsf{vk}, \mathsf{sk}, \cdot)$;
- there exists an efficient simulator that simulates the answers of $\Pi.\mathsf{Verify}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, \cdot, \mathsf{vk}, \mathsf{sk}, \cdot)$, which is not given sk but is instead given oracle access to $\mathcal{O}_1[\mathsf{sk}]$.

5.2 Instantiation

We now provide an instantiation of an oblivious DVNIZK suitable for our construction. At a high level, the secret witness relation we consider will be the one that checks, for triples (sk, x, w), that the message w is the one signed in the credential x (with respect to the secret key sk of the abstract MAC scheme defined in Section 3.1). Our construction heavily builds upon the DVNIZK proof system of [11]. Let $S = (S.\mathsf{KeyGen}, S.\mathsf{Enc}, S.\mathsf{Dec})$ denote a DVNIZK-friendly encryption scheme with plaintext space $\mathbb{Z}_{\mathsf{n}}$ and $M = (M.\mathsf{Setup}, M.\mathsf{KeyGen}, M.\mathsf{Sign}, M.\mathsf{Verify})$ be a MAC scheme, which we assume to have the abstract structure given in Section 3.1, over a group $\mathbb{G}$ of order n with generator G. We will consider the following witness relation: $R_{\mathsf{crs}}(\mathsf{sk}, x, w)$, given as input a vector $x = (U, (V_i)_{i \le \alpha}) \in \mathbb{G}^{\alpha+1}$ of group elements, a witness $w = (m_1, \cdots, m_\beta)$, and given sk, checks that $M.\mathsf{Ver}_{\mathsf{sk}}(m_1, \cdots, m_\beta, x) = 1$, where $\mathsf{sk} = (\mathbf{k}_i)_{i \le \alpha}$ is the MAC key. Since the purpose of the public word x_0 is mainly to allow more expressivity when considering

a more complex relation, and we focus here on the most basic relation (the scheme can be enhanced to work with more complex relations), we simply consider that $x = x_1$ is entirely a secret word. The scheme works as follows:

- $\Pi.\mathsf{Setup}(1^\lambda)$: compute $(\mathsf{ek}, \mathsf{dk}) \xleftarrow{\$} S.\mathsf{KeyGen}(1^\lambda)$ and $\mathsf{pp} \xleftarrow{\$} M.\mathsf{Setup}(1^\lambda)$. Output $\mathsf{crs} \leftarrow (\mathsf{ek}, \mathsf{pp})$. Note that ek defines a plaintext space $\mathbb{Z}_\mathsf{n}$ and a random source $\mathbb{Z}_R$. As the IND-CPA and strong additive properties of S require R to be unknown, we assume that a bound B on R is publicly available. We denote $\ell \leftarrow 2^\lambda \mathsf{n} B$.
- $\Pi.\mathsf{RelSetup}(\mathsf{crs})$: same as $M.\mathsf{KeyGen}$, namely: pick α vectors $(\mathbf{k}_i)_{i \leq \alpha} \in (\mathbb{Z}_\mathsf{n}^\beta)^\alpha$ (which can be either random or fixed) of length β, and α random group elements $(G_i)_{i \leq \alpha} \xleftarrow{\$} \mathbb{G}^\alpha$. Set $H_{i,j} \leftarrow \mathbf{k}_{i,j}^{-1} \bullet G$ for $i \in [1..\alpha], j \in [1..\beta]$, $G_i' \leftarrow \mathbf{k}_{i,0} \bullet G_i$ for $i \in [1..\alpha]$, and $\mathsf{ipp} \leftarrow ((H_{i,j})_{1 \leq j \leq \beta})_{i \leq \alpha}, (G_i, G_i')_{i \leq \alpha})$. Output $\mathsf{sk} = (\mathbf{k}_i)_{i \leq \alpha}$ and ipp.
- $\Pi.\mathsf{KeyGen}(\mathsf{crs})$: pick $e \leftarrow \mathbb{Z}_\ell$, set $\mathsf{pk} \leftarrow S.\mathsf{Enc}_\mathsf{ek}(0; e)$ and $\mathsf{vk} \leftarrow e$. Output $(\mathsf{pk}, \mathsf{vk})$.
- $\Pi.\mathsf{Prove}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, x, w)$: given $x = (U, (V_i)_{i \leq \alpha})$ and a witness $w = \boldsymbol{m}$, pick $(\boldsymbol{m}', \boldsymbol{t}, \boldsymbol{t}') \xleftarrow{\$} \mathbb{Z}_\mathsf{n}^\beta \times (\mathbb{Z}_\mathsf{n}^\alpha)^2$, $(\boldsymbol{r_m}, \boldsymbol{r_t}) \xleftarrow{\$} \mathbb{Z}_{2^\lambda B}^\beta \times \mathbb{Z}_{2^\lambda B}^\alpha$, $z \xleftarrow{\$} \mathbb{Z}_\mathsf{n}$. Let $(\boldsymbol{t_j'})_{j \leq \beta} = (t_{1,j}', \cdots, t_{\alpha,j}')_{j \leq \beta}$ denote uniformly random additive shares of $\boldsymbol{t}'$ over $\mathbb{Z}_\mathsf{n}^\alpha$. Compute

$$(U', (V_i')_{i \leq \alpha}) \leftarrow (z \bullet U, ((z \bullet V_i) \boldsymbol{+} (t_i \bullet G))_{i \leq \alpha})$$
$$W_{i,j} \leftarrow m_j' \bullet U' \boldsymbol{+} (t_{i,j}' \bullet H_{i,j}) \text{ for } i \in [1..\alpha], j \in [1..\beta]$$
$$(\boldsymbol{X_m}, \boldsymbol{X_t}) \leftarrow (S.\mathsf{Enc}_\mathsf{ek}(\boldsymbol{m}; \boldsymbol{r_m}), S.\mathsf{Enc}_\mathsf{ek}(\boldsymbol{t}; \boldsymbol{r_t})),$$
$$\boldsymbol{X_m'} \leftarrow S.\mathsf{Enc}_\mathsf{ek}(\boldsymbol{m}'; 0) \ominus (\boldsymbol{r_m} \odot \mathsf{pk}) = S.\mathsf{Enc}_\mathsf{ek}(\boldsymbol{m}'; -e \cdot \boldsymbol{r_m}),$$
$$\boldsymbol{X_t'} \leftarrow S.\mathsf{Enc}_\mathsf{ek}(\boldsymbol{t}'; 0) \ominus (\boldsymbol{r_t} \odot \mathsf{pk}) = S.\mathsf{Enc}_\mathsf{ek}(\boldsymbol{t}'; -e \cdot \boldsymbol{r_t}),$$

and output $\pi \leftarrow (U', (V_i')_{i \leq \alpha}, (W_{i,j})_{i \leq \alpha, j \leq \beta}, \boldsymbol{X_m}, \boldsymbol{X_t}, \boldsymbol{X_m'}, \boldsymbol{X_t'})$.
- $\Pi.\mathsf{Verify}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, \mathsf{vk}, \mathsf{sk}, \pi)$: parse π as

$$(U', (V_i')_{i \leq \alpha}, (W_{i,j})_{i \leq \alpha, j \leq \beta}, \boldsymbol{X_m}, \boldsymbol{X_t}, \boldsymbol{X_m'}, \boldsymbol{X_t'}).$$

Check that $e \odot \boldsymbol{X_m} \oplus \boldsymbol{X_m'}$ and $e \odot \boldsymbol{X_t} \oplus \boldsymbol{X_t'}$ are decodable, and decode them to vectors $\boldsymbol{d_m}, \boldsymbol{d_t}$. Reconstruct

$$(W_i')_{i \leq \alpha} \leftarrow \left(\sum_{j=1}^\beta \mathbf{k}_{i,j} \bullet W_{i,j} \right)_{i \leq \alpha}$$

and check that

$$(e \bullet (V_i' \boldsymbol{-} (\mathbf{k}_{i,0} \bullet U')) \boldsymbol{+} W_i')_{i \leq \alpha} = (\boldsymbol{d_m} \bullet (\mathbf{k}_{i,j} \bullet U')_{1 \leq j \leq \beta})_{i \leq \alpha} \boldsymbol{+} \boldsymbol{d_t} \bullet G.$$

Output 1 if and only if all checks succeeded.

Theorem 2. *The scheme Π is an oblivious designated-verifier zero-knowledge proof of knowledge for the family of secret witness relations $\{R_\mathsf{crs}\}_\mathsf{crs}$, whose*

oblivious zero-knowledge property reduces to the semantic security of the DVNIZK-*friendly encryption scheme S, and which satisfies statistical* $(\mathcal{O}_0, \mathcal{O}_1)$*-knowledge extractability for the oracle* $\mathcal{O}_0[\mathsf{sk}] \equiv M.\mathsf{Sign}_{\mathsf{sk}}$*, and an oracle* $\mathcal{O}_1[\mathsf{sk}]$ *which is either*

- *M*.$\mathsf{Verify}_{\mathsf{sk}}(\cdot, \cdot)$ *if* $\beta = 1$*, or*
- *M*.$\mathsf{Verify}_{\mathsf{sk}}(\cdot, \cdot)$ *together with* *M*.$\mathsf{Check}_{\mathsf{sk}}(\cdot, \cdot)$ *otherwise.*

5.3   Extensions and Optimizations

In itself, the above oblivious DVNIZK does not seem to provide a strong un-forgeability guarantee. Indeed, recall that the unforgeability of keyed-verification anonymous credential states (informally) that it should be infeasible to come up with a pair $(\boldsymbol{m}, \sigma)$ such that $M.\mathsf{Ver}_{\mathsf{sk}}(\boldsymbol{m}, \sigma) = 1$ and $\Phi(\boldsymbol{m}) = 1$, if all previous queries to the signing authority where on vectors $\boldsymbol{m}'$ such that $\Phi(\boldsymbol{m}') = 0$. The exact choice of Φ depends on the particular application; typically, $\Phi(\boldsymbol{m})$ could correspond to the statement that $\boldsymbol{m}$ is the value committed in some *pseudonym* known to the verifier; that way, the condition "all previous queries to the signing authority where on vectors $\boldsymbol{m}'$ such that $\Phi(\boldsymbol{m}') = 0$" boils down to the standard guarantee of anonymous credentials: it should be infeasible to come up with an accepting credential on a vector that was never signed before by the authority. But Φ can also check a more complex statement on the vector of attributes (e.g. it could check that the attribute "age" is above 18).

In the construction given above, we directly focus on enforcing $M.\mathsf{Ver}_{\mathsf{sk}}(\boldsymbol{m}, \sigma) = 1$; there is no additional Φ to, for example, bind $\boldsymbol{m}$ to a commitment. However, we observe that this typical choice of Φ is *for free* in our construction above. Indeed, a proof π does contain, by construction, a perfectly binding commitment (in fact, an encryption with S) of the vector $\boldsymbol{m}$, which is $\boldsymbol{X_m}$. Furthermore, it will immediately follow from the security analysis that the proof does not only guarantee the knowledge of a witness $w = \boldsymbol{m}$ (recovered by the extractor): it further guarantees that this witness is exactly the one encrypted in $\boldsymbol{X_m}$. Therefore, to bind the user to a pseudonym known to the verifier, it is unecessary to add a commitment to $\boldsymbol{m}$. Instead, the user can simply compute $(\boldsymbol{X_m}, \boldsymbol{X_t}, \boldsymbol{X_m'}, \boldsymbol{X_t'})$ in advance (observe that this does not require the knowledge of a credential) and send it to the verifier, who will simply define it to be the pseudonym of the user. Then, each time he wants to show possession of a credential $(U, (V_i)_{i \leq \alpha})$, the user only needs to compute the *missing part* of the proof, $(U', (V_i')_{i \leq \alpha}, (W_{i,j})_{i \leq \alpha, j \leq \beta})$. This significantly reduces the size of a proof of possession, and in scenario where Φ is only intended to check that the vector matches with a pseudonym, the basic construction suffices as is. Of course, it can be extended to more complex statements Φ, as long as they fit in the framework of statements handled by [11].

5.4   Security Analysis

Completeness follows from a straightforward (although tedious) inspection. We now establish oblivious zero-knowledge and $(\mathcal{O}_0, \mathcal{O}_1)$-knowledge extractability.

Oblivious Zero-Knowledge. We exhibit a simulator Sim which simulates a proof π given $(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, x_0, \mathsf{vk} = e, \mathsf{sk} = (\mathsf{k}_i)_{i \leq \alpha})$. Note that Sim is not given the witness w nor x_1. The simulator Sim proceeds as follows:

Pick $(\tilde{\boldsymbol{m}}, \boldsymbol{m}', \boldsymbol{d_m}, \boldsymbol{t}, \boldsymbol{t}', \boldsymbol{d_t}) \xleftarrow{\$} (\mathbb{Z}_{\mathsf{n}}^{\beta})^3 \times (\mathbb{Z}_{\mathsf{n}}^{\alpha})^3$, $(\boldsymbol{r_m}, \boldsymbol{r_t}) \xleftarrow{\$} \mathbb{Z}_{2^{\lambda}B}^{\beta} \times \mathbb{Z}_{2^{\lambda}B}^{\alpha}$. Let $(\boldsymbol{t}'_{\boldsymbol{j}})_{j \leq \beta} = (t'_{1,j}, \cdots, t'_{\alpha,j})_{j \leq \beta}$ denote uniformly random additive shares of $\boldsymbol{t}'$ over $\mathbb{Z}_{\mathsf{n}}^{\alpha}$. Compute

$(U', (V'_i)_{i \leq \alpha}) \xleftarrow{\$} \mathbb{G}^{\alpha+1}$

$(W'_i)_{i \leq \alpha} \leftarrow (\boldsymbol{d_m} \bullet (\mathsf{k}_{i,j} \bullet U')_{1 \leq j \leq \beta})_{i \leq \alpha} \boldsymbol{+} \boldsymbol{d_t} \bullet G \boldsymbol{-} e \bullet (V'_i \boldsymbol{-} (\mathsf{k}_{i,0} \bullet U'))_{i \leq \alpha}$

$(\boldsymbol{X_m}, \boldsymbol{X_t}) \leftarrow (S.\mathsf{Enc}_{\mathsf{ek}}(\tilde{\boldsymbol{m}}; \boldsymbol{r_m}), S.\mathsf{Enc}_{\mathsf{ek}}(\boldsymbol{t}; \boldsymbol{r_t}))$,

$(\boldsymbol{X'_m}, \boldsymbol{X'_t}) \leftarrow (S.\mathsf{Enc}_{\mathsf{ek}}(\boldsymbol{d_m} - e \cdot \tilde{\boldsymbol{m}}; -e \cdot \boldsymbol{r_m}), S.\mathsf{Enc}_{\mathsf{ek}}(\boldsymbol{d_t} - e \cdot \boldsymbol{t}; -e \cdot \boldsymbol{r_t}))$.

Then, for $i \in [1..\alpha], j \in [1..\beta]$, pick random $W_{i,j}$ conditioned on

$$W'_i = \sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet W_{i,j},$$

and output $\pi \leftarrow (U', (V'_i)_{i \leq \alpha}, (W_{i,j})_{i \leq \alpha, j \leq \beta}, \boldsymbol{X_m}, \boldsymbol{X_t}, \boldsymbol{X'_m}, \boldsymbol{X'_t})$.

We now show how to use an adversary Adv which outputs $((x_0, x_1), w, \mathsf{ipp}, \mathsf{sk})$ and distinguishes $\pi \leftarrow \Pi.\mathsf{Prove}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, (x_0, x_1), w)$ from $\pi \leftarrow \mathsf{Sim}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, , x_0, \mathsf{vk}, \mathsf{sk})$ conditioned on $R_{\mathsf{crs}}(\mathsf{sk}, (x_0, x_1), w) = 1$ to break the semantic security of S. The reduction obtains $\boldsymbol{m}$ from Adv, samples a random $\tilde{\boldsymbol{m}}$, and sends $(\boldsymbol{m}, \tilde{\boldsymbol{m}})$ to a challenger for the $\mathsf{IND\text{-}CPA}$ game of S. It receives a ciphertext $\boldsymbol{X_m}$. It samples $(\boldsymbol{m}', \boldsymbol{d_m}, \boldsymbol{t}, \boldsymbol{t}', \boldsymbol{d_t}) \xleftarrow{\$} (\mathbb{Z}_{\mathsf{n}}^{\beta})^2 \times (\mathbb{Z}_{\mathsf{n}}^{\alpha})^3$, $\boldsymbol{r_t} \xleftarrow{\$} \mathbb{Z}_{2^{\lambda}B}^{\alpha}$ as before, and sets $\boldsymbol{X'_m} \leftarrow S.\mathsf{Enc}_{\mathsf{ek}}(\boldsymbol{d_m}; 0) \ominus \boldsymbol{X_m} \odot e$. Finally, it computes $(U', (V'_i)_{i \leq \alpha})$, $(W_{i,j})_{i \leq \alpha, j \leq \beta}$, and $(\boldsymbol{X_t}, \boldsymbol{X'_t})$ as before. Observe that $(U', (V'_i)_{i \leq \alpha})$ are distributed identically in the real game and the simulated game; direct calculations show that when $\boldsymbol{X_m}$ encrypts $\boldsymbol{m}$, the proof π is distributed exactly as in the real game, while when $\boldsymbol{X_m}$ encrypts $\tilde{\boldsymbol{m}}$, the proof π is distributed exactly as in the simulated game.

$(\mathcal{O}_0, \mathcal{O}_1)$**-Knowedge-Extractability.** We now turn our attention to the $(\mathcal{O}_0, \mathcal{O}_1)$-knowledge extractability property. The extractor Ext proceeds as follows: given a proof $\pi = (U', (V'_i)_{i \leq \alpha}, (W_{i,j})_{i \leq \alpha, j \leq \beta}, \boldsymbol{X_m}, \boldsymbol{X_t}, \boldsymbol{X'_m}, \boldsymbol{X'_t})$, it computes $\boldsymbol{m} \leftarrow S.\mathsf{Dec}_{\mathsf{td}}(\boldsymbol{X_m})$, $\boldsymbol{t} \leftarrow S.\mathsf{Dec}_{\mathsf{td}}(\boldsymbol{X_t})$, and outputs $x \leftarrow (U', (V'_i \boldsymbol{-} t_i \bullet G)_{i \leq \alpha})$, and $w \leftarrow \boldsymbol{m}$. We now analyze the probability that $R_{\mathsf{crs}}(\mathsf{sk}, (x_0, x_1), w) = 0 \wedge \Pi.\mathsf{Verify}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, x_0, \mathsf{vk}, \mathsf{sk}, \pi) = 1$. To do so, we proceed in two steps:

Game 1. In this game, we modify the behavior of the oracle $\Pi.\mathsf{Verify}(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}, \mathsf{vk}, \cdot, \mathsf{sk}, \cdot)$ that Adv is given access to. Namely, the oracle is not given vk anymore. Rather, we generate vk as before, and set $e_R \leftarrow \mathsf{vk} \bmod R$. Each time Adv sends a query π to the oracle, we proceed as follows: we parse π as

$$\pi = (U', (V'_i)_{i \leq \alpha}, (W_{i,j})_{i \leq \alpha, j \leq \beta}, \boldsymbol{X_m}, \boldsymbol{X_t}, \boldsymbol{X'_m}, \boldsymbol{X'_t}),$$

and use td to decrypt $(\boldsymbol{X_m}, \boldsymbol{X_t}, \boldsymbol{X'_m}, \boldsymbol{X'_t})$, obtaining vectors $(\boldsymbol{m}, \boldsymbol{t}, \boldsymbol{m}', \boldsymbol{t}')$. Then, we perform the following checks:

1. we check that $-e_R \odot (\boldsymbol{X_m} \ominus \boldsymbol{m}) = \boldsymbol{X'_m} \ominus \boldsymbol{m'}$;
2. we check that $-e_R \odot (\boldsymbol{X_t} \ominus \boldsymbol{t}) = \boldsymbol{X'_t} \ominus \boldsymbol{t'}$;
3. we check $V'_i - t_i \bullet G = H_{\mathsf{k}_i}(\boldsymbol{m}) \bullet U'$ for every $i \leq \alpha$ (that is, we run $M.\mathsf{Ver}_{\mathsf{sk}}(\boldsymbol{m}, \sigma)$ on the MAC $\sigma = (U', (V'_i - t_i \bullet G)_{i \leq \alpha}))$;
4. we reconstruct $(W'_i)_{i \leq \alpha} \leftarrow \left(\sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet W_{i,j} \right)_{i \leq \alpha}$ and check $W'_i - t'_i \bullet G = \sum_{j=1}^{\beta} (\mathsf{k}_{i,j} \cdot m'_j) \bullet U'$ for every $i \leq \alpha$.

Note that this follows exactly the proof strategy of [11, Section 3.3]. It follows by the exact same argument that it is statistically infeasible to distinguish the simulated oracle in Game 1 from the real oracle, and the distinguishing advantage is at most $Q(\alpha+1)p$, where p is the smallest prime factor of n and Q is the number of queries of Adv to the oracle. Intuitively, the argument stems from the fact that if Adv ever submits a proof that would be accepted by the oracle, but not by the simulated oracle (or the converse), then this proof information-theoretically determines vk. However, even given $e_R = \mathsf{vk} \bmod R$, it follows from the chinese remainder theorem that the value $\mathsf{vk} \bmod \mathsf{n}$ remains *statistically hidden*, since vk was initially picked at random in $\mathbb{Z}_\ell$ and ℓ satisfies $\ell > 2^\lambda \mathsf{n} R$. Observe that game already suffices to establish that the probability of $R_{\mathsf{crs}}(\mathsf{sk}, (x_0, x_1), w) = 0 \wedge \Pi.\mathsf{Verify}(\mathsf{pk}, \mathsf{vk}, \mathsf{sk}, \pi) = 1$ must be negligible, since in this game the simulation of $\Pi.\mathsf{Verify}$ does in particular check that $R_{\mathsf{crs}}(\mathsf{sk}, (x_0, x_1), w) = 1$. However, the simulation of $\Pi.\mathsf{Verify}$ still uses sk; to establish the second property of the $(\mathcal{O}_0, \mathcal{O}_1)$-knowledge-extractability, we proceed with a second game.

Game 2. In this game, we further modify the simulated oracle, so that it does not use sk anymore. Instead, the simulation will itself rely on the MAC verification oracle. More precisely, the key sk is only used in the checks 3 and 4 of Game 1. The third check is straightforward given oracle access to $M.\mathsf{Verify}_{\mathsf{sk}}(\cdot, \cdot)$: just call $M.\mathsf{Verify}_{\mathsf{sk}}(\boldsymbol{m}, \sigma)$ with $\sigma = (U', (V'_i - t_i \bullet G)_{i \leq \alpha})$ (this is perfectly identical to the third check in the previous game).

The fourth check, however, is more problematic, since it's not clear how to reconstruct the $(W'_i)_{i \leq \alpha}$ without knowing sk. Rewriting a bit the fourth check, we need to check is

$$\sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet W_{i,j} - t'_i \bullet G = \sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet (m'_j \bullet U')$$

for every $i \leq \alpha$. Letting $(t'_{i,1}, \cdots, t'_{i,\beta})$ denote an arbitrary additive sharing of t'_i for every $i \leq \alpha$, this equation can be rewritten as

$$\sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet (W_{i,j} - t'_{i,j} \bullet H_{i,j}) = \sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet (m'_j \bullet U')$$

Now, we distinguish two cases:

- **Case 1.** If it holds that $\beta = 1$, corresponding to the case where the vector of attributes has length 1 (or, equivalently, we consider a simplified scenario

without attributes, and credentials computed directly on the identity of the user), then the equation becomes

$$\mathsf{k}_{i,1} \bullet (m_1' \bullet U' - W_{i,1}) = t_i' \bullet G.$$

Observe that this check can be performed efficiently: since we are given $H_{i,1} = \mathsf{k}_{i,1}^{-1} \bullet G$, this is perfectly equivalent to checking

$$m_1' \bullet U' - W_{i,1} = t_i' \bullet H_{i,1}$$

for every $i \leq \alpha$, which does not require the knowledge of sk.

- **Case 2.** In the general case, where β can be larger than 1, there is no immediate shortcut. In this case, we have to rely on a MAC with a stronger unforgeability property, the XUF-CMVA security property defined in Section 3, and we simulate the verification using the following two oracles:
  - $M.\mathsf{Verify}(m, \sigma)$ outputs $M.\mathsf{Verify}_{\mathsf{sk}}(m, \sigma)$;
  - $M.\mathsf{Check}((A_{i,j})_{i \leq \alpha, j \leq \beta}, (B_{i,j})_{i \leq \alpha, j \leq \beta})$ checks $\sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet A_{i,j} = \sum_{j=1}^{\beta} \mathsf{k}_{i,j} \bullet B_{i,j}$ for all $i \leq \alpha$, and outputs 1 iff all checks succeed,

  where the first oracle allows to check the third equation, and the second oracle allows to check the last equation.

In both cases, it is immediate to see that the answers of the simulated oracle are distributed exactly as in Game 1. Furthermore, the simulation only requires access to an oracle $\mathcal{O}_1[\mathsf{sk}]$, which is $M.\mathsf{Verify}$ in case 1, and the pair of oracles $M.\mathsf{Verify}, M.\mathsf{Check}$ in case 2.

6   A Construction of **NIKVAC** from Algebraic **MAC** and Oblivious **DVNIZK**

In this section, we will use the system introduced in section 5 to construct a NIKVAC scheme Θ.

6.1   Construction

Let M be a MAC and Φ a set of statements for attributes $m_1, .., m_l$. Let Π_Φ be an oblivious DVNIZK system which runs on a common Setup algorithm with M for the relation R_{crs}, for $\mathsf{crs} \xleftarrow{\$} M.\mathsf{Setup}(1^\lambda)$, defined as

$$R_{\mathsf{crs}}((x_0, x_1), (m_1, .., m_l), \mathsf{k}) = 1 \text{ iff } M.\mathsf{Verify}_{\mathsf{k}}((m_1, .., m_l), x_1) = 1 \wedge \Phi(m_1, .., m_l),$$

where x_0 is a public word needed to prove the statements Φ and $\Pi_\Phi.\mathsf{RelSetup} = M.\mathsf{KeyGen}$. We assume Π_Φ satisfies $(\mathcal{O}_0, \mathcal{O}_1)$-knowledge-extractability, where $\mathcal{O}_0[\mathsf{k}](\cdot) = M.\mathsf{Sign}_{\mathsf{k}}(\cdot)$ and $\mathcal{O}_1[\mathsf{k}]$ is either the MAC verification oracle, if M is UF-CMVA secure (and the attribute vectors are of length 1), or the MAC verification and additional check oracle, if M is XUF-CMVA secure. Since x_0 depends on the choice of Φ, we omit it entirely in the following and simply set $x = x_1$. Note that $\Pi_\Phi.\mathsf{Setup}, \Pi_\Phi.\mathsf{CredKeyGen}$ do not rely on the choice of Φ, so we simply write $\Pi.\mathsf{Setup}, \Pi.\mathsf{CredKeyGen}$. We now construct a NIKVAC scheme using $\{\Pi_\Phi\}_\Phi$.

- $\Theta.\mathsf{Setup}(1^\lambda)$, outputs $(\mathsf{pp}, \mathsf{td}) \xleftarrow{\$} \Pi.\mathsf{Setup}(1^\lambda)$, we assume that pp fixes the supported statements $\mathbf{\Phi}$, the universe of attributes $\mathcal{U}$ is the message space of M;
- $\Theta.\mathsf{CredKeyGen}(\mathsf{pp})$, runs $(\mathsf{pk}, \mathsf{vk}) \xleftarrow{\$} \Pi.\mathsf{KeyGen}(\mathsf{pp})$, $(\mathsf{k}, \mathsf{ipp}_M) \xleftarrow{\$} \Pi.\mathsf{RelSetup}(\mathsf{pp})$, outputs secret key $\mathsf{sk} \leftarrow (\mathsf{vk}, \mathsf{k})$ and issuer parameters $\mathsf{ipp} \leftarrow (\mathsf{pk}, \mathsf{ipp}_M)$, we assume that $\mathsf{CredKeyGen}$ satisfies key-parameter consistency;
- $\Theta.\mathsf{BlindIssue}(\mathsf{sk}, S) \leftrightarrow \Theta.\mathsf{BlindObtain}(\mathsf{ipp}, (m_1, ..., m_l))$, performs a secure two-party computation that issues a tag of M to the user on valid input, we assume that this protocol satisfies *blind issuance* property[2];
- $\Theta.\mathsf{Show}(\mathsf{ipp}, \mathsf{cred}, (m_1, ..., m_l), \Phi)$, parses ipp as $(\mathsf{pk}, \mathsf{ipp}_M)$ outputs $\pi \xleftarrow{\$} \Pi_\Phi.\mathsf{Prove}(\mathsf{pk}, \mathsf{ipp}_M, \mathsf{cred}, (m_1, ..., m_l))$;
- $\Theta.\mathsf{ShowVerify}(\mathsf{sk}, \pi, \Phi)$, parses sk as $(\mathsf{vk}, \mathsf{k})$ and ipp as $(\mathsf{pk}, \mathsf{ipp}_M)$, checks $\Pi_\Phi.\mathsf{Verify}(\mathsf{pk}, \mathsf{ipp}_M, \mathsf{vk}, \mathsf{k}, \pi)$.

6.2   Security Analysis

For Θ, the functions Issue and $\mathsf{CredVerify}$ are defined as follows:

- $\mathsf{Issue}(\mathsf{sk}, (m_1, .., m_l))$: for $\mathsf{sk} = (\mathsf{vk}, \mathsf{k})$ outputs $M.\mathsf{Sign}_\mathsf{k}(m_1, .., m_l)$;
- $\mathsf{CredVerify}(\mathsf{sk}, (m_1, .., m_l), \mathsf{cred})$: for $\mathsf{sk} = (\mathsf{vk}, \mathsf{k})$ outputs $M.\mathsf{Verify}_\mathsf{k}((m_1, .., m_l), \mathsf{cred})$.

Theorem 3 (Correctness). *The NIKVAC scheme Θ satisfies correctness if M is correct and Π_Φ is complete.*

Proof. Let $(\mathsf{pp}, \mathsf{td}) \leftarrow \Theta.\mathsf{Setup}(1^\lambda)$, $(m_1, .., m_l) \xleftarrow{\$} \mathcal{U}$, $(\mathsf{sk}, \mathsf{ipp}) \xleftarrow{\$} \Theta.\mathsf{CredKeyGen}(\mathsf{pp})$ and $\mathsf{credIssue}(\mathsf{sk}, (m_1, .., m_l))$. It follows that $\mathsf{CredVerify}(\mathsf{sk}, (m_1, .., m_l), \mathsf{cred}) = 1$ from the correctness of the MAC scheme M.

Now, let $(\mathsf{pp}, \mathsf{td}) \leftarrow \Theta.\mathsf{Setup}(1^\lambda)$, $\Phi \xleftarrow{\$} \mathbf{\Phi}$, $(m_1, .., m_l) \xleftarrow{\$} \mathcal{U}$ with $\Phi(m_1, .., m_l) = 1$, $(\mathsf{sk} = (\mathsf{vk}, \mathsf{k}), \mathsf{ipp}) \xleftarrow{\$} \Theta.\mathsf{CredKeyGen}(\mathsf{pp})$ and $\mathsf{cred} \xleftarrow{\$} \mathsf{Issue}(\mathsf{sk}, (m_1, .., m_l))$. Let $\pi \xleftarrow{\$} \Theta.\mathsf{Show}(\mathsf{ipp}, \mathsf{cred}, (m_1, .., m_l), \Phi)$. Note that $R_\mathsf{k}(\mathsf{cred}, (m_1, .., m_l)) = 1$ and thus $\Theta.\mathsf{ShowVerify}(\mathsf{sk}, \pi, \Phi) = 1$ by the completeness of Π_Φ.

Theorem 4 (Unforgeability). *The NIKVAC scheme Θ is unforgeable if M is unforgeable and Π_Φ is $(\mathcal{O}_0, \mathcal{O}_1)$-knowledge-extractable for all $\Phi \in \mathbf{\Phi}$.*

Proof. Let $\mathscr{A}$ be a PPT adversary on the unforgeability of Θ. We build an adversary $\mathscr{B}$ which either breaks the unforgeability of M (so either the UF-CMVA or XUF-CMVA security) or the $(\mathcal{O}_0, \mathcal{O}_1)$-knowledge-extractability of Π_Φ for some $\Phi \in \mathbf{\Phi}$.

$\mathscr{B}$ receives $(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}_M)$[3] and access to a proof verification oracle $\mathcal{V}_\Phi$ and an MAC issuing oracle $\mathcal{O}_0$ (defined in section 6.1) from the $(\mathcal{O}_0, \mathcal{O}_1)$-knowledge-extractability[4] game with Π_Φ for $\Phi \in \mathbf{\Phi}$. $\mathscr{B}$ sends $\mathsf{pp} \leftarrow \mathsf{crs}$, $\mathsf{ipp} \leftarrow (\mathsf{pk}, \mathsf{ipp}_M)$ to $\mathscr{A}$ and gives access to the following issuing and verification oracle $\mathcal{O}$:

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[2] The protocol depends highly on the chosen MAC scheme. Thus, we omit details in abstract instantiation.

[3] The parameters $(\mathsf{crs}, \mathsf{pk}, \mathsf{ipp}_M)$ are fixed for all $\Phi \in \mathbf{\Phi}$, since they do not depend on the particular choice of Φ.

[4] In this proof, this refers to the first property of definition 14.

- $\mathcal{O}.\mathsf{Issue}(m_1, .., m_l)$ sets $Q \leftarrow Q \cup \{m_1, .., m_l\}$ and outputs $\mathcal{O}_0(m_1, .., m_l)$;
- $\mathcal{O}.\mathsf{Verify}(\Phi, \pi)$ outputs $\mathcal{V}_\Phi(\pi)$.

By the second property of definition 14, $\mathcal{V}_\Phi(\cdot)$ can be simulated only using $\mathcal{O}_1$ without access to the secret key. Now, all answers to queries which require the secret MAC key can be computed using solely access to the MAC oracles. Note that $\mathscr{B}$ simulates the unforgeability game of Θ with overwhelming probability. At some point, if $\mathscr{A}$ is successful, he will output π, Φ such that the pair (π, Φ) verifies correclty and for all queried $(m_1, .., m_l) \in Q : \Phi(m_1, .., m_l) = 0$. Subsequently, $\mathscr{B}$ forwards π to the $(\mathcal{O}_0, \mathcal{O}_1)$-knowledge-extractability game for Π_Φ, which in turn forwards the extracted values (x_1, w) to the MAC unforgeability game.

We now analyze the success probability of $\mathscr{B}$ assuming $\mathscr{A}$ is successful. If $\mathscr{B}$ won the $(\mathcal{O}_0, \mathcal{O}_1)$-knowledge-extractability game, we are finished. In the other case, the MAC unforgeability game receives $(x_1 = \sigma, w = (m_1, .., m_l))$. Because $\mathscr{A}$ is successful, π verifies correctly with regards to Θ and thus also verifies correctly with regards to Π_Φ. Because $\mathscr{B}$ failed the first game, it necessarily holds that $R(\sigma, (m_1, .., m_l)) = 1$. Since $\forall (m'_1, .., m'_l) \in Q : \Phi(m'_1, .., m'_l) = 0$ and $\Phi(m_1, .., m_l) = 1$, it holds that $(m_1, .., m_l) \notin Q$ and σ verifies correctly. Thus, $\mathscr{B}$ breaks the unforgeability of M. □

Theorem 5 (Anonymity). *The* NIKVAC *scheme* Θ *is anonymous if* Π_Φ *satisfies oblivious zero-knowledge.*

Proof. Let $\mathscr{A}$ be an adversary on the anonymity of Θ. We construct an adversary $\mathscr{B}$ that breaks the oblivious zero-knowledge property of Π_Φ for some $\Phi \in \mathbf{\Phi}$ with overwhelming probability if $\mathscr{A}$ is successful.

$\mathscr{B}$ receives $\mathsf{crs}, \mathsf{pk}, \mathsf{vk}$ from the zero-knowledge game with Π_Φ for some arbitrary $\Phi \in \mathbf{\Phi}$. Note that these values are independent of the particular choice of Φ. $\mathscr{B}$ then runs $(\mathsf{k}, \mathsf{ipp}_M) \xleftarrow{\$} M.\mathsf{KeyGen}(\mathsf{crs})$ and sends $\mathsf{pp} \leftarrow \mathsf{crs}, \mathsf{ipp} \leftarrow (\mathsf{pk}, \mathsf{ipp}_M), \mathsf{sk} \leftarrow (\mathsf{vk}, \mathsf{k})$ to $\mathscr{A}$. In turn, $\mathscr{B}$ receives $(\Phi, \mathsf{cred}, (m_1, .., m_l))$ from $\mathscr{A}$. Next, $\mathscr{B}$ outputs $(\mathsf{cred}, (m_1, .., m_l), \mathsf{k}, \mathsf{ipp}_M)$ to the oblivious zero-knowledge game for the now fixed Φ and receives π in return which he forwards to $\mathscr{A}$. Note that $\mathscr{B}$ simulates the anonymity game with overwhelming probability. Also, $R_{\mathsf{crs}}(\mathsf{sk}, \mathsf{cred}, (m_1, .., m_l)) = 1 \iff \mathsf{CredVerify}(\mathsf{sk}, (m_1, .., m_l), \mathsf{cred}) = 1 \wedge \Phi(m_1, .., m_l)$. The simulation of π in the zero-knowledge game only uses $\mathsf{ipp}, \mathsf{sk}, \Phi$ and will thus be a simulation for the anonymity game. Otherwise, π is built honestly in both games and thus, if $\mathscr{A}$ is successful, $\mathscr{B}$ is successful with overwhelming probability. □

Missing Properties. The missing properties are *blind issuance* and *key-parameter consistency*. In practice, *key-parameter consistency* can easily be fulfilled by adding additional commitments to the components of the secret key and the two-party computation for *blind issuance* depends highly on the structure of the MAC scheme and can be implemented with any standard two party computation protocol; we briefly outline a possible candidate for an optimized version of our scheme in the full version [15].

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