

# CCA-Secure Proxy Re-Encryption without Pairings<sup>\*</sup>

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**Abstract.** In a proxy re-encryption scheme, a semi-trusted proxy can transform a ciphertext under Alice’s public key into another ciphertext that Bob can decrypt. However, the proxy cannot access the plaintext. Due to its transformation property, proxy re-encryption can be used in many applications, such as encrypted email forwarding. In this paper, by using signature of knowledge and Fujisaki-Okamoto conversion, we propose a proxy re-encryption scheme *without* pairings, in which the proxy can only transform the ciphertext in one direction. The proposal is secure against chosen ciphertext attack (CCA) and collusion attack in the *random oracle model* based on Decisional Diffie-Hellman (DDH) assumption over  $\mathbb{Z}_{N^2}^*$  and integer factorization assumption, respectively. To the best of our knowledge, it is the *first* unidirectional PRE scheme with CCA security and collusion-resistance.

**Keywords:** Unidirectional PRE, DDH, random oracle, CCA security, collusion-resistance

## 1 Introduction

In 1998, Blaze, Bleumer, and Strauss [6] proposed the concept of *proxy re-encryption* (PRE), where a semi-trusted proxy can transform a ciphertext for Alice into another ciphertext that Bob can decrypt.<sup>3</sup> However, the proxy cannot get the plaintext. According to the direction of transformation, PRE schemes

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<sup>3</sup> In almost all related papers, the concept of PRE is introduced as “PRE allows a semi-trusted proxy to convert a ciphertext under Alice’s public key to another ciphertext under Bob’s public key”. However, all existing unidirectional PRE schemes (including ours) do not exactly follow the definition. In particular, in these unidirectional PRE schemes, there are two kinds of ciphertexts, one is the original ciphertext, and

can be classified into two types, one is *bidirectional*, i.e., the proxy can transform from Alice to Bob and vice versa; the other is *unidirectional*, i.e., the proxy can only convert in one direction. Blaze *et al.* [6] also gave another method to classify PRE schemes: *multi-use*, i.e., the ciphertext can be transformed from Alice to Bob to Charlie and so on; and *single-use*, i.e., the ciphertext can be transformed only once.

Due to its transformation property, PRE can be used in many applications, including simplification of key distribution [6], key escrow [21], distributed file systems [2, 3], security in publish/subscribe systems [23], multicast [10], secure certified email mailing lists [24, 22], the DRM of Apple’s iTunes [36], interoperable architecture of DRM [34], access control [35], and privacy for public transportation [19]. Recently, Hohenberger *et al.* got a result of securely obfuscating re-encryption [20], which is the first positive result for obfuscating an encryption functionality and against a series of impossibility results [18, 16, 4].

Since the introduction of PRE by Blaze, Bleumer, and Strauss [6], there have been many papers [6, 21, 2, 3, 17, 9, 11, 25] that have proposed different PRE schemes with different security properties. Some of them are related to chosen ciphertext attack (CCA) security. Ivan and Dodis [21] proposed a CCA security model for PRE and a generic construction of single-use PRE in the security model. Nevertheless, their security model allows the delegatee (Bob) to make use of the proxy as an oracle. As a result, the schemes only secure in their security model are not enough for some applications. For example, in encrypted email forwarding, an adversary (Bob) might hope to gain access to the original encrypted email by re-forming it, sending it to the proxy, and then hoping that the proxy responds with, “Can you forward the following to me again? [Encrypted attachment.]”

To fix the problem, Green and Ateniese [17], Canetti and Hohenberger [9] proposed new CCA security models for ID-based PRE and PRE, respectively. In these two new security models, it requires that the proxy checks the validity of the ciphertext before transformation, which is called public verifiability. Following this intuition, the first CCA secure, single-use, unidirectional ID-based PRE scheme in the *random oracle model* and the first CCA secure, multi-use, bidirectional PRE scheme in the *standard model* are proposed in [17, 9], respectively. However, the scheme in [17] suffers from the attack in Remark 2. Furthermore, the generic construction of PRE in [21] cannot be proved secure in the CCA security model in [9]. (See Appendix A for details. Hereafter, we refer CCA security to the definition in [9] or Section 2 of this paper.) Chu and Tzeng [11] proposed a multi-use, unidirectional ID-based PRE scheme, and claimed that it was CCA secure in the standard model. However, we showed that it was not true [31], since its transformed ciphertext  $(C_{v1}, R, d'_1, d_2, d'_2)$  can be modified to another well-formed transformed ciphertext  $(C_{v1}, R, d'_1 F_2(vk)^r, d_2, d'_2 g^r)$  by

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the other is the transformed ciphertext. The transformed ciphertext is not exactly as the ciphertext under Bob’s public key, but Bob can decrypt the transformed ciphertext only by his secret key. To the best of our knowledge, only the bidirectional schemes in [6, 9] satisfy the definition.

anyone, where  $r$  is random number. Recently, Libert and Vergnaud [25] proposed a new unidirectional PRE scheme, which is replayable chosen ciphertext attack (RCCA) secure but *not* CCA-secure. It is fair to say that there is no CCA-secure unidirectional PRE scheme.<sup>4</sup> Furthermore, according to the results in [5, 29], the timing of a pairing computation is more than twice of that of a modular exponentiation computation. Hence, the CCA-secure unidirectional PRE schemes without pairings are desired.

Another important security notion on unidirectional PRE is collusion-resistance, which disallows Bob and the proxy to collude to reveal Alice’s (long term) secret key, but allows the recovery of Alice’s “weak” secret key only. In this case, Alice can delegate decryption rights, while keeping signing rights for the same public key. Till now, there are only a few PRE schemes [2, 3, 25] holding this security.<sup>5</sup>

Though many PRE schemes have been proposed, we find that no unidirectional PRE scheme without pairings but satisfying CCA security and collusion-resistance simultaneously, even in the random oracle model. In this paper, we attempt to propose such a unidirectional PRE scheme.

## 1.1 Our Contribution

We present a proxy re-encryption scheme *without* pairings, named scheme  $\mathfrak{U}$ , which is unidirectional and single-use, and proven CCA-secure and collusion resistant in the *random oracle model* based on Decisional Diffie-Hellman (DDH) assumption over  $\mathbb{Z}_{N^2}^*$  and integer factorization assumption, respectively. Here,  $N$  is a safe-prime modulus.

The difficulty in constructing a CCA secure PRE scheme is to add the *public verifiability* to original ciphertexts. This public verifiability can prevent malicious Bob from gaining some advantage by using the proxy as an oracle. In pairing setting, such as [9], we can use the gap Diffie-Hellman problem (decisional Diffie-Hellman problem is easy, but computational Diffie-Hellman problem is hard) to achieve this. In particular, the gap Diffie-Hellman problem allows us to check whether  $\log_g A = \log_h B$ . In this paper, we use *signature of knowledge* [8, 1] to provide  $\log_g A = \log_h B$ , hence obtaining public verifiability for original ciphertexts. In fact, using the signature of knowledge to provide public verifiability is due to Shoup and Gennaro [33]. Furthermore, we use Fujisaki-Okamoto conversion [14, 15] to provide the validity check of both original ciphertexts and re-encrypted ciphertexts for the decryptor (Alice or Bob).

<sup>4</sup> When we prepared the camera-ready version, we found another paper [13] dealing the similar problems, and getting the similar results with us. In [13], the authors use Schnorr signature [28] to make the original ciphertext be publicly verifiable, while we use signature of knowledge [8, 1]. In our submission version, we have a CCA-secure bidirectional PRE scheme, however, the bidirectional one in [13] beats ours in every aspects. Hence, in the current version, we removed our bidirectional one, which can be found in [30]. Furthermore, the unidirectional scheme in [13] suffers from the attack in Remark 2.

<sup>5</sup> The unidirectional PRE scheme in [13] suffers from the collusion attack.

Following the construction of the public key encryption scheme with double trapdoors in [7], scheme  $\mathfrak{U}$  holds collusion-resistance. In particular, the factors of  $N$  are the long term secret key, and an exponent is the “week” secret key, and revealing the exponent does not hurt the secrecy of the factors of  $N$ . To the best of our knowledge, scheme  $\mathfrak{U}$  is the *first* unidirectional PRE scheme holding CCA security and collusion-resistance simultaneously.

Finally, we extend scheme  $\mathfrak{U}$  to scheme  $\mathfrak{U}_T$ , where the delegator can revoke the proxy’s transformation ability. In particular, the proxy can only transform the ciphertext during a restricted time interval.

## 1.2 Organization

The remaining paper is organized as follows. In Section 2, we review the definitions related to our proposals. In what follows, we present scheme  $\mathfrak{U}$  and its security analysis, and scheme  $\mathfrak{U}_T$  and its security analysis, in Section 3 and Section 4, respectively. In Section 5 we compare scheme  $\mathfrak{U}$  with previous unidirectional PRE schemes. Finally, we conclude the paper in Section 6.

## 2 Preliminaries

In this section, we briefly review the definitions related to our proposals, some similar content can be found in [8, 1, 17, 9].

### 2.1 Public Key Encryption

**Definition 1 (Public Key Encryption (PKE)).** A public key encryption scheme PKE is a triple of PPT algorithms  $(\text{KeyGen}, \text{Enc}, \text{Dec})$ :

- $\text{KeyGen}(1^k) \rightarrow (pk, sk)$ . On input the security parameter  $1^k$ , the key generation algorithm  $\text{KeyGen}$  outputs a public key  $pk$  and a secret key  $sk$ .
- $\text{Enc}(pk, m) \rightarrow C$ . On input a public key  $pk$  and a message  $m$  in the message space, the encryption algorithm  $\text{Enc}$  outputs a ciphertext  $C$ .
- $\text{Dec}(sk, C) \rightarrow m$ . On input a secret key  $sk$  and a ciphertext  $C$ , the decryption algorithm  $\text{Dec}$  outputs a message  $m$  in the message space or  $\perp$ .

**Correctness.** The correctness property is that for any message  $m$  in the message space and any key pair  $(pk, sk) \leftarrow \text{KeyGen}(1^k)$ . Then the following condition must hold:  $\text{Dec}(sk, \text{Enc}(pk, m)) = m$ .

### 2.2 Unidirectional Proxy Re-Encryption

**Definition 2 (Unidirectional PRE).** A unidirectional proxy re-encryption scheme UniPRE is a tuple of PPT algorithms  $(\text{KeyGen}, \text{ReKeyGen}, \text{Enc}, \text{ReEnc}, \text{Dec})$ :

- $\text{KeyGen}, \text{Enc}, \text{Dec}$ : Identical to those in public key encryption.

- $\text{ReKeyGen}(sk_1, pk_2) \rightarrow rk_{1 \rightarrow 2}$ . On input a secret key  $sk_1$  and a public key  $pk_2$ , the re-encryption key generation algorithm  $\text{ReKeyGen}$  outputs a unidirectional re-encryption key  $rk_{1 \rightarrow 2}$ .
- $\text{ReEnc}(rk_{1 \rightarrow 2}, C_1) \rightarrow C_2$ . On input a re-encryption key  $rk_{1 \rightarrow 2}$  and a ciphertext  $C_1$ , the re-encryption algorithm  $\text{ReEnc}$  outputs a re-encrypted ciphertext  $C_2$  or  $\perp$ .

**Correctness.** A correct proxy re-encryption scheme should satisfy two requirements:

$$\text{Dec}(sk, \text{Enc}(pk, m)) = m,$$

and

$$\text{Dec}(sk', \text{ReEnc}(\text{ReKeyGen}(sk, pk'), C)) = m,$$

where  $(pk, sk), (pk', sk') \leftarrow \text{KeyGen}(1^k)$ , and  $C$  is the ciphertext of message  $m$  for  $pk$  from algorithm  $\text{Enc}$  or algorithm  $\text{ReEnc}$ .

### Chosen Ciphertext Security for Unidirectional Proxy Re-Encryption.

This security note is a modification of replayable chosen ciphertext security in [25], where the corrupted public keys are *not* decided before start of the Uni-PRE-CCA game, and the adversary is *allowed* adaptive corruption of users<sup>6</sup>, and proxies between corrupted and uncorrupted users. But unlike [25], we require that one well-formed ciphertext *cannot* be modified (but can be transformed) to be another well-formed ciphertext. In [25], anyone can modify the transformed ciphertext, such that  $(C_1, C_2', C_2'', C_2''', C_3, C_4, \sigma) \rightarrow (C_1, C_2'^t, C_2''^{t^{-1}}, C_2'''^t, C_3, C_4, \sigma)$ , where  $t$  is a random number from  $\mathbb{Z}_p$ .

Note that this security model is only for single-use scheme.

**Phase 1:** The adversary  $\mathcal{A}$  issues queries  $q_1, \dots, q_{n_1}$  where query  $q_i$  is one of:

- *Public key generation oracle*  $\mathcal{O}_{pk}$ : On input an index  $i$ ,<sup>7</sup> the Challenger takes a security parameter  $k$ , and responds by running algorithm  $\text{KeyGen}(1^k)$  to generate a key pair  $(pk_i, sk_i)$ , gives  $pk_i$  to  $\mathcal{A}$  and records  $(pk_i, sk_i)$  in table  $T_K$ .
- *Secret key generation oracle*  $\mathcal{O}_{sk}$ : On input  $pk$  by  $\mathcal{A}$ , where  $pk$  is from  $\mathcal{O}_{pk}$ , the Challenger searches  $pk$  in table  $T_K$  and returns  $sk$ .
- *Re-encryption key generation oracle*  $\mathcal{O}_{rk}$ : On input  $(pk, pk')$  by  $\mathcal{A}$ , where  $pk, pk'$  are from  $\mathcal{O}_{pk}$ , the Challenger returns the re-encryption key  $rk_{pk \rightarrow pk'} = \text{ReKeyGen}(sk, pk')$ , where  $sk$  is the secret key corresponding to  $pk$ .
- *Re-encryption oracle*  $\mathcal{O}_{re}$ : On input  $(pk, pk', C)$  by  $\mathcal{A}$ , where  $pk, pk'$  are from  $\mathcal{O}_{pk}$ , the re-encrypted ciphertext  $C' = \text{ReEnc}(\text{ReKeyGen}(sk, pk'), C)$  is returned by the Challenger, where  $sk$  is the secret key corresponding to  $pk$ .
- *Decryption oracle*  $\mathcal{O}_{dec}$ : On input  $(pk, C)$ , where  $pk$  is from  $\mathcal{O}_{pk}$ , the Challenger returns  $\text{Dec}(sk, C)$ , where  $sk$  is the secret key corresponding to  $pk$ .

<sup>6</sup> The security model in [13] does not allow such adaptive corruption.

<sup>7</sup> This index is just used to distinguish different public keys.

These queries may be asked adaptively, that is, each query  $q_i$  may depend on the replies to  $q_1, \dots, q_{i-1}$ .

**Challenge:** Once the adversary  $\mathcal{A}$  decides that Phase 1 is over, it outputs two equal length plaintexts  $m_0, m_1$  from the message space, and a public key  $pk^*$  on which it wishes to be challenged. There are three constraints on the public key  $pk^*$ , (i) it is from  $\mathcal{O}_{pk}$ ; (ii) it did not appear in any query to  $\mathcal{O}_{sk}$  in Phase 1; (iii) if  $(pk^*, \star)$  did appear in any query to  $\mathcal{O}_{rk}$ , then  $\star$  did not appear in any query to  $\mathcal{O}_{sk}$ . The Challenger picks a random bit  $b \in \{0, 1\}$  and sets  $C^* = \text{Enc}(pk^*, m_b)$ . It sends  $C^*$  as the challenge to  $\mathcal{A}$ .

**Phase 2:** The adversary  $\mathcal{A}$  issues more queries  $q_{n_1+1}, \dots, q_n$  where query  $q_i$  is one of:

- $\mathcal{O}_{pk}$ : The Challenger responds as in Phase 1.
- $\mathcal{O}_{sk}$ : On input  $pk$  by  $\mathcal{A}$ , if the following requirements are all satisfied, the Challenger responds as in Phase 1; otherwise, the Challenger terminates the game.
  - $pk$  is from  $\mathcal{O}_{pk}$ ;
  - $pk \neq pk^*$ ;
  - $(pk^*, pk)$  is not a query to  $\mathcal{O}_{rk}$  before;
  - $(pk', pk, C')$  is not a query to  $\mathcal{O}_{re}$  before, where  $(pk', C')$  is a **derivative**<sup>8</sup> of  $(pk^*, C^*)$ .
- $\mathcal{O}_{rk}$ : On input  $(pk, pk')$  by  $\mathcal{A}$ , if the following requirements are all satisfied, the Challenger responds as in Phase 1; otherwise, the Challenger terminates the game.
  - $pk, pk'$  are from  $\mathcal{O}_{pk}$ ;
  - if  $pk = pk^*$ , then  $pk'$  is not a query to  $\mathcal{O}_{sk}$ .
- $\mathcal{O}_{re}$ : On input  $(pk, pk', C)$  by  $\mathcal{A}$ , if the following requirements are all satisfied, the Challenger responds as in Phase 1; otherwise, the Challenger terminates the game.
  - $pk, pk'$  are from  $\mathcal{O}_{pk}$ ;
  - if  $(pk, C)$  is a derivative of  $(pk^*, C^*)$ , then  $pk'$  is not a query to  $\mathcal{O}_{sk}$ .
- $\mathcal{O}_{dec}$ : On input  $(pk, C)$ , if the following requirements are all satisfied, the Challenger responds as in Phase 1; otherwise, the Challenger terminates the game.
  - $pk$  is from  $\mathcal{O}_{pk}$ ;
  - $(pk, C)$  is not a derivative of  $(pk^*, C^*)$ .

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<sup>8</sup> Derivatives of  $(pk^*, C^*)$  are defined as follows [9]:

1.  $(pk^*, C^*)$  is a derivative of itself.
2. If  $(pk, C)$  is a derivative of  $(pk^*, C^*)$  and  $(pk', C')$  is a derivative of  $(pk, C)$ , then  $(pk', C')$  is a derivative of  $(pk^*, C^*)$ .
3. If  $\mathcal{A}$  has queried  $\mathcal{O}_{re}$  on input  $(pk, pk', C)$  and obtained  $(pk', C')$ , then  $(pk', C')$  is a derivative of  $(pk, C)$ .
4. If  $\mathcal{A}$  has queried  $\mathcal{O}_{rk}$  on input  $(pk, pk')$ , and  $C' = \text{ReEnc}(\mathcal{O}_{re}(pk, pk'), C)$ , then  $(pk', C')$  is a derivative of  $(pk, C)$ .

These queries may be also asked adaptively.

**Guess:** Finally, the adversary  $\mathcal{A}$  outputs a guess  $b' \in \{0, 1\}$  and wins the game if  $b = b'$ .

We refer to such an adversary  $\mathcal{A}$  as a Uni-PRE-CCA adversary. We define adversary  $\mathcal{A}$ 's advantage in attacking UniPRE as the following function of the security parameter  $k$ :  $\text{Adv}_{\text{UniPRE}, \mathcal{A}}(k) = |\Pr[b = b'] - 1/2|$ . Using the Uni-PRE-CCA game we can define chosen ciphertext security for unidirectional proxy re-encryption schemes.

**Definition 3 (Uni-PRE-CCA security).** *We say that a unidirectional proxy re-encryption scheme UniPRE is semantically secure against an adaptive chosen ciphertext attack if for any polynomial time Uni-PRE-CCA adversary  $\mathcal{A}$  the function  $\text{Adv}_{\text{UniPRE}, \mathcal{A}}(k)$  is negligible. As shorthand, we say that UniPRE is Uni-PRE-CCA secure.*

*Remark 1.* In [25], the authors considered this model as a static corruption model, since it does not capture some scenarios, such as the adversary generate public keys on behalf of corrupted parties. However, we think this model is an *adaptive* corruption model. Since *Adaptive Security* usually refers to the ability of the adversary to choose which parties to corrupt depending on the information gathered so far, *but* the Challenger still generates all parties' key pairs. Allowing adversaries to generate malicious parties' public keys on their own is usually called "chosen-key model" [26].<sup>9</sup>

Besides CCA security, there is another security notion, collusion resistance, for unidirectional PRE schemes.

**Definition 4 (Uni-PRE-CR security).**<sup>10</sup> *We say that a unidirectional proxy re-encryption scheme UniPRE is collusion resistant if for any polynomial bounded adversary  $\mathcal{A}$ , the following probability is negligible:*

$$\begin{aligned} & \Pr[(sk_1, pk_1) \leftarrow \text{KeyGen}(1^k), \{(sk_i, pk_i) \leftarrow \text{KeyGen}(1^k)\}, \\ & \quad \{rk_{i \rightarrow 1} \leftarrow \text{ReKeyGen}(sk_i, pk_1)\}, \\ & \quad \{rk_{1 \rightarrow i} \leftarrow \text{ReKeyGen}(sk_1, pk_i)\}, \\ & \quad \quad \quad i = 2, \dots, \\ & \quad \alpha \rightarrow \mathcal{A}(pk_1, \{pk_i, sk_i\}, \{rk_{1 \rightarrow i}\}, \{rk_{i \rightarrow 1}\}) : \\ & \quad \quad \quad \alpha = sk_1]. \end{aligned}$$

Due to its similarity with that of unidirectional PRE schemes, we put the definitions of unidirectional PRE schemes with temporary delegation in the Appendix.

### 2.3 Signature of Knowledge

In our proposal, we apply the following non-interactive zero-knowledge proof of knowledge, named signature of knowledge of equality of two discrete logarithms [8, 1, 32].

<sup>9</sup> We thank an anonymous reviewer of Indocrypt 2008 to point out this.

<sup>10</sup> This security notion is from [2, 3], called Master Secret Security.

**Definition 5.** Let  $y_1, y_2, g, h \in \mathbb{G}$ ,  $\mathbb{G}$  be a cyclic group of quadratic residues modulo  $N^2$  ( $N$  is a safe-prime modulus), and  $H(\cdot) : \{0, 1\}^* \rightarrow \{0, 1\}^k$  ( $k$  is the security parameter). A pair  $(c, s)$ , verifying  $c = H(y_1 || y_2 || g || h || g^s y_1^c || h^s y_2^c || m)$  is a signature of knowledge of the discrete logarithm of both  $y_1 = g^x$  w.r.t. base  $g$  and  $y_2 = h^x$  w.r.t. base  $h$ , on a message  $m \in \{0, 1\}^*$ .

The party in possession of the secret  $x$  is able to compute the signature, provided that  $x = \log_g y_1 = \log_h y_2$ , by choosing a random  $t \in \{0, \dots, 2^{|\mathbb{N}^2|+k} - 1\}$  ( $|n|$  is the bit-length of  $n$ ). And then computing  $c$  and  $s$  as:

$$c = H(y_1 || y_2 || g || h || g^t || h^t || m) \text{ and } s = t - cx.$$

We denote  $\text{SoK.Gen}(y_1, y_2, g, h, m)$  as the generation of the proof.

## 2.4 Complexity Assumption

The security of our proposal is based on the Decisional Diffie-Hellman assumption (DDH) over  $\mathbb{Z}_{N^2}^*$ .

**DDH Problem.** The DDH problem is as follows: Given  $\langle g, g^a, g^b \rangle$  for some  $a, b \in \text{ord}(\mathbb{G})$  and  $T \in \mathbb{G}$ , decide whether  $T = g^{ab}$ , where  $\mathbb{G}$  is a cyclic group of quadratic residues modulo  $N^2$  ( $N$  is a safe-prime modulus),  $g$  is a random number of  $\mathbb{G}$ . An algorithm  $\mathcal{A}$  has advantage  $\varepsilon$  in solving DDH problem if  $|\Pr[\mathcal{A}(g, g^a, g^b, g^{ab}) = 0] - \Pr[\mathcal{A}(g, g^a, g^b, T) = 0]| \geq \varepsilon$ , where the probability is over the random choices of  $a, b$  in  $\text{ord}(\mathbb{G})$ , the random choices of  $g, T$  in  $\mathbb{G}$ , and the random bits of  $\mathcal{A}$ .

**Definition 6 (DDH Assumption).** We say that the  $\varepsilon$ -DDH assumption holds if no PPT algorithm has advantage at least  $\varepsilon$  in solving the DDH problem.

Note that the DDH problem over  $\mathbb{Z}_{N^2}^*$  is easy if the factors of  $N$  is known [7].

## 2.5 The Public Key Encryption with Double Trapdoors

The basic public key encryption of our proposal is the public key encryption with double trapdoors in [7], named BCP03.

The following description is from [7]. Let  $N = pq$  be a safe prime modulus, such that  $p = 2p' + 1$ ,  $q = 2q' + 1$ , and  $p, p', q, q'$  are primes. Assume  $\mathbb{G}$  is the cyclic group of quadratic residues modulo  $N^2$ , then we have the order of  $\mathbb{G}$  is  $Np'q'$ .

- **KeyGen**( $1^k$ )  $\rightarrow (pk, sk)$ . Choose a random element  $\alpha \in \mathbb{Z}_{N^2}^*$ , a random value  $a \in [1, Np'q']$ , and set  $g = \alpha^2 \pmod{N^2}$  and  $h = g^a \pmod{N^2}$ . The public key is  $(N, g, h)$ , and the secret key is  $a$ .
- **Enc**( $pk, m$ )  $\rightarrow C$ . On input a public key  $pk$  and a message  $m \in \mathbb{Z}_N$ , the ciphertext  $(A, B)$  is computed as

$$A = g^r \pmod{N^2}, \quad B = h^r(1 + mN) \pmod{N^2},$$

where  $r$  is a random number from  $\mathbb{Z}_{N^2}$ .



- $\text{Dec}(sk, C) \rightarrow m$ . There are two methods to decrypt.
  - Knowing  $a$ , one can compute  $m$  by

$$m = \frac{B/(A^a) - 1 \bmod N^2}{N}.$$

- Knowing  $p', q'$ , one can compute  $m$  by

$$m = \frac{D - 1 \bmod N^2}{N} \cdot \pi \bmod N,$$

where  $D = \left(\frac{B}{g^{w_1}}\right)^{2p'q'}$ ,  $w_1 = ar \bmod N$ ,  $ar \bmod pqp'q' = w_1 + w_2N$ ,  $\pi$  is the inverse of  $2p'q' \bmod N$ .

Note the values of  $a \bmod N$  and  $r \bmod N$  can be computed when given  $h = g^a \bmod N^2$ ,  $A = g^r \bmod N^2$ , and  $p', q'$ , by the method in [27] (Theorem 1 in [27]).

### 3 New Unidirectional Proxy Re-Encryption Scheme without Pairings

The proposed unidirectional scheme  $\mathfrak{U}$  is based on the CPA secure and collusion resistant unidirectional PRE scheme in [2, 3] (the first attempt scheme in [2, 3]), and with the signature of knowledge [8, 1] and Fujisaki-Okamoto conversion [14, 15]. The basic public key encryption is scheme BCP03.

The intuition in scheme  $\mathfrak{U}$  is as follows. Firstly, since there are two trapdoors ( $a$  and the factorization of the modulus) in scheme BCP03, we can use the key sharing technique in [17] to share  $a$ . In particular, let  $a = r_1 + r_2$ , and sent the proxy  $r_1$  and the ciphertext of  $r_2$  under the delegatee's public key. Knowing  $a$  cannot hurt the secrecy of the factorization of the modulus, hence, collusion-resistance obtained. Secondly, scheme BCP03 is CPA-secure, hence, we use Fujisaki-Okamoto conversion to make scheme BCP03 be CCA-secure. Thirdly, we use the signature of knowledge to make the original ciphertext be publicly verifiable.

#### 3.1 Scheme $\mathfrak{U}$ with Single-Use

Scheme  $\mathfrak{U}$  contains three cryptographic hash functions for all users:  $H_1(\cdot) : \{0, 1\}^* \rightarrow \{0, 1\}^{k_1}$ ,  $H_2(\cdot) : \{0, 1\}^* \rightarrow \{0, 1\}^n$ , and  $H_3(\cdot) : \{0, 1\}^* \rightarrow \{0, 1\}^{k_2}$ , where  $k_1$  and  $k_2$  are the security parameter,  $n$  is the bit-length of messages to be encrypted. The details are as follows.

**KeyGen:** Choose a safe-prime modulus  $N = pq$ , three random numbers  $\alpha \in \mathbb{Z}_{N^2}^*$ ,  $a, b \in [1, pp'qq']$ , a hash function  $H(\cdot)$ , where  $p = 2p' + 1$ ,  $q = 2q' + 1$ ,  $p, p', q, q'$  are primes, and  $H(\cdot) : \{0, 1\}^* \rightarrow \mathbb{Z}_{N^2}$ . Furthermore, set  $g_0 = \alpha^2 \bmod N^2$ ,  $g_1 = g_0^a \bmod N^2$ , and  $g_2 = g_0^b \bmod N^2$ . The public key is  $pk = (H(\cdot), N, g_0, g_1, g_2)$ , the “weak” secret key is  $wsk = (a, b)$ , and the long term secret key is  $sk = (p, q, p', q')$ .

**ReKeyGen:** On input a public key  $pk_Y = (H_Y(\cdot), N_Y, g_{Y0}, g_{Y1}, g_{Y2})$ , a “weak” secret key  $wsk_X = a_X$ , and a long term secret key  $sk_X = (p_X, q_X, p'_X, q'_X)$ , it outputs the unidirectional re-encryption key  $rk_{X \rightarrow Y} = (rk_{X \rightarrow Y}^{(1)}, rk_{X \rightarrow Y}^{(2)})$ , where  $rk_{X \rightarrow Y}^{(1)} = (\dot{A}, \dot{B}, \dot{C})$ , and computed as follows:

- Choose two random numbers  $\dot{\sigma} \in \mathbb{Z}_N$ ,  $\dot{\beta} \in \{0, 1\}^{k_1}$ .
  - Compute  $rk_{X \rightarrow Y}^{(2)} = a_X - \dot{\beta} \bmod (p_X q_X p'_X q'_X)$ .
  - Compute  $r_{X \rightarrow Y} = H_Y(\dot{\sigma} || \dot{\beta})$ ,  $\dot{A} = (g_{Y0})^{r_{X \rightarrow Y}} \bmod (N_Y)^2$ ,  $\dot{C} = H_1(\dot{\sigma}) \oplus \dot{\beta}$ ,
- $$\dot{B} = (g_{Y2})^{r_{X \rightarrow Y}} \cdot (1 + \dot{\sigma} N_Y) \bmod (N_Y)^2. \quad (1)$$

**Enc:** On input a public key  $pk = (H(\cdot), N, g_0, g_1, g_2)$  and a message  $m \in \{0, 1\}^n$ , the encryptor does the following performances:

- Choose a random number  $\sigma \in \mathbb{Z}_N$ .
- Compute  $r = H(\sigma || m)$ ,  $A = (g_0)^r \bmod N^2$ ,  $C = H_2(\sigma) \oplus m$ ,  $D = (g_2)^r \bmod N^2$ ,

$$B = (g_1)^r \cdot (1 + \sigma N) \bmod N^2. \quad (2)$$

- Run  $(c, s) \leftarrow \text{SoK.Gen}(A, D, g_0, g_2, (B, C))$ , where the underlying hash function is  $H_3$ .
- Output the ciphertext  $K = (A, B, C, D, c, s)$ .

**ReEnc:** On input a re-encryption key  $rk_{X \rightarrow Y} = (rk_{X \rightarrow Y}^{(1)}, rk_{X \rightarrow Y}^{(2)})$  and a ciphertext  $K = (A, B, C, D, c, s)$  under key  $pk_X = (H_X(\cdot), N_X, g_{X0}, g_{X1}, g_{X2})$ , check whether  $c = H_3(A || D || g_{X0} || g_{X2} || (g_{X0})^s A^c || (g_{X2})^s D^c || (B || C))$ . If not hold, output  $\perp$  and terminate; otherwise, re-encrypt the ciphertext to be under key  $pk_Y$  as:

- Compute  $A' = A^{rk_{X \rightarrow Y}^{(2)}} = (g_{X0})^{r(a_X - \dot{\beta})} \bmod (N_X)^2$ .
- Output the new ciphertext  $(A, A', B, C, rk_{X \rightarrow Y}^{(1)}, \dot{A}, \dot{B}, \dot{C})$ .

**Dec:** On input a secret key and any ciphertext  $K$ , parse  $K = (A, B, C, D, c, s)$ , or  $K = (A, A', B, C, \dot{A}, \dot{B}, \dot{C})$ .

**Case  $K = (A, B, C, D, c, s)$ :** Check whether  $c = H_3(A || D || g_0 || g_2 || (g_0)^s A^c || (g_2)^s D^c || (B || C))$ , if not, output  $\perp$  and terminate; otherwise,

- if the input secret key is the “weak” secret key  $a$ , compute  $\sigma = \frac{B/(A^a) - 1 \bmod N^2}{N}$ .

- if the secret key is the long term secret key  $(p, q, p', q')$ , compute  $\sigma = \frac{(B/g_0^{w_1})^{2p'q'} - 1 \bmod N^2}{N} \cdot \pi \pmod{N}$ , where  $w_1$  is computed as that in scheme BCP03, and  $\pi$  is the inverse of  $2p'q' \bmod N$ .

Compute  $m = C \oplus H_2(\sigma)$ , if  $B = (g_1)^{H(\sigma || m)} \cdot (1 + \sigma N) \bmod N^2$  holds, output  $m$ ; otherwise, output  $\perp$  and terminate.

**Case  $K = (A, A', B, C, \dot{A}, \dot{B}, \dot{C})$ :** In this case, the decryptor should know the delegator’s (Alice’s) public key  $(H'(\cdot), N', g'_0, g'_1, g'_2)$ .

- If the input secret key is the “weak” secret key  $b$ , compute  $\dot{\sigma} = \frac{\dot{B}/(\dot{A}^b) - 1 \bmod N^2}{N}$ .

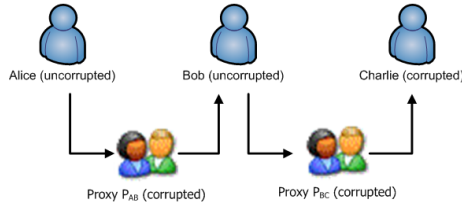
- If the input secret key is the long term secret key  $(p, q, p', q')$ , computes  $\dot{\sigma} = \frac{(\dot{B}/g_0^{w_1})^{2p'q'} - 1 \bmod N^2}{N} \cdot \pi \pmod{N}$ , where  $w_1$  is computed as that in scheme BCP03, and  $\pi$  is the inverse of  $2p'q' \bmod N$ .

Compute  $\dot{\beta} = \dot{C} \oplus H_1(\dot{\sigma})$ , if  $\dot{B} = (g_1)^{H(\dot{\sigma} \parallel \dot{\beta})} \cdot (1 + \dot{\sigma}N) \bmod N^2$  holds, then compute  $\sigma = \frac{B/(A' \cdot A^{\dot{\beta}}) - 1 \bmod N'^2}{N'}$ ,  $m = C \oplus H_2(\sigma)$ ; otherwise, output  $\perp$  and terminate. If  $B = (g_1')^{H'(\sigma \parallel m)} \cdot (1 + \sigma N') \bmod N'^2$  holds, then output  $m$ ; otherwise, output  $\perp$  and terminate.

Note that  $(H(\cdot), N, g_0, g_1, g_2)$  is the public key of the decryptor.

*Remark 2.* The values of  $\dot{B}$  and  $B$  are computed differently, in particular, in equation 1, the base is  $g_1$ , while in equation 2, the base is  $g_2$ . This difference aims to resist the following attack: Assume that there is the delegation relationship as in Fig. 1. Alice delegates her decryption rights to Bob via the proxy  $P_{AB}$ , and Bob delegates his decryption rights to Charlie via the proxy  $P_{BC}$ . Alice and Bob are uncorrupted, the rest parties are corrupted, and the target (challenged) user is Alice. This corruption situation is allowed in the security model in Section 2 (Note that the attacked scheme should be single-use). If the bases in equations (1) and (2) are both  $g_1$ , then the adversary can decrypt any ciphertext for Alice as follows. The proxy  $P_{BC}$  and Charlie collude to get Bob's weak secret key  $a_B$ , and then they collude with the proxy  $P_{AB}$  to get Alice's weak secret key  $a_A$ . As a result, the adversary can use  $a_A$  to decrypt any ciphertext for Alice. However, in scheme  $\mathfrak{U}$ , the proxy  $P_{BC}$  and Charlie cannot get Bob's weak secret key  $b_B$  (which is for decrypting partial re-encryption key), hence, they cannot collude with the proxy  $P_{AB}$  to get Alice's weak secret key  $a_A$  (which is for decrypting ciphertexts).

Note that the above attack is also allowed in the security model in [17, 13], since they only disallow the adversary to corrupt the proxy between the target user and the uncorrupted user. The unidirectional schemes in [17, 13] suffer from the above attack. To resist the above attack, we can use the same method in scheme  $\mathfrak{U}$ , in particular, every user has two public/secret key pairs, one is for decrypting ciphertexts of messages, and the other is for decrypting the partial re-encryption key.



**Fig. 1.** An example of delegation relationship.

**Correctness.** The correctness property is easily obtained by the correctness of scheme BCP03 [7] and Fujisaki-Okamoto conversion [14, 15].

Due to the limited space, we omit the proofs of the following two theorems here, and give them in the full version.

**Theorem 1 (Uni-PRE-CCA security).** *In the random oracle model, scheme  $\mathfrak{U}$  is CCA-secure under the assumptions that DDH problem over  $\mathbb{Z}_{N^2}^*$  is hard, and the signature of knowledge is secure.*

**Theorem 2 (Uni-PRE-CR security).** *In the random oracle, if  $N$  is hard to factor, then scheme  $\mathfrak{U}$  is collusion resistant.*

## 4 Scheme $\mathfrak{U}_T$ with Temporary Delegation

This section describes scheme  $\mathfrak{U}_T$ , a variant of scheme  $\mathfrak{U}$ , supporting temporary delegation. Like the temporary unidirectional PRE schemes in [2, 3, 25], the proxy is only allowed to transform ciphertexts from the delegator to the delegatee during a limited time period. The point of modifying scheme  $\mathfrak{U}$  to scheme  $\mathfrak{U}_T$  is to make different  $g_1$ 's for every time period.

Scheme  $\mathfrak{U}_T$  also contains three cryptographic hash functions for all users:  $H_1(\cdot) : \{0, 1\}^* \rightarrow \{0, 1\}^{k_1}$ ,  $H_2(\cdot) : \{0, 1\}^* \rightarrow \{0, 1\}^n$ , and  $H_3(\cdot) : \{0, 1\}^* \rightarrow \{0, 1\}^{k_2}$ , where  $k_1$  and  $k_2$  are the security parameter,  $n$  is the bit-length of messages to be encrypted. The details are as follows.

**KeyGen:** Choose a safe-prime modulus  $N = pq$ ,  $T + 2$  random numbers  $\alpha \in \mathbb{Z}_{N^2}^*$ ,  $a_1, \dots, a_T, b \in [1, pp'qq']$ , a hash function  $H(\cdot)$ , where  $p = 2p' + 1$ ,  $q = 2q' + 1$ ,  $p, p', q, q'$  are primes,  $T$  is the number of time intervals, and  $H(\cdot) : \{0, 1\}^* \rightarrow \mathbb{Z}_{N^2}$ . Furthermore, set  $g_0 = \alpha^2 \bmod N^2$ ,  $g_1^{(i)} = g_0^{a_i} \bmod N^2$  ( $i = 1, \dots, T$ ), and  $g_2 = g_0^b \bmod N^2$ . The public key is  $pk = (H(\cdot), N, g_0, g_1^{(i)} (i = 1, \dots, T), g_2)$ , the “weak” secret key is  $(a_i (i = 1, \dots, T), b)$ , and the long-term secret key is  $sk = (p, q, p', q')$ .

**ReKeyGen:** On input a public key  $pk_Y = (H_Y(\cdot), N_Y, g_{Y0}, g_{Y1}^{(1)}, \dots, g_{Y1}^{(T_Y)}, g_{Y2})$ , a “weak” secret key  $a_{X,j}$  for time period  $j \in \{1, \dots, T_X\}$ , and a secret key  $sk_X = (p_X, q_X, p'_X, q'_X)$ , it outputs the unidirectional re-encryption key  $rk_{X \rightarrow Y, j} = (rk_{X \rightarrow Y, j}^{(1)}, rk_{X \rightarrow Y, j}^{(2)})$  for the  $j$ -th time period, which is generated as follows.

- Choose two random numbers  $\dot{\sigma}_j \in \mathbb{Z}_N$ ,  $\dot{\beta}_j \in \{0, 1\}^{k_1}$ .
- Compute  $rk_{X \rightarrow Y, j}^{(2)} = a_{X,j} - \dot{\beta}_j \bmod (p_X q_X p'_X q'_X)$ .
- Compute

$$r_{X \rightarrow Y, j} = H_Y(\dot{\sigma}_j || \dot{\beta}_j), \quad \dot{A}_j = (g_{Y0})^{r_{X \rightarrow Y, j}} \bmod (N_Y)^2, \\ \dot{B}_j = (g_{Y2})^{r_{X \rightarrow Y, j}} \cdot (1 + \dot{\sigma}_j N_Y) \bmod (N_Y)^2, \quad \dot{C}_j = H_1(\dot{\sigma}_j) \oplus \dot{\beta}_j$$

- Set  $rk_{X \rightarrow Y, j}^{(1)} = (\dot{A}_j, \dot{B}_j, \dot{C}_j)$ .

**Enc:** On input a public key  $pk = (H(\cdot), N, g_0, g_1^{(1)}, \dots, g_1^{(T)}, g_2)$ , a time period  $j \in \{1, \dots, T\}$  and a message  $m \in \{0, 1\}^n$ , the encryptor does the following performances:

- Choose a random number  $\sigma_j \in \mathbb{Z}_N$ .

- Compute

$$r_j = H(\sigma_j \| m),$$

$$A_j = (g_0)^r \bmod N^2, \quad B_j = (g_1^{(j)})^r \cdot (1 + \sigma_j N) \bmod N^2,$$

$$C_j = H_2(\sigma_j) \oplus m, \quad D_j = (g_2)^r \bmod N^2.$$

- Run  $(c_j, s_j) \leftarrow \text{SoK.Gen}(A_j, D_j, g_0, g_2, (B_j, C_j))$ , where the underlying hash function is  $H_3$ .
- Output the ciphertext  $K_j = (A_j, B_j, C_j, D_j, c_j, s_j)$  for the  $j$ -th time period .

**ReEnc:** On input a re-encryption key  $rk_{X \rightarrow Y, j} = (rk_{X \rightarrow Y, j}^{(1)}, rk_{X \rightarrow Y, j}^{(2)})$  and a ciphertext  $K_j = (A_j, B_j, C_j, D_j, c_j, s_j)$  under key  $pk_X = (H_X(\cdot), N_X, g_{X0}, g_{X1}^{(1)}, \dots, g_{X1}^{(T_X)}, g_{X2})$ , where  $j \in \{1, \dots, T_X\}$ , the proxy checks whether  $c_j = H_3(A_j \| D_j \| g_{X0} \| g_{X2} \| (g_{X0})^{s_j} (A_j)^{c_j} \| (g_{X2})^{s_j} (D_j)^{c_j} \| (B_j \| C_j))$ . If not hold, output  $\perp$  and terminate; otherwise, re-encrypt the ciphertext to be under key  $pk_Y$  as:

- Compute  $A'_j = (A_j)^{rk_{X \rightarrow Y, j}^{(2)}} = (g_{X0})^{r(a_{X, j} - \beta_j)}$  mod  $(N_X)^2$ .
- Output the new ciphertext

$$(A_j, A'_j, B_j, C_j, rk_{X \rightarrow Y, j}^{(1)}) = (A_j, A'_j, B_j, C_j, \dot{A}_j, \dot{B}_j, \dot{C}_j).$$

**Dec:** On input a secret key and any ciphertext  $K_j$  for the  $j$ -th time period, where  $j \in \{1, \dots, T\}$ , the decryptor parses  $K_j = (A_j, B_j, C_j, D_j, c_j, s_j)$ , or  $K_j = (A_j, A'_j, B_j, C_j, \dot{A}_j, \dot{B}_j, \dot{C}_j)$ .

**Case  $K_j = (A_j, B_j, C_j, D_j, c_j, s_j)$ :** Check whether  $c_j = H_3(A_j \| D_j \| g_0 \| g_2 \| (g_0)^{s_j} (A_j)^{c_j} \| (g_2)^{s_j} D_j^{c_j} \| (B_j \| C_j))$ , if not, output  $\perp$  and terminate; otherwise,

- if the input secret key is the “weak” secret key  $a_j$ , compute  $\sigma_j = \frac{B_j / ((A_j)^{a_j}) - 1 \bmod N^2}{N}$ .

- if the secret key is the long term secret key  $(p, q, p', q')$ , compute  $\sigma_j = \frac{(B_j / (g_0)^{w_1})^{2p'q'} - 1 \bmod N^2}{N} \cdot \pi(\bmod N)$ , where  $w_1$  is computed as that in scheme BCP03, and  $\pi$  is the inverse of  $2p'q' \bmod N$ .

Compute  $m = C_j \oplus H_2(\sigma_j)$ , if  $B_j = (g_1)^{H(\sigma_j \| m)} \cdot (1 + \sigma_j N) \bmod N^2$  holds, output  $m$ ; otherwise, output  $\perp$  and terminate.

**Case  $K_j = (A_j, A'_j, B_j, C_j, \dot{A}_j, \dot{B}_j, \dot{C}_j)$ :** In this case, the decryptor should

know the delegator’s (Alice’s) public key  $(H'(\cdot), N', g'_0, g'_1, \dots, g'_1, g'_2)$ .

- If the input secret key is the “weak” secret key  $b$ , compute  $\dot{\sigma}_j = \frac{\dot{B}_j / ((\dot{A}_j)^b) - 1 \bmod N^2}{N}$ .

- If the input secret key is the long term secret key  $(p, q, p', q')$ , computes  $\dot{\sigma}_j = \frac{(\dot{B}_j / g_0^{w_1})^{2p'q'} - 1 \bmod N^2}{N} \cdot \pi(\bmod N)$ , where  $w_1$  is computed as that in scheme BCP03, and  $\pi$  is the inverse of  $2p'q' \bmod N$ .

Compute  $\dot{\beta}_j = \dot{C}_j \oplus H_1(\dot{\sigma}_j)$ , if  $\dot{B}_j = (g_1)^{H(\dot{\sigma}_j \| \dot{\beta}_j)} \cdot (1 + \dot{\sigma}_j N) \bmod N^2$

holds, then compute  $\sigma_j = \frac{B_j / (A'_j \cdot (A_j)^{\beta_j}) - 1 \bmod N^2}{N'}$ ,  $m = C_j \oplus H_2(\sigma_j)$ ;

otherwise, output  $\perp$  and terminate. If  $B_j = (g_1^{(j)'})^{H'(\sigma_j \| m)} \cdot (1 + \sigma_j N') \bmod N'^2$  holds, then output  $m$ ; otherwise, output  $\perp$  and terminate.

Note that  $(H(\cdot), N, g_0, g_1^{(i)}, \dots, g_1^{(T)}, g_2)$  is the public key of the decryptor.

**Correctness.** The correctness property is easily obtained by the same method for scheme  $\mathfrak{U}$ .

Due to the limited space, we omit the proofs of the following two theorems here, and give them in the full version.

**Theorem 3 (Uni-PRETD-CCA security).** *In the random oracle model, scheme  $\mathfrak{U}_T$  is CCA-secure under the assumptions that DDH problem over  $\mathbb{Z}_{N^2}^*$  is hard, and the signature of knowledge is secure.*

**Theorem 4 (Uni-PRETD-CR security).** *In the random oracle, if  $N$  is hard to factor, then scheme  $\mathfrak{U}_T$  is collusion resistant.*

## 5 Comparison

In this section, we compare scheme  $\mathfrak{U}$  with the previous CCA-secure unidirectional PRE schemes. Since as mentioned above, the unidirectional PRE schemes in [21, 17, 11, 13] are not CCA-secure, we only compare scheme  $\mathfrak{U}$  with the scheme in [25] (named LV08).

In Table 1, we denote  $t_p$ ,  $t_{eb}$ ,  $t_{eN}$ ,  $t_s$ , and  $t_v$  as the computational cost of a bilinear pairings, an exponentiation over a bilinear group, an exponentiation over  $\mathbb{Z}_{N^2}^*$  ( $N$  is a safe-prime modulus), a one-time signature and verification, respectively.  $\mathbb{G}_e$  and  $\mathbb{G}_T$  are the bilinear groups used in scheme LV08.  $N_X$  and  $N_Y$  are the safe-prime modulus corresponding to the delegator and the delegatee, respectively.  $svk$  and  $\sigma$  are the one-time signature's public key and signature. Note that we only consider the case of using weak secret key to decrypt in Dec algorithm of scheme  $\mathfrak{U}$ .

From Table 1, we can see that scheme LV08 is a little bit more efficient than scheme  $\mathfrak{U}$ . In order to guarantee that  $N$  is hard to factor,  $N$  should be 1024-bit at least, which makes scheme  $\mathfrak{U}$  need more time for an exponentiation and more storage for a ciphertext. However, we emphasize that scheme  $\mathfrak{U}$  is CCA-secure and based on the well-studied DDH assumption, while scheme LV08 is RCCA-secure and based on the less-studied 3-quotient decision Bilinear Diffie-Hellman (3-QDBDH) assumption.

## 6 Conclusions

In this paper, by using signature of knowledge and Fujisaki-Okamoto conversion, we proposed the *first* CCA-secure and collusion resistant unidirectional PRE scheme without pairings, which solves a problem proposed in [9, 25].

There are still many open problems to be solved, such as designing more efficient CCA-secure, collusion resistant unidirectional PRE schemes without pairings, and CCA-secure multi-use unidirectional PRE schemes [9, 25].

**Table 1.** Comparison between scheme  $\mathcal{U}$  and scheme LV08.

Schemes	LV08	$\mathcal{U}$		
Comput. Cost	ReKeyGen	$2t_{eb}$	$2t_{eN}$	
	Enc	$3.5t_{eb} + 1t_s$	$5t_{eN}$	
	ReEnc	$2t_p + 4t_{eb} + 1t_v$	$4t_{eN}$	
	Dec	Original	$3t_p + 2t_{eb} + 1t_v$	$5t_{eN}$
		Transformed	$5t_p + 2t_{eb} + 1t_v$	$4t_{eN}$
Ciphertext Size	Original	$1 svk  + 2 \mathbb{G}_e  + 1 \mathbb{G}_T  + 1 \sigma $	$2k + 3 N_X^2  +  m $	
	Transformed	$1 svk  + 4 \mathbb{G}_e  + 1 \mathbb{G}_T  + 1 \sigma $	$k_1 + 3 N_X^2  + 2 N_Y^2  +  m $	
Security	Security Level	collusion resistant, RCCA	collusion resistant, CCA	
	Standard model	Yes	No	
	Underlying Assumptions	3-QDBDH	DDH	

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## A Analysis on Ivan-Dodis Construction

### A.1 Ivan-Dodis Construction

The Ivan-Dodis construction is based on any CCA-secure PKE. The details are as follows.

- UniPRE.KGen:** On input the security parameter  $1^k$ , it outputs two key pairs  $(pk_1, sk_1)$  and  $(pk_2, sk_2)$ .
- UniPRE.RKGen:** On input the delegator’s key pairs  $(pk_1, sk_1)$  and  $(pk_2, sk_2)$ , the delegator sends  $sk_1$  as the re-encryption key to the proxy via a secure channel, and sends  $sk_2$  to the delegatee as the partial key via another secure channel.
- UniPRE.Enc:** On input public keys  $(pk_1, pk_2)$  and a message  $m$ , it outputs  $\text{PKE.Enc}(pk_1, \text{PKE.Enc}(pk_2, m))$ .
- UniPRE.ReEnc:** On input a re-encryption key  $sk_1$  and a ciphertext  $C$ , it outputs a re-encrypted ciphertext  $C' = \text{PKE.Dec}(sk_1, C)$ .
- UniPRE.Dec:** On input secret keys  $(sk_1, sk_2)$ , a partial key  $sk'_2$  from its delegator and a ciphertext  $C$ , **UniPRE.Dec** does:
- If  $C$  is an original ciphertext, then it outputs  $\text{PKE.Dec}(sk_2, \text{PKE.Dec}(sk_1, C))$ .
  - If  $C$  is a re-encrypted ciphertext, then it outputs  $\text{PKE.Dec}(sk'_2, C)$ .

Note that the partial key  $sk_2$  can be encrypted by the delegatee’s public key, and forwarded to Bob by the proxy. In this case, the delegatee does not require to store extra secrets for every delegation [2, 3].

### A.2 Chosen Ciphertext Attacks on the Ivan-Dodis Construction

In this subsection, we will show that the adversary always wins the Uni-PRE-CCA game with the Ivan-Dodis construction’s Challenger.

**Phase 1:** The adversary does not need to make any query in this phase.

**Challenge:** The adversary outputs two equal length plaintexts  $m_0, m_1$  from the message space, and an uncorrupted public key  $pk^* = (pk_1^*, pk_2^*)$ .

The Challenger will follow the Uni-PRE-CCA game’s specification, i.e., pick a random bit  $b \in \{0, 1\}$  and sets  $C^* = \text{UniPRE.Enc}(pk^*, m_b)$ . It sends  $C^*$  as the challenge ciphertext to  $\mathcal{A}$ .

**Phase 2:** The adversary performs as follows.

1. The adversary queries  $\mathcal{O}_{re}$  with  $(pk^*, pk, C^*)$ , such that  $pk$  is uncorrupted. Then as the Uni-PRE-CCA game's specification, the adversary can get the re-encrypted ciphertext  $C'$  such that  $C' = \text{PKE}.\text{Dec}(sk_1^*, C^*)$ ,  $sk_1^*$  is the key corresponding to  $pk_1^*$ .
2. The adversary computes  $\hat{C} = \text{PKE}.\text{Enc}(pk_1^*, C')$ . Note that  $\hat{C} \neq C^*$  since PKE is CCA-secure, such as the underlying PKE scheme is the Cramer-Shoup scheme [12].
3. The adversary queries  $\mathcal{O}_{de}$  with  $(pk^*, \hat{C})$  and gets a message  $m$ . Note that  $(pk^*, \hat{C})$  is not a derivative of  $(pk^*, C^*)$ , hence this query is valid.

**Guess:** If  $m = m_0$ , the adversary  $\mathcal{A}$  outputs  $b' = 0$ ; otherwise, output  $b' = 1$ .

Since  $\hat{C}$  and  $C^*$  are corresponding to the same message, we always have  $b = b'$ . As a result, the Ivan-Dodis construction is not CCA-secure for the security model in Section 2.

## B Definitions of Unidirectional PRE Schemes with Temporary Delegation

**Definition 7 (Unidirectional PRE with Temporary Delegation).** A unidirectional proxy re-encryption scheme UniPRE with temporary delegation is a tuple of PPT algorithms  $(\text{KeyGen}, \text{ReKeyGen}, \text{Enc}, \text{ReEnc}, \text{Dec})$ :

- $\text{KeyGen}(1^k) \rightarrow (pk, sk, T)$ . On input the security parameter  $1^k$ , the key generation algorithm  $\text{KeyGen}$  outputs a public/secret key pair  $(pk, sk)$ , and the number of time intervals  $T$ .
- $\text{Enc}(pk, m, j) \rightarrow C_j$ . On input a public key  $pk$ , a message  $m$  in the message space, and the time period  $j \in \{1, \dots, T\}$ , the encryption algorithm  $\text{Enc}$  outputs a ciphertext  $C_j$  for the  $j$ -th time period.
- $\text{ReKeyGen}(sk_1, pk_2, j) \rightarrow rk_{1 \rightarrow 2, j}$ . On input a secret key  $sk_1$ , a public key  $pk_2$ , and the time period  $j \in \{1, \dots, T_1\}$ , where  $T_1$  is the number of time intervals corresponding to the delegator. The re-encryption key generation algorithm  $\text{ReKeyGen}$  outputs a unidirectional re-encryption key  $rk_{1 \rightarrow 2, j}$  for the  $j$ -th time period.
- $\text{ReEnc}(rk_{1 \rightarrow 2, j}, C_1^{(j)}) \rightarrow C_2^{(j)}$ . On input a re-encryption key  $rk_{1 \rightarrow 2}$ , and a ciphertext  $C_1^{(j)}$  for the  $j$ -th time period, where  $j \in \{1, \dots, T_1\}$ ,  $T_1$  is the number of time intervals corresponding to the delegator. The re-encryption algorithm  $\text{ReEnc}$  outputs a re-encrypted ciphertext  $C_2^{(j)}$  for the  $j$ -th time period or  $\perp$ .
- $\text{Dec}(sk, C_j) \rightarrow m$ . On input a secret key  $sk$  and a ciphertext  $C_j$  for the  $j$ -th time period, where  $j \in \{1, \dots, T\}$ ,  $T$  is the number of time intervals corresponding to the decryptor. The decryption algorithm  $\text{Dec}$  outputs a message  $m$  in the message space or  $\perp$ .

**Correctness.** A correct proxy re-encryption scheme should satisfy two requirements:  $\text{Dec}(sk, \text{Enc}(pk, m, j)) = m$ , and  $\text{Dec}(sk', \text{ReEnc}(\text{ReKeyGen}(sk, pk', j), C_j)) = m$ , where  $(pk, sk, T), (pk', sk', T') \leftarrow \text{KeyGen}(1^k)$ ,  $C_j$  is the ciphertext of message  $m$  for  $pk$  and the  $j$ -th time period from algorithm  $\text{Enc}$  or algorithm  $\text{ReEnc}$ , and  $j \in \{1, \dots, T\}$ .

**Chosen Ciphertext Security for Unidirectional Proxy Re-Encryption with Temporary Delegation.** Following the method in [25], we extend Uni-PRE-CCA game to Uni-PRETD-CCA game, which is described as follows.

**Phase 1:** The adversary  $\mathcal{A}$  issues queries  $q_1, \dots, q_{n_1}$  where query  $q_i$  is one of:

- $\mathcal{O}_{pk}, \mathcal{O}_{sk}$ : Identical to those Uni-PRE-CCA game.
- *Re-encryption key generation oracle*  $\mathcal{O}_{rk}$ : On input  $(pk, pk', j)$  by  $\mathcal{A}$ , where  $pk, pk'$  are from  $\mathcal{O}_{pk}$ , and the time period  $j \in \{1, \dots, T\}$ , the Challenger returns the re-encryption key  $rk_{pk \rightarrow pk', j} = \text{ReKeyGen}(sk, pk', j)$ , where  $sk$  is the secret key corresponding to  $pk$ .
- *Re-encryption oracle*  $\mathcal{O}_{re}$ : On input  $(pk, pk', C, j)$  by  $\mathcal{A}$ , where  $pk, pk'$  are from  $\mathcal{O}_{pk}$ , and the time period  $j \in \{1, \dots, T\}$ , the re-encrypted ciphertext  $C' = \text{ReEnc}(\text{ReKeyGen}(sk, pk', j), C)$  is returned by the Challenger, where  $sk$  is the secret key corresponding to  $pk$ .
- *Decryption oracle*  $\mathcal{O}_{dec}$ : On input  $(pk, C, j)$ , where  $pk$  is from  $\mathcal{O}_{pk}$ , and the time period  $j \in \{1, \dots, T\}$ , the Challenger returns  $\text{Dec}(sk, C)$ , where  $sk$  is the secret key corresponding to  $pk$ .

These queries may be asked adaptively, that is, each query  $q_i$  may depend on the replies to  $q_1, \dots, q_{i-1}$ .

**Challenge:** Once the adversary  $\mathcal{A}$  decides that Phase 1 is over, it outputs two equal length plaintexts  $m_0, m_1$  from the message space, a public key  $pk^*$ , and the time period  $j^*$  on which it wishes to be challenged. There are some constraints on the public key  $pk^*$  and  $j^*$ : (i)  $pk^*$  is from  $\mathcal{O}_{pk}$ ; (ii)  $pk^*$  did not appear in any query to  $\mathcal{O}_{sk}$  in Phase 1; (iii) if  $(pk^*, \star, j^*)$  did appear in any query to  $\mathcal{O}_{rk}$ , then  $\star$  did not appear in any query to  $\mathcal{O}_{sk}$ . The Challenger picks a random bit  $b \in \{0, 1\}$  and sets  $C^* = \text{Enc}(pk^*, m_b, j)$ . It sends  $C^*$  as the challenge to  $\mathcal{A}$ .

**Phase 2:** The adversary  $\mathcal{A}$  issues more queries  $q_{n_1+1}, \dots, q_n$  where query  $q_i$  is one of:

- $\mathcal{O}_{pk}$ : The Challenger responds as in Phase 1.
- $\mathcal{O}_{sk}$ : On input  $pk$  by  $\mathcal{A}$ , if the following requirements are all satisfied, the Challenger responds as in Phase 1; otherwise, the Challenger terminates the game.
  - $pk$  is from  $\mathcal{O}_{pk}$ ;
  - $pk \neq pk^*$ ;
  - $(pk^*, pk, j^*)$  is not a query to  $\mathcal{O}_{rk}$  before;
  - $(pk', pk, C', j^*)$  is not a query to  $\mathcal{O}_{re}$  before, where  $(pk', C', j^*)$  is a derivative<sup>11</sup> of  $(pk^*, C^*, j^*)$ .

<sup>11</sup> Derivatives of  $(pk^*, C^*, j^*)$  are defined similarly with that in Section 2.2, and just add  $j^*$  into every input/output.

- $\mathcal{O}_{rk}$ : On input  $(pk, pk', j)$  by  $\mathcal{A}$ , if the following requirements are all satisfied, the Challenger responds as in Phase 1; otherwise, the Challenger terminates the game.
  - $pk, pk'$  are from  $\mathcal{O}_{pk}$ ;
  - if  $pk = pk^*$  and  $j = j^*$ , then  $pk'$  is not a query to  $\mathcal{O}_{sk}$ .
- $\mathcal{O}_{re}$ : On input  $(pk, pk', C, j)$  by  $\mathcal{A}$ , if the following requirements are all satisfied, the Challenger responds as in Phase 1; otherwise, the Challenger terminates the game.
  - $pk, pk'$  are from  $\mathcal{O}_{pk}$ ;
  - if  $(pk, C, j)$  is a derivative of  $(pk^*, C^*, j^*)$ , then  $pk'$  is not a query to  $\mathcal{O}_{sk}$ .
- $\mathcal{O}_{dec}$ : On input  $(pk, C, j)$ , if the following requirements are all satisfied, the Challenger responds as in Phase 1; otherwise, the Challenger terminates the game.
  - $pk$  is from  $\mathcal{O}_{pk}$ ;
  - $(pk, C, j)$  is not a derivative of  $(pk^*, C^*, j^*)$ .

These queries may be also asked adaptively.

**Guess:** Finally, the adversary  $\mathcal{A}$  outputs a guess  $b' \in \{0, 1\}$  and wins the game if  $b = b'$ .

We refer to such an adversary  $\mathcal{A}$  as a Uni-PRETD-CCA adversary. We define adversary  $\mathcal{A}$ 's advantage in attacking UniPRE as the following function of the security parameter  $k$ :  $\text{Adv}_{\text{UniPRE}, \mathcal{A}}(k) = |\Pr[b = b'] - 1/2|$ . Using the Uni-PRE-CCA game we can define chosen ciphertext security for unidirectional proxy re-encryption schemes.

**Definition 8 (Uni-PRETD-CCA security).** *We say that a unidirectional proxy re-encryption scheme UniPRE with temporary delegation is semantically secure against an adaptive chosen ciphertext attack if for any polynomial time Uni-PRETD-CCA adversary  $\mathcal{A}$  the function  $\text{Adv}_{\text{UniPRE}, \mathcal{A}}(k)$  is negligible. As shorthand, we say that UniPRE is Uni-PRETD-CCA secure.*

**Definition 9 (Uni-PRETD-CR security).** *We say that a unidirectional proxy re-encryption scheme UniPRE with temporary delegation is collusion resistant if for any polynomial bounded adversary  $\mathcal{A}$ , the following probability is negligible:*

$$\begin{aligned}
& \Pr[(sk_1, pk_1, T_1) \leftarrow \text{KeyGen}(1^k), \{(sk_i, pk_i, T_i) \leftarrow \text{KeyGen}(1^k)\}, \\
& \quad \{rk_{i \rightarrow 1, j} \leftarrow \text{ReKeyGen}(sk_i, pk_1, j)\} (j = 1, \dots, T_i), \\
& \quad \{rk_{1 \rightarrow i, j} \leftarrow \text{ReKeyGen}(sk_1, pk_i, j)\} (j = 1, \dots, T_1), \\
& \quad \quad \quad i = 2, \dots, \\
& \quad \alpha \rightarrow \mathcal{A}(pk_1, \{pk_i, sk_i\}, \{rk_{1 \rightarrow i, j}\}, \{rk_{i \rightarrow 1, j}\}) : \\
& \quad \quad \quad \alpha = sk_1].
\end{aligned}$$