Improved Linear Sieving Techniques with Applications to Step-Reduced LED-64

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Abstract. In this paper, we present advanced meet-in-the-middle (MITM) attacks against the lightweight block cipher LED-64, improving the best known attacks on several step-reduced variants of the cipher in both single-key and related-key models. In particular, we present a known-plaintext attack on 2-step LED-64 with complexity of 2^{48} and a related-key attack on 3-step LED-64 with complexity of 2^{49}. In both cases, the previously known attacks have complexity of 2^{60}, i.e., only 16 times faster than exhaustive key search.

While our attacks are applied to the specific scheme of LED-64, they contain several general methodological contributions: First, we present the *linear key sieve* technique, which allows to exploit linear dependencies between key bits to obtain filtering conditions in MITM attacks on block ciphers. While similar ideas have been previously used in the domain of hash functions, this is the first time that such a technique is applied in block cipher cryptanalysis. As a second contribution, we demonstrate for the first time that a *splice-and-cut* attack (which so far seemed to be an inherently chosen-plaintext technique) can be used in the known-plaintext model, with data complexity which is significantly below the code-book size. Finally, we extend the differential MITM attack on AES-based designs, and apply it independently in two stages from both sides of the cipher, while using the linear key sieve and other enhancements.

Keywords: Cryptanalysis, LED, AES, Even-Mansour, Meet-in-the-middle attack, Splice-and-cut, Known plaintext splice-and-cut

1 Introduction

Meet-in-the-middle (MITM) attacks on block ciphers were first introduced more than 30 years ago [18]. A block cipher is vulnerable to such attacks if it is possible

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to independently compute a variable of its inner state from the encryption and decryption sides without having to guess the full key. The value of this inner variable (or variables) is used to efficiently sieve the key suggestions obtained from both sides and mount an efficient attack. This motivated block cipher designers to incorporate relatively complex key schedule algorithms into the design, thus assuring very quick diffusion of the full key into the state and making the cipher resistant to MITM attacks. On the other hand, complex key schedule algorithms are difficult to implement in resource-constrained environments (such as RFID tags and wireless sensors). Thus, in recent years, with the rise of lightweight cryptography, designers have proposed many schemes with simplified key schedule algorithms. At the extreme end of the scale lie block ciphers such as LED-64 [13], Zorro [11] and PRINCE [4] which have no key schedule at all, and simply XOR the key to the internal state of the cipher several times during the encryption process.

Naturally, the tendency to simplify the key schedule of block ciphers was accompanied by the development of interesting new techniques in MITM attacks in order to break these schemes. One of the most notable techniques is splice-and-cut [1, 20], initially applied to hash functions, but quickly shown to be applicable to block ciphers as well. Splice-and-cut attacks are adaptations of Merkle and Hellman's attack on 2K-3DES [18] to single encryption. The main idea is to obtain the encryptions of several chosen plaintexts in order to view the first and the last rounds of the cryptosystem as consecutive rounds. As a result, the adversary can split the cipher into two parts in an unconventional way, and mount an efficient MITM attack in cases where such an attack seems difficult otherwise.

Another important technique used in several MITM attacks exploits the ability to independently and efficiently compute linear combinations of variables of the inner state (rather than the actual variables) of the cipher from the encryption and decryption sides. The MITM attack is then applied through a linear layer of the block cipher, in the same way as in a number of attacks on SHA-1, SHA-2 and AES-based designs, e.g. [2, 5, 14, 19]. This technique is often referred to as indirect partial matching; it also corresponds to the linear case of the sieve-in-the-middle method, described in [6].

In this paper, we also exploit some additional filtering conditions derived from the linear dependencies in the key suggestions that are computed from both sides of the MITM attack. Similar ideas where previously used in MITM attacks against SHA-1 in [2, 15] (which exploited the linear message schedule in the hash function to obtain filtering conditions). However, this is the first time that the technique is used in the domain of block ciphers, and we call it a *linear key sieve.*

Although we do not expect such linear dependencies to exist in block ciphers with complex non-linear key schedules, they are much more likely to occur in lightweight designs with simple key schedules. We apply the linear key sieve technique to LED-64 [13], and use it to improve some of the best previously known attacks on step-reduced variants of this block cipher in both the single-key and related-key models.

The lightweight block cipher LED was presented at CHES 2011 [13], and due to its elegant AES-based design, it has been the target of significant cryptanalytic effort in the past few years. In the single-key chosen plaintext model, the best previously known attack on 2 steps of LED-64 (reduced from the full 8) was presented in [14], and we reduce its time complexity from 2^{56} to 2^{48}. Both the previous attack and our new attack apply the splice-and-cut technique in order to mount a MITM attack on the cipher. The main element that enables us to improve the previous attack of [14] is the linear key sieve, which we use in order to filter the key suggestions obtained during the attack in a more efficient way.

In addition to the chosen plaintext attack, we describe a known plaintext attack on 2-step LED-64 which improves both the time and memory complexities of the previous best attack[1] (presented at Asiacrypt 2013 [8]) from 2^{60} to 2^{48}. The main novelty of this attack is that it uses, for the first time, the splice-and-cut technique (which seems to require chosen messages in an inherent way[2]) in the *known plaintext model*. Once again, in this attack we use the linear key sieve technique, and it enables our known plaintext attack to maintain the same running time as our chosen plaintext attack.

Finally, in the stronger related-key model, we analyze 3-step LED-64, on which the best previously known attack[3] used a classical differential method [17]. In this model, we extend the differential MITM attack[4] on AES-based designs of [5], and apply it in two independent stages from both sides of the cipher. We use this technique, it in addition to the linear key sieve, in order to improve the previous attack of [17] in all the complexity parameters of time/memory/data from 2^{60} to 2^{49}.

The paper is organized as follows: in Section 2, we briefly describe LED-64, and in Section 3, we describe the notations and conventions that are used in this paper. Our new chosen plaintext, known plaintext and related-key attacks are described in Sections 4, 5 and 6, respectively. Finally, we conclude in Section 7.

2 Description of LED-64

LED [13] is a 64-bit block cipher built using several public permutations, interleaved with round-key additions over $GF(2)$ (i.e., XOR operations). This

[1] We also mention the attack on 3-step LED-64 given in [8]. However, despite its theoretical significance, the attack is non-practical with respect to the memory complexity (which is 2^{60}) and it is only about 16 times faster than exhaustive search.

[2] As explicitly required in the first generic application of the splice-and-cut technique to block ciphers [22], and in subsequent splice-and-cut attacks on concrete block ciphers such as in [14].

[3] We also mention the related-key attack on 4-step LED-64 given in [17]. However, despite its theoretic interest, it is very marginal with respect to the time, memory and data complexities, which are all about 2^{63}.

[4] Differential MITM is a classical technique used, e.g., in [9], and was recently formalized in [15].

Reference	Model	Steps	Time	Data	Memory
[17]	Single-key	2	2^{60}	2^{60} CP	2^{60}
[14]	Single-key	2	2^{56}	2^{8} CP	2^{8}
Section 4.2	Single-key	2	2^{48}	2^{16} CP	2^{17}
Section 5	Single-key	2	2^{48}	2^{48} KP	2^{48}
[8]	Single-key	3	$2^{60.2}$	2^{49} KP	2^{60}
[17]	Related-Key	3	2^{60}	2^{60} CP	2^{60}
Section 6.2	Related-Key	3	2^{49}	2^{49} CP	2^{49}
[17]	Related-Key	4	2^{63}	2^{63} CP	2^{63}

The data complexity is given in chosen plaintexts (CP), or in known plaintexts (KP).

Table 1. Attacks of Step-Reduced LED-64

construction is generally known as iterated Even-Mansour (see Figure 1), which generalizes the original one-round construction [10]. In the case of LED, the public permutations are called steps, and each step is composed of 4 rounds. A round of LED uses an AES-like design, where given a 64-bit input X, it is treated as a concatenation of 16 four-bit nibbles $X[0] \| X[1] \| \dots \| X[15]$, which are (conceptually) arranged in a 4×4 array:

$X[0]$	$X[1]$	$X[2]$	$X[3]$
$X[4]$	$X[5]$	$X[6]$	$X[7]$
$X[8]$	$X[9]$	$X[10]$	$X[11]$
$X[12]$	$X[13]$	$X[14]$	$X[15]$

The round function uses 4 AES-like mappings AddConstants (AC), SubCells (SC), ShiftRows (SR), and MixColumnsSerial (MCS). The structural properties of these mappings (given below) are similar to those of the AES mappings AddRoundKey, SubBytes, ShiftRows and MixColumns, respectively, and these are the only properties which are exploited by our attacks. For the complete implementation details of the LED mappings, refer to [13].

1. AddConstants adds (over GF(2)) a round-dependent constant to each cell of the first two columns.
2. SubCells applies a 4-bit Sbox to every cell of the internal state.
3. ShiftRows rotates each cell located in row i by i positions to the left.
4. MixColumnsSerial independently applies an MDS (Maximum Distance Separable) matrix over $GF(2^4)$ to each column.

LED has two main variants, LED-64 and LED-128, which differ according to the key size. In this paper, we are mainly interested in the 64-bit version, which uses 32 rounds (or 8 steps). The key schedule of LED-64 simply adds the 64-bit key K before rounds $4i + 1$ for $i = 0, 1, \dots, 7$, and again after the final round.

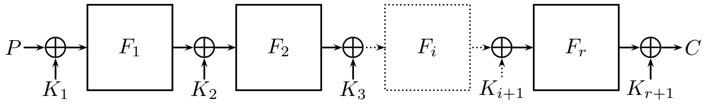


Fig. 1. Iterated Even-Mansour

3 Notations and Conventions

Notations We denote by R_r the public function of round r of LED-64 (without the key addition), i.e., given a 64-bit state X, $R_r(X) \triangleq MCS(SR(SC(AC(X)))$. We denote by F_i the public function of step i, i.e., given a 64-bit state X, $F_i(X) \triangleq R_{4i+4}(R_{4i+3}(R_{4i+2}(R_{4i+1}(X))))$. The functions R_r^{-1} and F_i^{-1} are defined as the inverses of R_r and F_i, respectively.

Given a plaintext-ciphertext pair (P, C), we denote the state after r encryption rounds by X_r (e.g., $X_0 = P$ and X_1 is the state after one round of LED-64). In order to simplify our notation, we define $\hat{X}_{4i} = X_{4i} \oplus K$, and so $F_i(\hat{X}_{4i}) = X_{4(i+1)}$. In some of our attacks, in addition to obtaining plaintext-ciphertext pairs, we independently evaluate the public step function F_i (for some i) on some input states $\hat{Y}_{4i}$, and we define $F_i(\hat{Y}_{4i}) = Y_{4(i+1)}$ (i.e., $\hat{Y}_{4i} = Y_{4i} \oplus K_i$).

We denote the j'th column of X_i by $X_{i,|j|}$, i.e., $X_{i,|1|}$ is composed of nibbles $\{1, 5, 9, 13\}$. Similarly, we denote by $X_{i,|j,l|}$ columns j and l of X_i. We define two more column-related sets: the first is the *diagonal* $X_{i,/j/}$ which is composed of the nibbles in X_i corresponding to the places after the ShiftRows operation on column j, e.g., $X_{i,/1/}$ is composed of nibbles {1,4,11,13}. The second set is *inverse diagonal* $X_{i,\backslash j \backslash}$ which is composed of the nibbles in the positions of column j after having applied the inverse ShiftRows operation (see Figure 2).

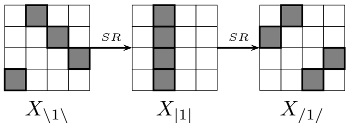


Fig. 2. An Inverse Diagonal, a Column and a Diagonal

Conventions Throughout this paper, we use the standard conventions and calculate the time complexity of our attacks in terms of evaluations of the full cipher, while calculating their memory complexity in terms of 64-bit words (since the block size of LED-64 is 64 bits). Some of the attacks presented in this paper involve basic linear algebra algorithm (such as solving a system of linear equations with a few dozen variables[5] over $GF(2)$). Since our attacks execute these basic linear algebra algorithms no more than a few times per evaluation of the full cipher, we can ignore them in our time complexity analysis.

4 The Linear Key Sieve Technique – A Chosen Plaintext Attack on 2-Step LED-64

In this section, we introduce the linear key sieve technique, and apply it (combined with splice-and-cut) to 2-step LED-64. Our attack improves the time complexity of the previously best known attack on 2-step LED-64 [14] (which is based on the techniques of [19]) from 2^{56} to 2^{48}.

To simplify the description of the technique, before presenting the full 2-step attack, we introduce the linear key sieve in a simple example of a basic MITM attack on 1-step LED-64.

4.1 A Meet-in-the-Middle Attack on 1-Step LED-64

We describe a basic MITM attack on 4-round (1-step) LED-64, using a single known plaintext-ciphertext pair ($P = X_0, C = \hat{X}_4$). The attack is based on a few simple and well-known observations on AES-based constructions:

1. The order of the linear operations ARK and MCS is interchangeable, i.e., $MCS^{-1}(ARK^{-1}(C)) = ARK'^{-1}(MCS^{-1}(C))$, where ARK' adds the key $K' \triangleq MCS^{-1}(K)$ to the state. As in many attacks on AES-based constructions, we can thus apply MCS^{-1} to the ciphertext C, and "peel-off" the last-round MCS operation.
2. Given an inverse diagonal $X_{r,\backslash i\backslash}$ (at the beginning of any round r), we can fully compute the diagonal $SR(SC(AC(R_r(X_r))))_{/i/} = MCS^{-1}(X_{r+2})_{/i/}$ after the first 7 operations. Similarly, given a diagonal $MCS^{-1}(X_{r+2})_{/i/}$, we can fully compute the inverse diagonal $X_{r,\backslash i\backslash}$ after the 7 inverse operations. Such a permutation, mapping 4 nibbles to 4 nibbles of the state through a "round and a half" is called a "Super-Sbox" of LED (a term which was originally defined for AES [7]).
3. Given knowledge of any b_1 bits of the state X, we can compute the values of b_1 linear combinations (over $GF(2)$) on the bits of the state $MCS(X)$.

Observation 2 implies that, given any two inverse diagonals $\hat{X}_{0,\backslash i,j\backslash}$, we can compute 32 bits of $MCS^{-1}(X_2)_{/i,j/}$, namely the two diagonals of indices i and j. Combined with Observation 3, they correspond to 32 linear combinations of the bits of the state X_2, spanning a subspace of dimension 32. Similarly, from the decryption side, the knowledge of three diagonals of $MCS^{-1}(ARK^{-1}(C))_{/l,m,n/}$

[5] In order to reduce the $O(n^3)$ bit operations required to solve a system of n linear equations $Ax = b$ in the online phase of the attack, we compute A^{-1} offline. Given the vector b in the online phase, we simply compute $x = A^{-1}b$ in $O(n^2)$ bit operations.

gives us the knowledge of 48 bits of X_2, namely, the corresponding inverse diagonals $X_{2,\backslash l,m,n\backslash}$, as shown in Figure 3.

Since the full state contains 64 bits, the intersection of these two subspaces is a linear subspace of dimension[6] $32+48-64=16$. The basis of this subspace gives rise to 16 linearly independent combinations in the bits of the state X_2 (denoted by A, as shown in Figure 3) whose values are computable independently from the known 2 inverse diagonals (from the encryption side), and from the known 3 diagonals (from the decryption side).

The computation of the $|A|=16$ joint linear combinations, requires the knowledge of $16 \cdot 2 = 32$ bits of K from one side and $16 \cdot 3 = 48$ bits of $K' = MCS^{-1}(K)$ from the other side. For the correct guess of these bits, the values of the joint linear combintations in A match with probability 1, whereas for an arbitrary incorrect suggestion, based on standard randomness assumptions, the values of the linear combinations in A match with probability $2^{-|A|} = 2^{-16}$. Therefore, incorrect suggestions in the MITM attack are discarded (as in the related attacks of [5, 14, 19]).

In standard MITM attacks, if one treats the keys K and K' as independent, there are $2^{32+48} = 2^{80}$ suggestions for the key from both sides. Given the 16 bits of the sieve on the state, we expect about 2^{64} suggestions to remain, which we need to further analyze. Thus, in its current form, this attack is not faster than exhaustive search.

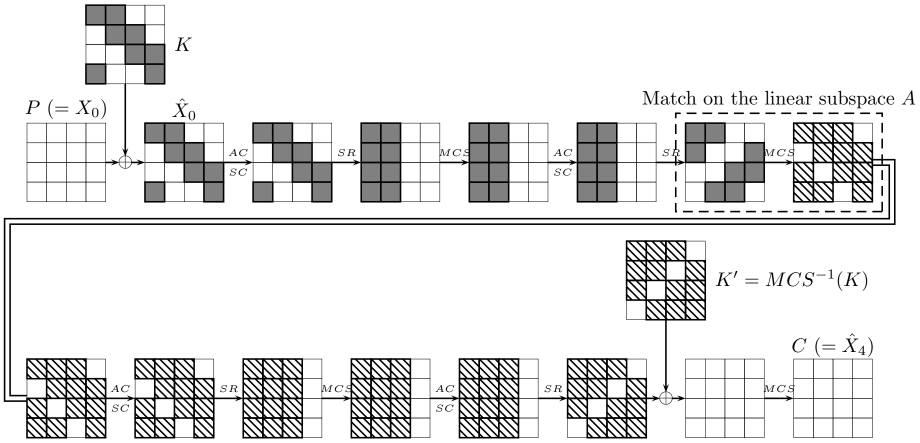


Fig. 3. A Meet-in-the-Middle Attack on 1-Step LED-64

[6] In general, the dimension of the intersection can be bigger. However, in AES-based constructions (where the MixColumns operation is implemented using an MDS matrix), the dimension of the intersection is exactly $32+48-64=16$.

The Linear Key Sieve A possible solution to the problem of insufficient filtering is to use an additional plaintext-ciphertext pair, which will offer an additional 16-bit filtering condition. However, we now introduce the *linear key sieve* which provides these 16 bit-conditions with *no additional data*, by exploiting the linear dependency of K and K'. This observation (exploited by [2] in the domain of hash functions, but which [14] did not use) is at the basis of our improved attack on 2-step LED-64 (described in the next section), and all the other attacks presented in the paper.

Recall that the MITM attack requires 32 bits of K and 48 bits of $K' = MCS^{-1}(K)$, which are linear combinations in the bits of K. Just as the state subspaces intersect (and allow us to obtain the sieve on the state), so do the two linear subspaces spanned by the linear combinations in the bits of K and K' that we guess. The intersection is a linear subspace of dimension $32 + 48 - 64 = 16$, giving rise to 16 linearly independent combinations in the bits of K (denoted by B_3), whose values are computable independently from both sides. The linear combinations in B_3 are used in order to filter our wrong key guesses (for the right key they agree with probability 1, and for wrong key guesses they agree with probability $2^{-|B_3|}$), and thus we call this set of linear combinations a *linear key sieve.*

Let B_1 be additional 16 linear combinations of K needed for the attack[7] (i.e, B_1 and B_3 determine the partial encryption of two inverse diagonals), and let B_2 be additional 32 linear combinations of K' needed for the attack (i.e, B_2 and B_3 determine the partial decryption of three diagonals). Our MITM attack is composed of an outer loop, iterating over the linear subspace spanned by B_3, where in each iteration, we independently iterate over the linear subspaces spanned by B_1 and B_2.[8] Thus, we *force* the key suggestions obtained from both sides of the attack to agree on B_3 (rather than randomly achieving agreement). The resultant attack (described for two arbitrarily columns i, j in the forward direction and three arbitrary columns l, m, n in the backward direction) is as follows:

1. For each value of the 16 linear combinations of B_3:
   (a) For each value of the 16 linear combinations of B_1:
      i. Compute $K_{\backslash i,j\backslash}$, and use it to compute $\hat{X}_{0,\backslash i,j\backslash}$.
      ii. Compute the values of the 16 linear combinations of A, and store them in a sorted list L, next to the value of the 16 linear combinations of B_1.
   (b) For each value of the 32 linear combinations of B_2:
      i. Compute $K'_{/l,m,n/}$, and use it to compute $MCS^{-1}(ARK^{-1}(C))_{/l,m,n/}$.
      ii. Compute the values of the 16 linear combinations of A, and search for matches in the list L.
      iii. For each match:

[7] There are many options for the basis B_1, and we choose one arbitrarily.

[8] We note that the approach of taking out shared bits to an outer loop is a very common practice in saving memory. The main improvement in this attack compared to [14] is the fact that we take out shared *linear combinations.*

A. Obtain the value of the 16 linear combinations of B_1.
B. Compute K using linear algebra, given the values of B_1, B_2 and B_3.
C. Test K using a trial encryption, and if it succeeds, return the key.

The list L contains 2^{16} values, and thus we expect a single match for each value of the 16 linear combinations of A in Step 1.(b).ii. This implies that the expected time complexity of each iteration of Step 1 is about 2^{32}, and thus the expected time complexity of the whole attack is 2^{48}, which is faster than exhaustive search by a factor of 2^{16}. The memory complexity of the attack is about 2^{16}, which is required for storing the list L. Note that the memory needed for storing L in each iteration of Step 1 can be reused.

4.2 The Improved Chosen Plaintext Single-Key Attack on 2-Step LED-64

Our attack on 2-Step LED-64 follows the same general structure as the previous one of [14, 19]. We use the splice-and-cut technique on 4 rounds (1 step) of the cipher. The advantage of our attack comes from the linear key sieve (missing from [14]), i.e., using the linear relations between K and $K' = MCS^{-1}(K)$.

In order to apply splice-and-cut to 2-step LED-64, we (as in the previous attack [14]) partition the indices of 64-bit state into two lexicographically ordered sets, S_1 and S_2. The attack requires the encryptions of $2^{|S_1|}$ plaintexts $P^1, P^2, \ldots$ in which all the bits of S_2 are fixed to zero,[9] and the bits of S_1 range over all the possible values. Independently, we evaluate the first 4 key-less rounds of LED-64 (i.e., F_1) on $2^{|S_2|}$ inputs $\hat{Y}_0^1, \hat{Y}_0^2, \ldots$ in which the bits of S_1 are fixed to zero, and the bits of S_2 range over all the possible values, and obtain the corresponding outputs $Y_4^1, Y_4^2, \ldots$.

The aim of the splice-and-cut technique is to find a plaintext $P^i = X_0^i$ and an internal state $\hat{Y}_0^j$ such that $\hat{X}_0^i = X_0^i \oplus K = \hat{Y}_0^j$. This occurs if and only if P^i and K "agree" on the bits of S_1 and $\hat{Y}_0^j$ and K "agree" on the bits of S_2, or formally $P^i_{|S_1} = K_{|S_1}$ and $\hat{Y}^j_{0|S_2} = K_{|S_2}$ (where $W_{|S}$ denotes the $|S|$-bit value of the word W on the indices of the ordered set S.). In other words, each plaintext P^i is associated with a potential value of $K_{|S_1}$, and each state $\hat{Y}_0^j$ is associated with a potential value of $K_{|S_2}$. For each value of K, there is only one such correct pair, and we denote its plaintext by P, and its evaluated state by $\hat{Y}_0 = P \oplus K$. Thus, finding the pair (i, j) is equivalent to recovering the key K.

Consider the correct pair $(P, \hat{Y}_0)$. Applying F_1 to $\hat{Y}_0$ gives $Y_4 = X_4$. Thus, if we consider all $F_1(\hat{Y}_0^j) = Y_4^j$ values, one of them is indeed X_4. As a result, given the ciphertext C that corresponds to P and X_4, the splice-and-cut technique reduces the problem to attacking 4 rounds of LED-64. Hence, as shown in Figure 4, we choose the bits of S_2 to be the 48 bits of 3 inverse diagonals (and thus S_1 contains the 16 bits of the remaining inverse diagonal). As a result, we

[9] The technique can be applied in a similar way for any fixed value of the bits of S_2.

can take each Y_4^j value (associated with a suggestion for 3 inverse diagonals of K) and continue its partial encryption (as per Observation 2), resulting in the knowledge of three diagonals before the MCS operation of round 6, i.e., three diagonals of $MCS^{-1}(Y_6)$. This knowledge gives rise to suggestions for the values of 48 linear combinations of Y_6. Independently, we try all possible values of 32 bits of two diagonals of K', and partially decrypt all the ciphertexts to obtain suggestions of 32 bits of X_6.

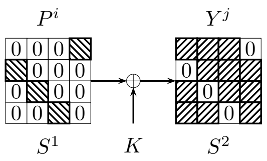


Fig. 4. The Sets S_1 and S_2

Now, we can apply the 4-round attack (as $Y_6 = X_6$), obtaining a sieve of the state of $48 + 32 - 64 = 16$ linear combinations on the state bits (each independently computed from a different side), denoted by A. To obtain more filtering conditions, we again use the linear key sieve: each ciphertext is associated with 16 bits of K, and thus each suggestion for the 32 bits of X_6 is associated with the values of $16 + 32 = 48$ linear combinations on the bits of K. Since each suggestions for Y_6 depends on 48 bits of K, we have $48 + 48 - 64 = 32$ linear combinations which we use as a linear key sieve (denoted by B_3). Similarly to our basic MITM attack, we complement the 32 linear combinations of B_3 to a basis of the subspace spanned by the 48 bits of S_2 using 16 additional linear combinations, denoted by B_2. Similarly, we complement B_3 to a basis of the 48-dimensional subspace spanned by the 16 bits of S_1 and the 32 bits of the 2 diagonals of K', using 16 additional linear combinations, denoted by B_1.

The attack proceeds as follows (see figure 5):

1. Request the encryptions of the 2^{16} plaintexts P^i such that $P^i_{\backslash 0,1,2\backslash} = 0$, and store all plaintext-ciphertext pairs.
2. For each value of the 32 linear combinations of B_3:
   (a) For each value of the 16 linear combinations of B_2:
      i. Using the values of B_2 and B_3, compute a suggestion for $K_{\backslash 0,1,2\backslash}$.
      ii. Let $\hat{Y}_0^j$ be the state such that $Y^j_{0,\backslash 3\backslash} = 0$ and $Y^j_{0,\backslash 0,1,2\backslash} = K_{\backslash 0,1,2\backslash}$.
      iii. Compute $F_1(\hat{Y}_0^j) = Y_4^j$, and use the partial knowledge of $K_{\backslash 0,1,2\backslash}$ to obtain a suggestion for the values of the 16 linear combinations of A. Store the suggestion for the values of A in a sorted list L, next to the value of B_2.
   (b) For each value of the 16 linear combinations of B_1:

i. Using the values of B_1 and B_3, compute a suggestion for $K_{\backslash 3 \backslash}$ and $K'_{/0,1/}$.

ii. Let (P^i, C^i) be the plaintext-ciphertext pair such that $P^i_{\backslash 3 \backslash} = K_{\backslash 3 \backslash}$ (recall that $P^i_{\backslash 0,1,2 \backslash} = 0$).

iii. Compute a suggestion for the values of the 16 linear combinations of A using C^i and $K'_{/0,1/}$.

iv. Search for the suggestion for the values of A in the list L.

v. For each match, obtain the value of B_2, use it to obtain a suggestion for the key K and test it using a trial encryption. If the trial succeeds, return the key.

The data complexity of the attack is 2^{16} chosen plaintexts. The memory complexity of the attack is about 2^{17}, required in order to store the plaintext-ciphertext pairs, and in order to store the list L. Since L contains 2^{16} values per iteration of Step 2.(a), we expect one match in Step 2.(b).iv, and a total of 2^{16} matches per iteration of Step 2. Thus, the time complexity of an iteration of Step 2 is equivalent to about 2^{16} 2-step LED-64 encryptions, and the total time complexity of the attack is about 2^{48} encryptions.

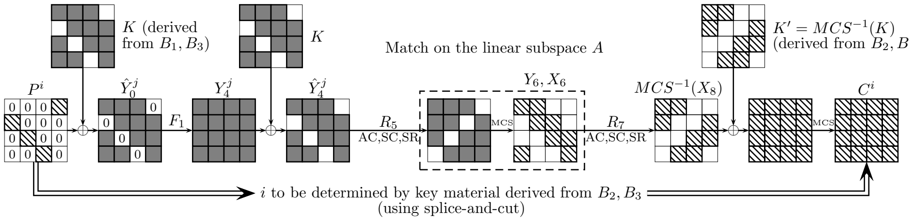

K (derived from B_1, B_3)
K
Match on the linear subspace A
$K' = MCS^{-1}(K)$ (derived from B_2, B_3)
P^i
$\hat{Y}_0^j$
Y_4^j
$\hat{Y}_4^j$
Y_6, X_6
$MCS^{-1}(X_8)$
C^i
0
F_1
R_5
AC,SC,SR
MCS
R_7
AC,SC,SR
MCS
i to be determined by key material derived from B_2, B_3
(using splice-and-cut)

Fig. 5. Our Improved Chosen Plaintext Attack on 2-Step LED-64

5 Known-Plaintext Splice-and-Cut Attack – Application to 2-Step LED-64

In this section, we describe the first splice-and-cut attack (which was believed to be inherently a chosen message technique) in the known plaintext model. In particular, we improve the time and memory complexities of the best attack on 2-step LED-64 [8] in the known plaintext model from 2^{60} to 2^{48}. In fact, due to the efficient sieving techniques, this attack has the same time complexity as our chosen plaintext attack, presented in the previous section.

The classical chosen-plaintext splice-and-cut attack considers two sets: a set of plaintexts P^i of size 2^m and a set of states $\hat{Y}_0^j$ of size 2^{n-m} such that the set $P^i \oplus \hat{Y}_0^j$ covers F_2^n, i.e., there exists a unique pair (i, j) such that $P^i \oplus \hat{Y}_0^j = K$.

However, the coverage of F_2^n by $P^i \oplus \hat{Y}_0^j$ cannot be guaranteed with probability 1 if the set of 2^m plaintexts (such that $1 \gg 2^m \gg 2^n$) is composed of arbitrary elements. Consequently, in our known-plaintext variant, the existence of some pair (i, j) such that $P^i \oplus \hat{Y}_0^j = K$ is now guaranteed with a good probability by the birthday paradox (assuming that the plaintexts are uniformly distributed).[10]

The main difficulty in applying our splice-and-cut technique in the known plaintext model, is that $P_{|S_2}$ is not fixed, and thus we "lose" the association of an input $\hat{Y}_0^j$ to a potential value of $K_{|S_2}$. Consequently, for each $\hat{Y}_0^j$, we need to guess additional key bits in order to obtain the necessary filtering conditions on Y_6. Since an input $\hat{Y}_0^j$ is now associated with many key guesses, we are forced to evaluate fewer such inputs in order to obtain an efficient attack. Indeed, while in the chosen plaintext attack, we evaluated 2^{48} such inputs, here we evaluate only 2^{16} inputs, and for each one we guess the 32 bits of $K_{\backslash 0,1 \backslash}$ and obtain a suggestion for the values of 32 linear combinations on Y_6. According to the birthday paradox, we need 2^{48} known plaintexts-ciphertext pairs (P^i, C^i) $(i \in \{1, 2, \ldots, 2^{48}\})$ to obtain with good probability a pair (i, j), such that $P^i \oplus \hat{Y}_0^j = K$.

The values $\hat{Y}_0^1, \hat{Y}_0^2, \ldots$ which we evaluate are defined by the 16-dimensional linear subspace $\{\hat{Y}_0^j | MCS^{-1}(\hat{Y}_0^j)_{/1,2,3/} = 0\}$. Thus, a plaintext P^i is implicitly associated with a partial key value $P^i_{/1,2,3/} = K'_{/1,2,3/}$. This implies that we can partially decrypt C^i in order to obtain the values of 48 bits of X_6^i without additional key guesses. Note that this is not the traditional way in which splice-and-cut is applied, as all previous attacks (including our previous chosen plaintext attack) directly partitioned the bits of the state into two groups S_1 and S_2. Instead, in this attack we work with linear subspaces constructed to exploit the linear dependency between K and K' in order to be able to partially decrypt C^i without additional key guesses (whereas our previous chosen plaintext attack did not directly exploit this dependency).

As in the chosen plaintext attack, we have a sieve on the state of $32 + 48 - 64 = 16$ linear combinations on the bits of $X_6 = Y_6$ which are independently computable from each side, and we denote it by A_1. Each value of the linear combinations of A_1, computed from an input $\hat{Y}_0^j$, is associated with a suggestion for $K_{\backslash 0,1 \backslash}$, and as a result, we can also compute $Y^j_{0,\backslash 0,1 \backslash} = \hat{Y}^j_{0,\backslash 0,1 \backslash} \oplus K_{\backslash 0,1 \backslash}$. Since $P^i = X_0^i$, the 32-bit value of $Y^j_{0,\backslash 0,1 \backslash}$ can be directly matched with each plaintext, and we denote this sieve by A_2. We note that since $MCS^{-1}(\hat{Y}_0^j)_{/1,2,3/} = 0$, then Y^j (and $Y^j_{0,\backslash 0,1 \backslash}$), can only attain 2^{16} values, and thus effectively, the 32 bits of A_2 give only 16 bits of filtering.

From the decryption side, each value computed from (P^i, C^i) is associated with a suggestion for $K'_{/1,2,3/}$. Thus, we can identify $32 + 48 - 64 = 16$ linear combinations (i.e., a linear key sieve) which are independently computable from each side, and we denote this sieve by B. In total, we have 48 bits of filtering,

[10] We note that in case that the plaintexts are not uniformly distributed, we can apply a similar attack assuming that the ciphertexts are uniformly distributed (by exchanging the roles of encryption and decryption).

as A_1 gives us 16 bits, A_2 (effectively) gives us 16 bits of filtering, and B gives us additional 16 bits.

The attack proceeds as follows:

1. For each of the 2^{16} possible values of $\hat{Y}_0^j$ such that $MCS^{-1}(\hat{Y}_0^j)_{/1,2,3/} = 0$:
   (a) Compute $F_1(\hat{Y}_0^j) = Y_4^j$
   (b) For each of the 2^{32} values of $K_{\backslash 0,1\backslash}$:
      i. Compute the values of the 16 linear combinations of A_1, the values of the bits of A_2, and the values of the 16 linear combinations of B. Store these values in a sorted list L, next to the values of $K_{\backslash 0,1\backslash}$.
2. For each plaintext-ciphertext pair (P^i, C^i):
   (a) Assume that $K'_{/1,2,3/} = MCS^{-1}(P^i)_{/1,2,3/}$, compute the values of A_1, A_2 and B, and search the list L for matches.
   (b) For each match, obtain $K_{\backslash 0,1\backslash}$, compute a suggestion for the full key K, and test it using a trial encryption. If the trial succeeds return the key.

Since we evaluate 2^{16} inputs $\hat{Y}_0^j$, we expect that after obtaining the encryptions of about 2^{48} arbitrary plaintexts, we will have a pair $(P^i, \hat{Y}_0^j)$ satisfying $P^i \oplus K = \hat{Y}_0^j$, which will enable us to recover the correct key.[11] Thus, the expected data complexity of the attack is 2^{48} known plaintexts. Since the size of L is 2^{48}, and (effectively) we have 48 bits of filtering conditions, we expect one match for each plaintext in Step 2.(a), and thus the time and memory complexities of the attack are 2^{48} as well.

Data/Memory/Time Tradeoffs The attack can be applied by evaluating $\hat{Y}_0^1, \hat{Y}_0^2, \ldots$ in a linear subspace $\{\hat{Y}_0^j | MCS^{-1}(\hat{Y}_0^j)_{/1,2,3/} = \mathit{const}\}$ for *any* value of *const*, with the appropriate changes to the algorithm. This suggests data/memory/time tradeoffs for the attack, in which we first obtain the plaintext-ciphertext pairs and store them in memory (thus exchanging the order of some computations performed in Steps 1 and 2 above). Then, we perform the computations of Step 1 (previously used in order to build the list L) "on-the-fly", and search the stored data for matches according to the filtering conditions.

By using this revised algorithm, we can reduce the amount of known plaintexts by a factor of 2, repeating the attack once with $\mathit{const} = 0$, and once for an arbitrary value $\mathit{const} \neq 0$. In this case, the time complexity of the attack is increased by a factor of 2, and its memory complexity is reduced by a factor of 2. Similarly, for any $d \leq 16$, given 2^{48-d} known plaintexts, the time complexity of the attack increases to 2^{48+d}, and its memory complexity decreases to 2^{48-d}. In other words, we can attack the scheme (with a similar success rate) given $D \leq 2^{48}$ known plaintexts and time complexity T such that $DT = 2^{96}$ (with memory $M = D$).

[11] We note that unlike our chosen plaintext attack (that succeeds in finding the key with probability 1), our known plaintext attack succeeds with probability of about 0.63, which is the probability suggested by the birthday paradox (given 2^{48} known plaintext-ciphertext pairs).

6 Enhanced Differential Meet-in-the-Middle – A Related-Key Attack on 3-Step LED

In this section, we describe a related-key attack on 3-step LED-64 using two related keys. The attack improves the previously best known attack on this scheme, described in [17], in all the complexities parameters of time/memory/data from 2^{60} to 2^{49}. The 3-step attack uses the linear key sieve technique on top of a rather involved differential MITM attack. Before describing the full attack, we describe a simple differential MITM attack on 1-step LED-64 in the single-key model, which serves as background to our 3-step related-key attack. We note that in the case of 1-step LED-64, our simple attack is closely related to the attack on Pelican-MAC described in [5].

6.1 A Differential Single-Key Meet-in-the-Middle Attack on 1-Step LED-64

The simple differential MITM attack on 4-round (1-step) LED-64 requires 2 chosen plaintexts, and its memory and time complexities are slightly more than 2^{16}. In order to obtain an efficient attack, we compute and use the difference distribution tables for the LED Super-Sboxes (spanning the third round and part of the fourth round), using similar technique to those published in [7, 12, 16]. Namely, given an entry $[\delta_{in}, \delta_{out}]$, specifying a 16-bit input/output difference to the Super-Sbox, the table stores the actual pairs of values that conform to this entry. A single full table can be easily computed during preprocessing in 2^{32} simple operations, and it requires about 2^{32} words of memory. However, in this simple attack, the output difference δ_{out} to each Super-Sbox is fixed by the ciphertexts, and thus we only need a single column in each table. Such a column is computed in the online phase (after obtaining the encryptions of the plaintexts) in 2^{16} time, using 2^{16} storage.

The details of the attack are given below.

1. Obtain the encryptions of P^1 and P^2, chosen such that $(P^1 \oplus P^2)_{\backslash 0,1,2 \backslash} = 0$.

Denote $\Delta_r \triangleq X_r^1 \oplus X_r^2$, i.e., Δ_r is the 64-bit state difference after round r.

3. Compute 3 columns in the difference distribution tables of the LED Super-Sboxes, corresponding to the output differences specified by the three diagonals $MCS^{-1}(\Delta_4)_{/1,2,3/}$.
4. For each value of $K'_{/0/}$:
   (a) Use C^1 to compute $X^1_{2,\backslash 0 \backslash}$ and C^2 to compute $X^2_{2,\backslash 0 \backslash}$, and calculate $\Delta_{2,\backslash 0 \backslash}$.
   (b) Given $\Delta_{2,\backslash 0 \backslash}$, and the fact that $MCS^{-1}(\Delta_2)_{/0,1,2/} = 0$, calculate the full Δ_2 by solving a system of linear equations.
   (c) Given the input difference Δ_2 and $MCS^{-1}(\Delta_4)$, use the Super-Sbox (partial) difference distribution tables to obtain the possible values for $ARK'(MCS^{-1}(C^1))_{/1,2,3/}$, and use these values to obtain suggestions for the full K', thus obtaining suggestions for K.

(d) Test each suggestion for K using a trial encryption, and if it succeeds return the key.

Since we expect, on average, a single suggestion for K in Step 3.(c), the time complexity of Step 3 is about 2^{16}, which is the time complexity of the full attack. The memory complexity is about 2^{16}, required in order to store the columns of the difference distribution tables for the Super-Sboxes.

We note that this attack is faster than the attack of Section 4.1 since the collision on the diagonals of P allows us to obtain a suggestion for the full key after guessing only 16 key bits (enabling us to compute the full state difference Δ_2). This observation will be further exploited in the next section.

6.2 The Improved Related-Key Attack on 3-Step LED-64

In this section, we describe the details of our related-key attack on 12-round (3-step) LED-64 which assumes that we can obtain the encryptions of plaintexts with keys K_1 and K_2, such that $K_1 \oplus K_2 = \Delta$ is known (in fact, as explained below, we only need the ability to partially choose the value of Δ). During the online phase of the attack, we request the encryptions of 2^{48} chosen plaintexts encrypted with K_1 and 2^{48} (different) chosen plaintexts encrypted with K_2. The time and memory complexities of the attack are about 2^{49}.

Our attack uses the basic framework of [17] for related-key attacks on iterated Even-Mansour schemes. Namely, we ask for the encryptions of pairs of plaintexts $P^{i,1}$ and $P^{i,2} = P^{i,1} \oplus \Delta$, encrypted with K_1 and $K_2 = K_1 \oplus \Delta$, respectively. Considering the encryption process of these two plaintexts, the input difference to the public F_1 function is zero, which implies that the output difference of F_1 is zero, and after the second key addition, the input difference to F_2 is $\hat{\Delta}_4 = \Delta$ (namely, $\hat{X}_4^{i,1} \oplus \hat{X}_4^{i,2} = \hat{\Delta}_4 = \Delta$). At this point, our algorithm diverges from [17] (which assumes that the function F_2 has some high-probability differential characteristic).

Our attack is based on the 4-round differential MITM attack of the previous section. Here, we apply a similar attack to F_3 by processing plaintext pairs whose ciphertexts collide on a diagonal (before the MCS operation). However, unlike the 4-round attack, we do not know the input difference to the public function on which we perform the MITM attack (F_3 in this case). Thus, we preprocess F_2 by computing and storing pairs of inputs to this function with an input difference of Δ, and we use a birthday argument to claim that one of the input pairs will collide with a plaintext pair with high probability. However, this is insufficient, as we were not able to find parameters for which the differential MITM algorithm yields an efficient attack, under the constraint that we store sufficiently many pairs of inputs to F_2 (in order to obtain a collision with a processed plaintext pair). The problem is that we need to guess too many key bits before we can compute filtering conditions and eliminate some key guesses.

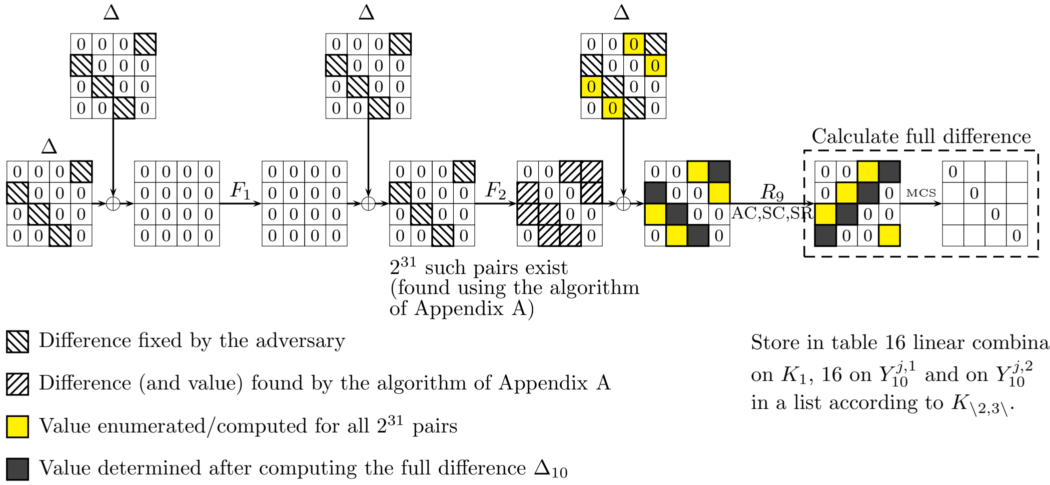
In order to reduce the number of key guesses, we again exploit collisions on (inverse) diagonals, but this time at the input of F_3. Namely, we require that the difference at the output of F_2 on some (inverse) diagonals cancels out after the key addition. More specifically, we preprocess F_2, and find 2^{31} pairs of inputs to this function, ($\hat{Y}_4^{j,1}$ and $\hat{Y}_4^{j,2} = \hat{Y}_4^{j,1} \oplus \Delta$), such that their output difference is equal to Δ in two inverse diagonals (i.e., $(Y_8^{j,1} \oplus Y_8^{j,2})_{\backslash 0,1 \backslash} = \Delta_{\backslash 0,1 \backslash}$, implying that $(\hat{Y}_8^{j,1} \oplus \hat{Y}_8^{j,2})_{\backslash 0,1 \backslash} = 0$). We expect that 2^{31} such pairs indeed exist, since there are 2^{63} unordered input pairs to F_2 with an input difference of Δ, and based on standard randomness assumptions, about 2^{31} of them satisfy the 32-bit condition on the output difference (a slightly smaller number will only slightly increase the complexity of the attack). The trivial algorithm to find these pairs is to exhaustively enumerate all the 2^{63} input pairs, however, this is wasteful as it requires 2^{64} time (and our model does not allow free precomputation). Instead, we use yet again a MITM approach, and devise an auxiliary preprocessing algorithm that finds the required pairs in about 2^{48} time. In order to run efficiently, our algorithm requires that 48 specific bits of Δ are zero, and thus we have to assume that we can partially choose the key difference[12] Δ. The details of this preprocessing step are specified in Appendix A.

The full preprocessing algorithm (which calls the algorithm of Appendix A) is given below and depicted in Figure 6.[13] In this attack, we assume that we have computed during preprocessing the full difference distribution tables for the LED Super-Sboxes, using about 2^{32} simple operations and 2^{32} memory.

1. Use the auxiliary preprocessing algorithm of Appendix A to obtain 2^{31} pairs $(\hat{Y}_4^{j,1}, \hat{Y}_4^{j,2} = \hat{Y}_4^{j,1} \oplus \Delta)$, such that $(Y_8^{j,1} \oplus Y_8^{j,2})_{\backslash 0,1 \backslash} = \Delta_{\backslash 0,1 \backslash}$.
2. For each of the 2^{31} pairs $(\hat{Y}_4^{j,1}, \hat{Y}_4^{j,2})$:
   (a) For each value of $(K_1)_{\backslash 2 \backslash}$:
      i. Compute $(K_2)_{\backslash 2 \backslash} = (K_1)_{\backslash 2 \backslash} \oplus \Delta_{\backslash 2 \backslash}$. Assume that $\hat{Y}_4^{j,1}$ is encrypted with K_1 and $\hat{Y}_4^{j,2}$ is encrypted with K_2, and let $\Delta_r = Y_r^{j,1} \oplus Y_r^{j,2}$. Use the LED Super-Sbox to compute $MCS^{-1}(\Delta_{10})_{/2/} = MCS^{-1}(Y_{10}^{j,1})_{/2/} \oplus MCS^{-1}(Y_{10}^{j,2})_{/2/}$.
      ii. Since $MCS^{-1}(\Delta_{10})_{/0,1/} = 0$ and $MCS^{-1}(\Delta_{10})_{/2/}$ is known, we know 48 bits of $MCS^{-1}(\Delta_{10})$. We now assume that $\Delta_{10,\backslash 0 \backslash} = 0$, and compute the full $MCS^{-1}(\Delta_{10})$ using linear algebra (as MCS is a linear operation).
      iii. Use the difference distribution table for the LED Super-Sbox, and the knowledge of $\hat{\Delta}_8 \triangleq \hat{Y}_8^{j,1} \oplus \hat{Y}_8^{j,2}$ and $MCS^{-1}(\Delta_{10})$ to compute suggestions for the actual $MCS^{-1}(Y_{10}^{j,1})_{/3/}$, $MCS^{-1}(Y_{10}^{j,2})_{/3/}$ and $(K_1)_{\backslash 3 \backslash}$.
      iv. We now have suggestions for 32 bits of K_1, 32 bits of $MCS^{-1}(Y_{10}^{j,1})$ and 32 bits of $MCS^{-1}(Y_{10}^{j,2})$. During the online phase, we will obtain suggestions for (specific) 48 bits of K'_1, 48 bits of $X_{10}^{i,1}$ and 48 bits of

[12] The related key attack of [17] required that we can choose the full 64 bits of Δ, so our attack is slightly more generic.

[13] Since the preprocessing algorithm of this attack is more involved than in the previous attacks, we describe it separately from the online algorithm.


Δ
Δ
Δ
Δ
F_1
F_2
R_9
AC,SC,SR
MCS
Calculate full difference
2^{31} such pairs exist (found using the algorithm of Appendix A)
Difference fixed by the adversary
Difference (and value) found by the algorithm of Appendix A
Value enumerated/computed for all 2^{31} pairs
Value determined after computing the full difference Δ_{10}

Store in table 16 linear combinations on K_1, 16 on $Y_{10}^{j,1}$ and on $Y_{10}^{j,2}$ in a list according to $K_{\backslash 2,3\backslash}$.

Fig. 6. The Preprocessing Algorithm (depicting differences)

$X_{10}^{i,2}$ (the encryption values of some $P^{i,1}$ and $P^{i,2}$ after 10 rounds). Thus, we can compute the values of a total of 48 linear combinations to serve as filtering bits: 16 linear combinations on the bits of K_1 (a linear key sieve), 16 linear combinations on $Y_{10}^{j,1}$ and 16 linear combinations on $Y_{10}^{j,2}$. We store the values of these linear combinations in a sorted list L, next to $(K_1)_{\backslash 2,3\backslash}$.

As described in Appendix A, the time complexity of Step 1 is about 2^{48} evaluations of 1-step LED. On average, we expect a single suggestion for the values computed in Step 2.(a).iii (using the difference distribution table of the LED Super-Sbox). Thus, we perform only a few simple operations for each of the 2^{31} pairs (computed in Step 1) and the 2^{16} possible values of $(K_1)_{\backslash 2\backslash}$, implying that the total time complexity of the preprocessing algorithm is about 2^{48} evaluations of 1-step LED. In order to slightly reduce the data complexity of the online algorithm (at the expense of using slightly more memory), we repeat Step 2.(a) twice for each ordered pair, exchanging the roles of $\hat{Y}_4^{j,1}$ and $\hat{Y}_4^{j,2}$ (i.e., by assuming that $\hat{Y}_4^{j,1}$ is encrypted with K_2). Thus, the time complexity of the preprocessing algorithm is about 2^{49} evaluations of 1-step LED, and its memory complexity is about 2^{49}.

The online algorithm of the attack is given below (and depicted in Figure **??**.

1. For 2^{48} arbitrary values of the plaintext $P^{i,1}$:
   (a) Ask for the encryption of $P^{i,1}$ under the key K_1, and for the encryption of $P^{i,2} = P^{i,1} \oplus \Delta$ under the key $K_2 = K_1 \oplus \Delta$. Let $\Lambda_r \triangleq X_r^{j,1} \oplus X_r^{j,2}$.

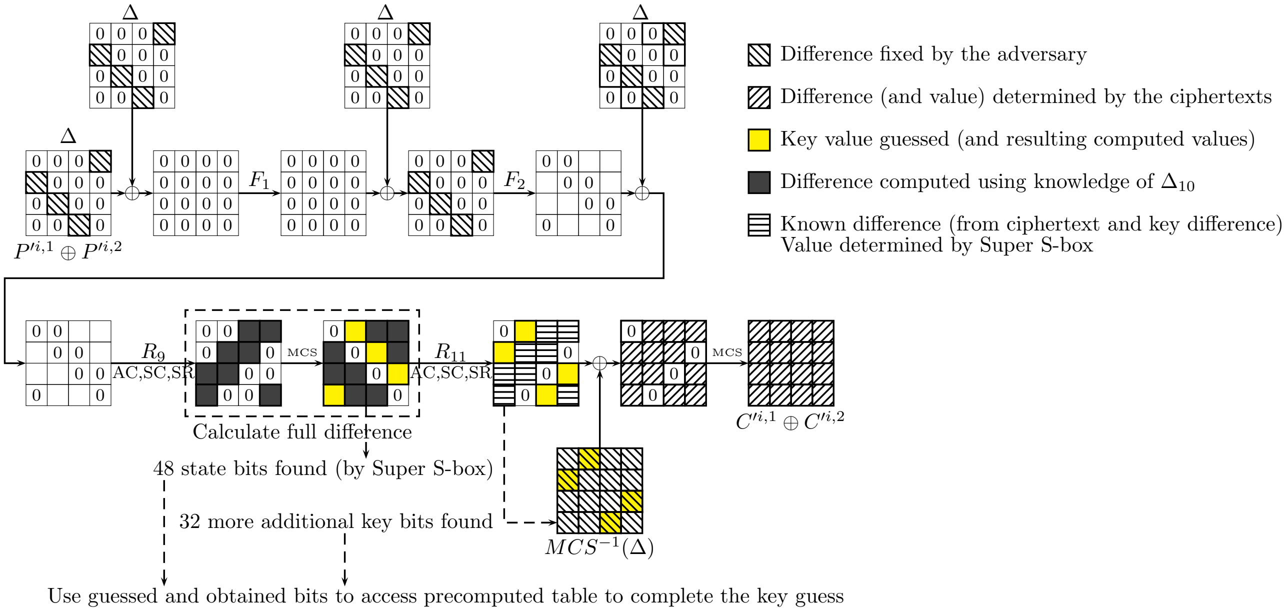

Δ
Δ
Δ
Δ
$P'^{i,1} \oplus P'^{i,2}$
F_1
F_2
▧ Difference fixed by the adversary
▨ Difference (and value) determined by the ciphertexts
🟨 Key value guessed (and resulting computed values)
■ Difference computed using knowledge of Δ_{10}
☰ Known difference (from ciphertext and key difference)
Value determined by Super S-box
R_9
AC,SC,SR
MCS
R_{11}
AC,SC,SR
MCS
$C'^{i,1} \oplus C'^{i,2}$
Calculate full difference
48 state bits found (by Super S-box)
32 more additional key bits found
$MCS^{-1}(\Delta)$
Use guessed and obtained bits to access precomputed table to complete the key guess

Fig. 7. The Preprocessing Algorithm (depicting differences)

(b) Compute $C'^{i,1} = MCS^{-1}(C^{i,1})$ and $C'^{i,2} = MCS^{-1}(C^{i,1})$. Check if $(C'^{i,1} \oplus C'^{i,2})_{/0/} = 0$, and if not, return to Step 1.

(c) For each value of $(K'_1)_{/1/}$:

i. Compute $(K'_2)_{/1/} = (K'_1)_{/1/} \oplus MCS^{-1}(\Delta)_{/1/}$ and use the LED Super-Sbox to compute $\Lambda_{10,\backslash 1\backslash} = X^{i,1}_{10,\backslash 1\backslash} \oplus X^{i,2}_{10,\backslash 1\backslash}$.

ii. Assume that $MCS^{-1}(\Lambda_{10})_{/0,1/} = 0$, and compute Λ_{10} (using the fact that $\Lambda_{10,\backslash 0\backslash} = 0$ and $\Lambda_{10,\backslash 1\backslash}$ is known).

iii. Use the difference distribution table for the LED Super-Sbox, and the knowledge of Λ_{10} and $(C'^{j,1} \oplus C'^{j,2})$ to compute suggestions for $X^{i,1}_{10,\backslash 2\backslash}$, $X^{i,2}_{10,\backslash 2\backslash}$ and $(K'_1)_{/2/}$, and similarly compute suggestions for $X^{i,1}_{10,\backslash 3\backslash}$, $X^{i,2}_{10,\backslash 3\backslash}$ and $(K'_1)_{/3/}$.

iv. From the knowledge of 48 bits of K'_1, 48 bits of $X^{i,1}_{10}$ and 48 bits of $X^{i,2}_{10}$, compute the filtering values of the 48 linear combinations, and look for matches in the list L.

v. For each match, obtain $(K_1)_{\backslash 2,3\backslash}$, compute a suggestion for K_1 and test it.

A pair $((P^{i,1}, C^{i,1}), (P^{i,2}, C^{i,2}))$ passes the 16-bit filtering condition of Step 1.(b) with probability 2^{-16}, and thus we expect to process about $2^{48-16} = 2^{32}$ pairs in Step 1.(c). As we store 2^{32} ordered input pairs of values for F_2 with an input difference of Δ, and each of the processed 2^{32} plaintext pairs has a difference of Δ at the input to F_2, we expect a collision between these two groups of pairs. For such a collision, the assumptions made during the online and preprocessing algorithms hold ($\Delta_{10,\backslash 0\backslash} = 0$ is assumed in preprocessing Step 2.(a).ii, and $MCS^{-1}(\Lambda_{10})_{/0,1/} = 0$ is assumed in online Step 1.(c).ii). Thus, this collision will yield a match in Step 1.(c).iv, suggesting the correct value of K_1.

Since we expect a single suggestion for the values computed in Step 1.(c).iii, we perform a few simple operations for each of the 2^{32} processed pairs in Step 1.(b) and the 2^{16} possible values of $(K_1')_{/1/}$, on which we iterate in Step 1.(c). Thus, the total time complexity of the online algorithm is about 2^{49} evaluations of 1-step LED. Including the preprocessing time, the total time complexity of the attack is about 2^{49} evaluations of 2-step LED (which is a bit less than 2^{49} evaluations of the full 3-step scheme). The data complexity of the attack is 2^{49}, and its memory complexity is 2^{49}, required in order to store the list L.

A Single-Key Attack on a Variant of 2-Step LED with Independent Keys Consider a variant of 8-round LED, where the three round keys K_1, K_2 and K_3 are independent. We now show how to adapt the above attack to this scheme with about the same time/memory/data complexities. We note that this construction has a similar structure to the block cipher AES^2 [3], which is composed of two key-less AES-128 permutations such that K_1 and K_2 are added before and after the first permutation and K_3 is added after the second permutation. Since our techniques only exploit the AES structure of the LED step function, this attack can also be applied to AES^2, reduced from 20 rounds to 8 rounds (with the complexity of the attack adjusted to the 128-bit cipher).

In this attack, we select Δ in a similar way to the related-key attack above. However, in the single-key attack we cannot inject a pair of different messages such that they have a zero difference at the input to F_1. Thus, we now preprocess F_1 (instead of F_2) and find 2^{31} pairs of inputs to this function, $\hat{Y}_0^{j,1}, \hat{Y}_0^{j,2} = \hat{Y}_0^{j,1} \oplus \Delta$, such that $(\hat{Y}_4^{j,1} \oplus \hat{Y}_4^{j,2})_{\backslash 0,1\backslash} = 0$. Consequently, in the online algorithm, we request the encryptions of 2^{48} pairs of plaintexts with an input difference of Δ, and apply the differential MITM technique to F_2.

Another difference between this attack and the previous related-key attack on LED-64, is that now K_1, K_2 and K_3 are independent, and thus we do not have any filtering conditions on the key when matching the suggestions during the MITM phase (i.e., we do not have a linear key sieve). This implies that if we use only one pair of plaintexts, the complexity of the attack will be at least 2^{64}. In order to speed up the attack, we use the same idea used in [8] in the attack on AES^2: we choose an arbitrary non-zero difference $\Delta' \neq \Delta$, and ask for the encryption of another plaintext $P^{i,1} \oplus \Delta'$ for each pair $P^{i,1}$ and $P^{i,1} \oplus \Delta$. Similarly, we attach another evaluation of $\hat{Y}_0^{j,1} \oplus \Delta'$ to each evaluated pair $\hat{Y}_0^{j,1}$ and $\hat{Y}_0^{j,1} \oplus \Delta$. This allows us to obtain the required filtering values such that the attack has similar time/memory/data complexities to the related-key attack. The full details of the filtering technique are found in [8].

7 Conclusions

In this paper, we introduced various techniques in MITM attacks including the linear key sieve technique, a known plaintext splice-and-cut attack, and new techniques for differential MITM. We applied these techniques to step-reduced LED-64 and obtained the best known results on this block cipher, both in the single-key and related-key models. Although our techniques are mainly applied to LED-64, we believe that they will be useful in the analysis of other cryptosystems, in particular AES-based cryptosystems and lightweight block ciphers.

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A The Auxiliary Preprocessing Algorithm of the Related-Key Attack on 3-Step LED-64

Our goal in the auxiliary preprocessing algorithm is to find (about) 2^{31} pairs $(\hat{Y}_4^{j,1}, \hat{Y}_4^{j,2} = \hat{Y}_4^{j,1} \oplus \mathit{\Delta})$, such that $(Y_8^{j,1} \oplus Y_8^{j,2})_{\backslash 0,1\backslash} = \mathit{\Delta}_{\backslash 0,1\backslash}$. In order to run in

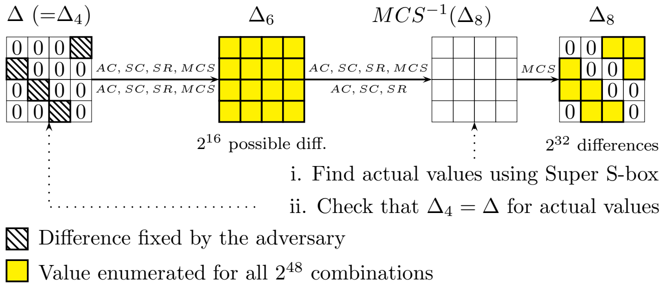
Δ (=Δ4)
Δ6
MCS−1(Δ8)
Δ8
AC, SC, SR, MCS
AC, SC, SR, MCS
AC, SC, SR, MCS
AC, SC, SR
MCS
0
2^16 possible diff.
2^32 differences
i. Find actual values using Super S-box
ii. Check that Δ4 = Δ for actual values
Difference fixed by the adversary
Value enumerated for all 2^48 combinations

Fig. 8. The Auxilary Algorithm Used in Preprocessing (depicting differences)

time of about 2^{48} evaluations of 1-step LED, we assume that $\varDelta_{\backslash 0,1,2\backslash} = 0$, and the value of $\varDelta_{\backslash 3\backslash}$ is arbitrary (but non-zero).

Let $\varDelta_r = Y_r^{j,1} \oplus Y_r^{j,2}$. The difference $\varDelta_8$ can obtain $2^{32} - 1$ non-zero values (since we require that $\varDelta_{8,\backslash 0,1\backslash} = (Y_8^{j,1} \oplus Y_8^{j,2})_{\backslash 0,1\backslash} = \varDelta_{\backslash 0,1\backslash} = 0$), and in addition $MCS^{-1}(\varDelta_6)$ can obtain (at most) $2^{16} - 1$ non-zero values, implying that $\varDelta_6$ can obtain $2^{16} - 1$ non-zero values. The algorithm is given below.

1. For each of the possible $2^{16} - 1$ non-zero value of $\varDelta_6$:
   (a) For each non-zero value of $\varDelta_8$ such that $\varDelta_{8,\backslash 0,1\backslash} = 0$:
      i. Calculate $MCS^{-1}(\varDelta_8)$. Given $\varDelta_6$, use the difference distribution tables for the LED Super-Sboxes to obtain the actual values $Y_8^{j,1}, Y_8^{j,2}$.
      ii. Compute $\hat{Y}_4^{j,1} = F_2^{-1}(Y_8^{j,1})$ and $\hat{Y}_4^{j,2} = F_2^{-1}(Y_8^{j,2})$, and if $(\hat{Y}_4^{j,1} \oplus \hat{Y}_4^{j,2})_{\backslash 3\backslash} = \varDelta_{\backslash 3\backslash}$, store the pair.

Since we obtain an average of one value for $Y_8^{j,1}$ and $Y_8^{j,2}$ in Step 1.(a).i (when considering ordered pairs), the expected time complexity of the algorithm is 2^{48}. The condition of Step 1.(a).ii holds for about 2^{-16} of the pairs, and thus we expect to return 2^{32} ordered pairs, or 2^{31} unordered pairs as claimed. We note that it is possible to implement the algorithm such that it always returns (at least) 2^{31} pairs. This can be achieved by storing all the 2^{48} pairs in Step 1.(a).ii, and finally setting the value of $\varDelta_{\backslash 3\backslash}$ to a value for which there is a maximal number of pairs.