

Match Me if You Can: Matchmaking Encryption and its Applications

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Abstract. We introduce a new form of encryption that we name *matchmaking encryption* (ME). Using ME, sender S and receiver R (each with its own attributes) can both specify policies the other party must satisfy in order for the message to be revealed. The main security guarantee is that of privacy-preserving policy matching: During decryption nothing is leaked beyond the fact that a match occurred/did not occur.

ME opens up new ways of secretly communicating, and enables several new applications where both participants can specify fine-grained access policies to encrypted data. For instance, in social matchmaking, S can encrypt a file containing his/her personal details and specify a policy so that the file can be decrypted only by his/her ideal partner. On the other end, a receiver R will be able to decrypt the file only if S corresponds to his/her ideal partner defined through a policy.

On the theoretical side, we define security for ME, as well as provide generic frameworks for constructing ME from functional encryption.

These constructions need to face the technical challenge of simultaneously checking the policies chosen by S and R, to avoid any leakage.

On the practical side, we construct an efficient identity-based scheme for equality policies, with provable security in the random oracle model under the standard BDH assumption. We implement and evaluate our scheme and provide experimental evidence that our construction is practical. We also apply identity-based ME to a concrete use case, in particular for creating an anonymous bulletin board over a Tor network.

Keywords. Secret handshake, attribute-based encryption, social matchmaking, Tor.

1 Introduction

Intelligence operations often require secret agents to communicate with other agents from different organizations. When two spies meet to exchange secrets, they use a type of secret handshake to ensure that the parties participating in the exchange are the ones intended. For example, an FBI agent may want to communicate only with CIA agents, and if this is not the case, the communication should drop without revealing membership information and why the communication failed. This form of *live drop* communication,¹ when parties are

¹ See https://en.wikipedia.org/wiki/Dead_drop.

online and interact, has been implemented in cryptography and it is referred to as secret handshake (SH) protocol [9]. In SH, two parties agree on the same secret key only if they are both from the same group. Privacy is preserved in the sense that, if the handshake fails, nobody learns anything relevant other than the participants are not in the same group. In SH with dynamic matching [6], groups and roles can even be determined just before the protocol execution.

SH can be thought of as an evolution of traditional key exchange protocols, where protecting privacy of the participants assumes an essential role. As any other key agreement protocol, SH is inherently interactive and its purpose is for the parties to converge on a secret key. A natural question is whether there exists a non-interactive version of SH, in a similar way as ElGamal public-key encryption can be interpreted as a non-interactive version of the classical Diffie-Hellman key exchange. This new cryptographic primitive would allow senders to encrypt messages offline given only the public key of the receiver, thus getting rid of real-time interactions, while at the same time providing strong privacy guarantees for time-delayed communications such as email. Non-interactivity mitigates or prevents *traffic analysis* which affects all SH protocols when deployed within a network environment (see, e.g., [6]). In particular, increased traffic between nodes may signal to an adversary that the SH protocol was successful, even though the nodes' group affiliations and roles remain private.

Non-interactive SH is even more relevant if we consider that the most common method of espionage tradecraft is the *dead drop* one,¹ which maintains operational security by using a secret location for the exchange of information, thus relieving the agents from meeting in person. Unfortunately, dead-drop communication cannot be captured by any existing cryptographic primitive, since it requires a form of expressiveness that is not currently provided by encryption and its more advanced forms.

Matchmaking encryption. In this paper, we are revamping the encryption primitive and introducing a new concept termed "*Matchmaking Encryption*", or ME. In ME, a trusted authority generates encryption and decryption keys associated, respectively, to attributes of the sender and the receiver. The authority also generates an additional decryption key for the receiver, associated to an arbitrary policy of its choice. The sender of the message can specify on the fly an arbitrary policy the receiver must satisfy in order for the message to be revealed. The guarantee is now that the receiver will obtain the message if and only if a match occurs (i.e., the sender's attributes match the receiver's policy and vice-versa). Nothing beyond that is leaked; furthermore, the sender's attributes are certified by the authority, so that no malicious sender can forge a valid ciphertext which embeds fake attributes.

For instance, the sender, during encryption, can specify that the message is intended for an FBI agent that lives in NYC. The receiver, during decryption, can also specify that he wants to read messages only if they come from CIA agents. If any of these two policies is not satisfied, the message remains secret, but nobody learns which policy failed. In this vein, ME can be seen as a non-interactive version of SH, but with much more enhanced functionality. Indeed,

an SH works only for groups and roles, while attribute-based key agreements [25] do not consider privacy. We refer the reader to §1.3 for a comparison between ME and other primitives in the realm of attribute-based cryptography.

Other killer applications of ME are those where the receiver must be sheltered from the actual content of messages to avoid liability, inconvenience or inappropriateness. ME naturally tackles *social matchmaking* confidentiality, where potential partners open files intended for them but only if they contain the traits of the desired person; if decryption fails, nobody learns why, so that privacy is preserved. Encrypting bids (or votes) under ME provides an exciting twist to well-studied problems. Bidders send private bids to a collector and specify the conditions under which the encryption should be opened. The collector opens only the bids that match specific requirements. If decryption fails, the collector does not learn why, and the actual bid (or vote) remain sealed. ME avoids exposing information connected to unlooked-for bids which could influence the receiver and adversely affect the bidding process outcome.

ME also supports marginalized and dissident communities in authoritarian countries. It can act as an enabler for journalists, political activists and minorities in free-speech technical applications such as SecurePost ([35]) that provides verified group anonymity. Indeed, in their thorough study [35], the authors reveal that, in authoritarian countries, anonymous communication may not be credible and cannot be trusted since sources are unknown.² ME provides a comprehensive technical solution for censorship-resistant communication while providing source authenticity and strong privacy guarantees that cannot be obtained with existing tools. For instance, the ability to check ciphertexts against a policy before decryption allows journalists or activists to vet messages and avoid exposure to unwanted information that would make them liable. To this end, in Section §6, we introduce and implement a privacy-preserving bulletin board that combines Tor hidden services with ME to allow parties to collect information from anonymous but authentic sources.

1.1 Our Contributions

We initiate a systematic study of ME, both in terms of definitions and constructions. Our main contributions are summarized below.

Syntax of ME. In ME, a trusted authority publishes a master public key mpk , associated to a master secret key msk . The master secret key msk is used by the authority to generate three types of keys: (i) An encryption key ek_σ , associated with attributes σ for the sender (created using an algorithm SKGen); (ii) A decryption key dk_ρ , associated with attributes ρ for the receiver (created using an algorithm RKGen); (iii) A decryption key $\text{dk}_\mathbb{S}$, associated to a policy \mathbb{S} that the sender's attributes should satisfy, but that is chosen by the receiver (created using an algorithm PolGen).

² See <https://www.news.ucsb.edu/2019/019308/anonymous-yet-trustworthy>

A sender with attributes σ , and corresponding encryption key ek_σ obtained from the authority, can encrypt a plaintext m by additionally specifying a policy \mathbb{R} (chosen on the fly), thus yielding a ciphertext c that is associated with both σ and \mathbb{R} . Finally, the receiver can attempt to decrypt c using keys dk_ρ and $\text{dk}_\mathbb{S}$: In case of a match (i.e., the attributes of both parties satisfy the counterparty’s policy), the receiver obtains the plaintext, and otherwise an error occurs.

Security of ME. We consider two properties termed *privacy*, and *authenticity*. On rough terms, privacy looks at the secrecy of the sender w.r.t. the plaintext m , the chosen policy \mathbb{R} , and its attributes σ , whenever a malicious receiver, possessing decryption keys for several attributes ρ and policies \mathbb{S} :

- Can’t decrypt the ciphertext (“mismatch condition”), i.e., either the sender’s attributes do not satisfy the policies held by the receiver ($\mathbb{S}(\sigma) = 0$), or the receiver’s attributes do not satisfy the policy specified by the sender ($\mathbb{R}(\rho) = 0$).
- Can decrypt the ciphertext (“match condition”), i.e., both the sender’s and the receiver’s attributes satisfy the corresponding policy specified by the counterpart ($\mathbb{R}(\rho) = 1$ and $\mathbb{S}(\sigma) = 1$). Of course, in such a case the receiver is allowed to learn the plaintext.

On the other hand, authenticity says that an attacker not possessing attributes σ should not be able to create a valid ciphertext (i.e., a ciphertext not decrypting to \perp) w.r.t. any access policy that is satisfied by σ .

Black-box constructions. It turned out that building matchmaking encryption is quite difficult. While a compiler turning key agreement into public-key encryption exists (e.g., Diffie-Hellman key exchange into ElGamal public-key encryption), there is no obvious way of building ME from SH, even by extending the model of SH to include attributes and policies in order to achieve something akin to attribute-based key agreement protocols. The main technical challenge is to ensure that the policies established by the sender and receiver are simultaneously checked to avoid any leakage. This simultaneity requirement is so elusive that even constructions combining attribute-based encryption (ABE) with authentication mechanisms fail to achieve it (more on this later).

Our first technical contribution is a construction of an ME for arbitrary policies based on three tools: (i) an FE scheme for randomized functionalities [1] (rFE), (ii) digital signatures, and (iii) non-interactive zero-knowledge (NIZK) proofs. When using the rFE scheme from [1], we can instantiate our scheme assuming the existence of *either* semantically secure public-key encryption schemes and low-depth pseudorandom generators, *or* concrete assumptions on multi-linear maps, *or* polynomially-secure indistinguishability obfuscation (iO).

This construction satisfies only security against bounded collusions, where there is an a-priori upper bound on the number of queries a malicious receiver can make to oracles RGen and PolGen. We additionally give a simpler construction of ME for arbitrary policies that even achieves full security (i.e., security against unbounded collusions), albeit under stronger assumptions. In particular,

we replace rFE with 2-input functional encryption (2FE) [24]. When using the 2FE scheme by Goldwasser *et al.* [24], we can instantiate this construction based on sub-exponentially secure iO.

Being based on strong assumptions, the above constructions should be mainly understood as feasibility results showing the possibility of constructing ME for arbitrary policies. It is nevertheless worth pointing out a recent construction of iO based on LWE, bilinear maps, and weak pseudorandomness [4], which avoids multi-linear maps. Additionally, Fisch *et al.* [20] show how to implement efficiently FE and 2FE using Intel’s Software Guard Extensions (SGX), a set of processors allowing for the creation of isolated execution environments called *enclaves*. At a high level, in their practical implementation, a functional decryption key sk_f consists of a signature on the function f , while messages are encrypted using standard PKE. In order to run the decryption algorithm, a client sends sk_f together with ciphertext c to a *decryption enclave*, which first checks if the signature is valid (i.e., the function evaluation has been authorized by the authority), and if so it decrypts c by using the corresponding secret key, and outputs the function f evaluated on the plaintext. Lastly, the enclave erases its memory. This approach can be applied directly to FE, 2FE, and even rFE for arbitrary functionalities, which, thanks to our results, makes ME for arbitrary policies practical in the trusted hardware setting.

The identity-based setting. Next, we turn to the natural question of obtaining efficient ME in restricted settings. In particular, we focus on the identity-based setting where access policies are simply bit-strings representing identities (as for standard identity-based encryption). This yields identity-based ME (IB-ME). For this setting, we provide an efficient construction that we prove secure in the random oracle model (ROM), based on the standard bilinear Diffie-Hellman assumption (BDH) over bilinear groups.

Recall that in ME the receiver needs to obtain from the authority a different key for each access policy \mathbb{S} . While this requirement is perfectly reasonable in the general case, where the policy might consist of the conjunction of several attributes, in the identity-based setting a receiver that wants to receive messages from several sources must obtain one key for each source. As this would not scale well in practice, we change the syntax of IB-ME and remove the PolGen algorithm. In particular, the receiver can now specify on the fly an identity string snd (playing the role of the access policy \mathbb{S}) that is directly input to the decryption algorithm (together with the secret key associated to the receiver’s identity).

While the above modification yields much more efficient IB-ME schemes, it comes with the drawback that an adversary in the privacy game can try to unlock a given ciphertext using different target identities snd chosen on the fly. The latter yields simple attacks that required us to tweak the definition of privacy in the identity-based setting slightly. We refer the reader to §5 for more details.

	Type	Privacy Authenticity		Assumptions
§4	ME	\checkmark^{\ddagger}	\checkmark^{\ddagger}	rFE + Signatures + NIZK
§5	IB-ME	\checkmark^{\dagger}	\checkmark^{\dagger}	BDH (RO model)
[5]	ME	\checkmark	\checkmark	2FE + Signatures + NIZK
[5]	A-ME	\checkmark	\checkmark	FE + Signatures + NIZK

Table 1. Results achieved in this work ([5] is the full version of this paper). \dagger Security only holds in the identity-based setting. \ddagger Security only holds in case of bounded collusions.

Concrete use case and implementation. We give evidence of the practical viability of our IB-ME construction by providing a prototype implementation in Python. Our experimental evaluation can be found in §6. There, we also detail a concrete use case where IB-ME is used in order to realize a prototype of a new privacy-preserving bulletin board that is run on the Tor network [43]. Our system allows parties to communicate privately, or entities such as newspapers or organizations to collect information from anonymous sources.

A public bulletin board is essentially a broadcast channel with memory. Messages can be encrypted under ME so that their content is revealed only in case of a policy match. The privacy-preserving feature of ME ensures that, if decryption fails, nobody learns which policies were not satisfied. This effectively creates secure and private virtual rooms or sub-channels.

Arranged ME. In ME a receiver can obtain independent decryption keys for its attributes and policies. Note that these keys can be arbitrarily combined during decryption. For this reason, we also consider an alternative flavor of ME, called *arranged* matchmaking encryption (A-ME), where there is a single decryption key $\text{dk}_{\rho, \mathbb{S}}$ that describes simultaneously the receiver’s attributes ρ and the policy \mathbb{S} chosen by the receiver. Thus, an A-ME scheme does not come with a PolGen algorithm. This feature makes sense in applications where a receiver has many attributes, each bearing different restrictions in terms of access policies. A-ME is simpler to construct, in fact we show how to obtain A-ME for arbitrary policies from FE for deterministic functionalities, digital signatures, and NIZK proofs.

See Tab. 1 for a summary of our constructions in terms of assumptions and for different flavors of ME.

1.2 Technical Approach

Below, we describe the main ideas behind our constructions of ME. We start by presenting two unsuccessful attempts, naturally leading to our secure constructions. Both attempts are based on FE. Recall that FE allows us to generate decryption keys dk_f associated to a functionality f , in such a way that decrypting a ciphertext c , with underlying plaintext x , under dk_f , yields $f(x)$ (and nothing more). Note that FE implies both ciphertext-policy ABE [12] (CP-ABE) and key-policy ABE [28] (KP-ABE).

First attempt. A first natural approach would be to construct an ME scheme by combining two distinct FE schemes. The idea is to apply sequentially two functionalities f^1 and f^2 , where the first functionality checks whether the sender’s policy \mathbb{R} is satisfied, whereas the second functionality checks whether the receiver’s policy \mathbb{S} is satisfied. More in details, let f^1 and f^2 be the following functions:

$$f_\rho^1(\mathbb{R}, c) = \begin{cases} c, & \text{if } \mathbb{R}(\rho) = 1 \\ \perp, & \text{otherwise} \end{cases} \quad f_{\mathbb{S}}^2(\sigma, m) = \begin{cases} m, & \text{if } \mathbb{S}(\sigma) = 1 \\ \perp, & \text{otherwise} \end{cases}$$

where $\mathbb{R}(\rho) = 1$ (resp. $\mathbb{S}(\sigma) = 1$) means that receiver’s attributes ρ (resp. sender’s attributes σ) satisfy the sender’s policy \mathbb{R} (resp. receiver’s policy \mathbb{S}). A sender now encrypts a message m under attributes σ by first encrypting (σ, m) under the second FE scheme, and thus it encrypts the corresponding ciphertext concatenated with the policy \mathbb{R} under the first FE scheme. The receiver first decrypts a ciphertext using secret key dk_ρ associated with function f_ρ^1 , and then it decrypts the obtained value using secret key $\text{dk}_{\mathbb{S}}$ associated with function $f_{\mathbb{S}}^2$.

While “semantic security” of the underlying FE schemes computationally hides the plaintext of the resulting ME scheme, privacy is not guaranteed completely: In fact, when the first encrypted layer decrypts correctly (resp. does not decrypt correctly), a receiver infers that the sender’s attributes σ match (resp. do not match) the policy \mathbb{S} .

Second attempt. One could think to salvage the above construction as follows. Each function f^i returns a random key r_i in case the corresponding policy (i.e., the policy checked by function f^i) is satisfied, and otherwise it returns a random value generated by running a secure PRF F . Both partial keys r_1, r_2 are then needed to unmask the string $r_1 \oplus r_2 \oplus m$, which is included in the ciphertext.

More precisely, consider functions $f_\rho^1(\mathbb{R}, r_1, k_1)$ and $f_{\mathbb{S}}^2(\sigma, r_2, k_2)$, such that $f_\rho^1(\mathbb{R}, r_1, k_1)$ (resp. $f_{\mathbb{S}}^2(\sigma, r_2, k_2)$) returns r_1 (resp. r_2) if ρ satisfies \mathbb{R} (resp. σ satisfies \mathbb{S}); otherwise, it returns $F_{k_1}(\rho)$ (resp. $F_{k_2}(\mathbb{S})$), where k_1 (resp. k_2) is a key for the PRF F . An encryption of message m w.r.t. attributes σ and policy \mathbb{R} would now consist of three values (c_1, c_2, c_3) , where c_1 is an encryption of (\mathbb{R}, r_1, k_1) under the first FE scheme, c_2 is an encryption of (σ, r_2, k_2) under the second FE scheme, and finally $c_3 = r_1 \oplus r_2 \oplus m$. A receiver (with keys dk_ρ and $\text{dk}_{\mathbb{S}}$ associated to functions f_ρ^1 and $f_{\mathbb{S}}^2$ as before) would decrypt c_1 and c_2 using dk_ρ and $\text{dk}_{\mathbb{S}}$, and finally xor the outputs between them and with c_3 .

As before, “semantic security” still follows from the security of the two FE schemes. Furthermore, it might seem that privacy is also satisfied because, by security of the PRF, it is hard to distinguish whether the decryption of each c_i yields the random string r_i (i.e., there was a match) or an output of F_{k_i} (i.e., there was no match). However, a malicious receiver possessing distinct attributes ρ and ρ' , such that both satisfy the policy \mathbb{R} , is able to figure out whether the sender’s policy is matched by simply decrypting c_1 twice (using attributes ρ and ρ') and comparing if the decryption returns twice the same value (i.e., r_1). A similar attack can be carried out using two different keys for distinct policies \mathbb{S} and \mathbb{S}' , such that both policies are satisfied by the attributes σ .

ME from 2FE. Intuitively, in order to avoid the above attacks, we need to check simultaneously that $\mathbb{S}(\sigma) = 1$ and $\mathbb{R}(\rho) = 1$. 2FE comes handy to solve this problem, at least if one is willing to give up on authenticity. Recall that in a 2FE scheme we can associate secret keys with 2-ary functionalities, in such a way that decrypting ciphertexts c_0, c_1 computed using independent keys ek_0, ek_1 , and corresponding to plaintexts x_0, x_1 , yields $f(x_0, x_1)$ (and nothing more).

Wlog., we reserve the 1st slot to the sender, while the 2nd slot is reserved to the receiver; the administrator gives the key ek_0 to the sender. The sender now encrypts a message m under attributes σ and policy \mathbb{R} by computing $\text{Enc}(\text{ek}_0, (\sigma, \mathbb{R}, m))$, which yields a ciphertext c_0 for the first input of the function f . The receiver, as usual, has a pair of decryption keys $\text{dk}_\rho, \text{dk}_\mathbb{S}$ obtained from the administrator; here, $\text{dk}_\mathbb{S} = \text{Enc}(\text{ek}_1, \mathbb{S}) = c_1$ is an encryption of \mathbb{S} under key ek_1 . Hence, the receiver runs $\text{Dec}(\text{dk}_\rho, c_0, c_1)$, where dk_ρ is associated to the function $f_\rho((m, \sigma, \mathbb{R}), \mathbb{S})$ that returns m if and only if both $\mathbb{R}(\rho) = 1$ and $\mathbb{S}(\sigma) = 1$ (i.e., a match occurs).

On rough terms, privacy follows by the security of the underlying 2FE scheme, which guarantees that the receiver learns nothing more than the output of f . Unfortunately, this construction does not immediately satisfy authenticity. To overcome this limitation, we tweak it as follows. First, we let the sender obtain from the authority a signature s on its own attributes σ ; the signature is computed w.r.t. a verification key that is included in the public parameters of the scheme. Second, during encryption, the sender computes the ciphertext c_0 as above, but now additionally proves in zero knowledge that it knows a valid signature for the attributes that are hidden in the ciphertext. As we show, this modification allows us to prove authenticity, while at the same time preserving privacy. We refer the reader to the full version [5] for the formal proof.

ME from rFE. In §4, we give an alternative solution that combines rFE and FE (and thus can be instantiated from weaker assumptions). Recall that rFE is a generalization of FE that supports randomized functionalities. In what follows, we write f^1 for the randomized functionality supported by the rFE scheme, and f^2 for the deterministic functionality supported by the plain FE scheme. The main idea is to let the sender encrypt (m, σ, \mathbb{R}) under the rFE scheme. We then consider the randomized function f_ρ^1 that checks if ρ satisfies \mathbb{R} : In case a match occurs (resp. does not occur), it returns an encryption of (m, σ) (resp. of (\perp, \perp) , where \perp denotes garbage) for the second function $f_\mathbb{S}^2$ that simply checks whether the policy \mathbb{S} is satisfied or not. The receiver decryption keys are the keys $\text{dk}_\rho, \text{dk}_\mathbb{S}$ associated to the functions f_ρ^1 and $f_\mathbb{S}^2$.

Roughly speaking, since the randomized function f^1 passes encrypted data to f^2 , a malicious receiver infers nothing about the satisfiability of policy \mathbb{R} . On the other hand, the satisfiability of \mathbb{S} remains hidden, as long as the FE scheme for the function f^2 is secure.

While the above construction does not directly satisfy authenticity, we can show that the same trick explained above for the 2FE-based scheme works here as well.

A-ME from FE. Recall that the difference between ME and A-ME lies in the number of decryption keys: While in ME there are two distinct keys (one for the policy \mathbb{S} , and one for the attributes ρ), in A-ME there is a single decryption key $\text{dk}_{\rho,\mathbb{S}}$ that represents both the receiver’s attributes ρ and the policy \mathbb{S} .

As a result, looking at our construction of ME from 2FE, we can now hard-code the policy \mathbb{S} (together with the attributes ρ) into the function, which allows us to replace 2FE with plain FE. This way, each A-ME decryption key $\text{dk}_{\rho,\mathbb{S}}$ is the secret key corresponding to the function $f_{\rho,\mathbb{S}}$ for the FE scheme. The security proof, which appears in the full version [5], only requires FE with game-based security [12], which in turn can be instantiated under much weaker assumptions.

IB-ME. Above, we mentioned that the natural construction of ME where a ciphertext masks the plaintext m with two distinct pads r_1, r_2 —where r_1, r_2 are re-computable by the receiver as long as a match occurs—is insecure. This is because the expressiveness of ME allows us to have two distinct attributes ρ and ρ' (resp. two distinct policies \mathbb{S} and \mathbb{S}') such that both satisfy the sender’s policy \mathbb{R} (resp. both are satisfied by the sender’s attributes σ).

The main idea behind our construction of IB-ME (cf. §5) under the BDH assumption is that the above attack does not work in the identity-based setting, where each receiver’s policy \mathbb{S} (resp. receiver’s policy \mathbb{R}) is satisfied only by the attribute $\sigma = \mathbb{S}$ (resp. $\rho = \mathbb{R}$). This means that an encryption $m \oplus r_1 \oplus r_2$ yields an efficient IB-ME as long as the random pad r_2 (resp. r_1) can be re-computed by the receiver if and only if its policy \mathbb{S} is satisfied (resp. its attributes ρ satisfy the sender’s policy). On the other hand, if \mathbb{S} is not satisfied (resp. ρ does not satisfy the sender’s policy), the receiver obtains a pad r'_2 (resp. r'_1) that is completely unrelated to the real r_2 (resp. r_1). In our scheme, we achieve the latter by following a similar strategy as in the Boneh-Franklin IBE construction [11].

1.3 Related Work

Secret handshakes. Introduced by Balfanz *et al.* [9], an SH allows two members of the same group to secretly authenticate to each other and agree on a symmetric key. During the protocol, a party can additionally specify the precise group identity (e.g., role) that the other party should have.

SH preserves the privacy of the participants, meaning that when the handshake is successful they only learn that they both belong to the same group (yet, their identities remain secret), whereas they learn nothing if the handshake fails. Subsequent work in the area [29,42,6,13,46,49,32,47,31,30,41] focused on improving on various aspects of SH, including members’ privacy and expressiveness of the matching policies (i.e., attribute-based SH).

In this vein, ME can be thought of as a *non-interactive* SH. Indeed, ME gives privacy guarantees similar to that of SH, but it provides a more efficient way to communicate (being non-interactive) and, at the same time, it is more flexible since a party is not constrained to a group.

Attribute-based encryption. The concept of ABE was first proposed by Sahai and Waters [40] in the setting of fuzzy identity-based encryption, where users are identified by a single attribute (or identity string), and policies consist of a single threshold gate. Afterwards, Bethencourt *et al.* [10] generalized this idea to the case where users are described by multiple attributes. Their ABE scheme is a CP-ABE, i.e., a policy is embedded into the ciphertext, whereas the attributes are embedded into the receiver’s decryption keys. The first CP-ABE with non-monotonic access structures was proposed by Ostrovsky *et al.* [37]. Goyal *et al.* [28], instead, introduced KP-ABE, where ciphertexts contain the attributes, whereas the policy is embedded in the decryption keys. Several other CP-ABE and KP-ABE schemes have been proposed in the literature, see, among others, [16,27,48,53,14,15,51,38,50,52,36,8].

In ABE, only one party can specify a policy, and thus only one entity has the power to select the source (or the destination) of an encrypted message. Motivated by this limitation, Attrapadung and Imai [7] introduced dual-policy ABE. Here, the sender encrypts a message by choosing both a policy and a set of attributes. The receiver can decrypt the ciphertext using a single decryption key that describes both the receiver’s policy and attributes. Similarly to ME, if both policies are satisfied by the respective counterpart, the message is revealed.

Dual-policy ABE and ME differ in several aspects. First, on the syntactical level, in ME there are two distinct decryption keys: One for the attributes and one for the policy specified by the receiver. This yields improved flexibility, as receivers are allowed to choose attributes and policies independently. (Indeed, the syntax of dual-policy ABE is more similar to that of A-ME.) Second, on the security level, both ME and A-ME provide much stronger privacy guarantees than dual-policy ABE. In fact, the security definition for dual-policy ABE only protects the secrecy of the plaintext. Additionally, the actual constructions in [7,8] are easily seen not to preserve privacy w.r.t. the sender’s attributes/policy whenever a match does not occur. Intuitively, this is because the procedure that checks, during decryption, whether a match occurred or not, is not an atomic operation. Also note that dual-policy ABE does not directly provide authenticity, which instead is a crucial property for ME and A-ME (those being a type of non-interactive SH).

Attribute-based key exchange. Gorantla *et al.* [25] introduced attribute-based authenticated key exchange (AB-AKE). This is essentially an interactive protocol which allows sharing a secret key between parties whose attributes satisfy a fixed access policy. Note that the policy must be the same for all the parties, and thus it must, e.g., be negotiated before running the protocol.

In a different work, Kolesnikov *et al.* [34] built a different AB-KE without bilateral authentication. In their setting, a client with some attributes (certificated by an authority) wants to authenticate himself to a server according to a fixed policy. The server will share a secret key with the client if and only if the client’s attributes satisfy the server’s policy.

Note that in ME both senders and receivers can choose their own policies, a feature not present in attribute-based key exchange protocols.

Access control encryption. Access control encryption (ACE) [19,33,21,44] is a novel type of encryption that allows fine-grained control over information flow. The actors are a set of senders, a set of receivers, and a sanitizer. The goal is to enforce *no-read* and *no-write* rules (described by a policy) over the communication, according to the sender’s and receiver’s identities.

The flow enforcement is done by the sanitizer, that applies a randomized algorithm to the incoming ciphertexts. The result is that only receivers allowed to communicate with the source will be able to decrypt the sanitized ciphertext correctly, obtaining the original message (*no-read* rule). On the other hand, if the source has not the rights to communicate with a target receiver (e.g., the sender is malicious), then the latter will receive a sanitized ciphertext that looks like an encryption of a random message (*no-write* rule).

ACE and ME accomplish orthogonal needs: The former enables cryptographic control over information flow within a system, whereas the latter enables both the sender and the receiver to specify fine-grained access rights on encrypted data. Furthermore, ACE inherently requires the presence of a trusted sanitizer, whereas ME involves no additional actor (besides the sender and the receiver).

2 Preliminaries

2.1 Notation

We use the notation $[n] \stackrel{\text{def}}{=} \{1, \dots, n\}$. Capital boldface letters (such as \mathbf{X}) are used to denote random variables, small letters (such as x) to denote concrete values, calligraphic letters (such as \mathcal{X}) to denote sets, and serif letters (such as \mathbf{A}) to denote algorithms. All of our algorithms are modeled as (possibly interactive) Turing machines; if algorithm \mathbf{A} has oracle access to some oracle \mathbf{O} , we often implicitly write $\mathcal{Q}_{\mathbf{O}}$ for the set of queries asked by \mathbf{A} to \mathbf{O} .

For a string $x \in \{0, 1\}^*$, we let $|x|$ be its length; if \mathcal{X} is a set, $|\mathcal{X}|$ represents the cardinality of \mathcal{X} . When x is chosen randomly in \mathcal{X} , we write $x \leftarrow_s \mathcal{X}$. If \mathbf{A} is an algorithm, we write $y \leftarrow_s \mathbf{A}(x)$ to denote a run of \mathbf{A} on input x and output y ; if \mathbf{A} is randomized, y is a random variable and $\mathbf{A}(x; r)$ denotes a run of \mathbf{A} on input x and (uniform) randomness r . An algorithm \mathbf{A} is *probabilistic polynomial-time* (PPT) if \mathbf{A} is randomized and for any input $x, r \in \{0, 1\}^*$ the computation of $\mathbf{A}(x; r)$ terminates in a polynomial number of steps (in the input size).

Negligible functions. Throughout the paper, we denote by $\lambda \in \mathbb{N}$ the security parameter and we implicitly assume that every algorithm takes as input the security parameter. A function $\nu : \mathbb{N} \rightarrow [0, 1]$ is called *negligible* in the security parameter λ if it vanishes faster than the inverse of any polynomial in λ , i.e. $\nu(\lambda) \in O(1/p(\lambda))$ for all positive polynomials $p(\lambda)$. We sometimes write $\text{negl}(\lambda)$ (resp., $\text{poly}(\lambda)$) to denote an unspecified negligible function (resp., polynomial function) in the security parameter.

2.2 Signature Schemes

A signature scheme is made of the following polynomial-time algorithms.

- KGen**(1^λ): The randomized key generation algorithm takes the security parameter and outputs a secret and a public key (sk, pk).
- Sign**(sk, m): The randomized signing algorithm takes as input the secret key sk and a message $m \in \mathcal{M}$, and produces a signature s .
- Ver**(pk, m, s): The deterministic verification algorithm takes as input the public key pk , a message m , and a signature s , and it returns a decision bit.

A signature scheme should satisfy two properties. The first property says that honestly generated signatures always verify correctly. The second property, called unforgeability, says that it should be hard to forge a signature on a fresh message, even after seeing signatures on polynomially many messages. See the full version [5] for formal definitions.

2.3 Functional Encryption

Functional Encryption for Randomized Functionalities A functional encryption scheme for randomized functionalities [26] (rFE) $f : \mathcal{K} \times \mathcal{X} \times \mathcal{R} \rightarrow \mathcal{Y}$ consists of the following polynomial-time algorithms.³

- Setup**(1^λ): Upon input the security parameter, the randomized setup algorithm outputs a master public key mpk and a master secret key msk .
- KGen**(msk, k): The randomized key generation algorithm takes as input the master secret key msk and an index $k \in \mathcal{K}$, and outputs a secret key sk_k for f_k .
- Enc**(mpk, x): The randomized encryption algorithm takes as input the master public key mpk , an input $x \in \mathcal{X}$, and returns a ciphertext c_x .
- Dec**(sk_k, c_x): The deterministic decryption algorithm takes as input a secret key sk_k and a ciphertext c_x , and returns a value $y \in \mathcal{Y}$.

Correctness of rFE intuitively says that decrypting an encryption of $x \in \mathcal{X}$ using a secret key sk_k for function f_k yields $f_k(x; r)$, where $r \leftarrow \mathcal{R}$. Since $f_k(x)$ is a random variable, the actual definition requires that whenever the decryption algorithm is invoked on a fresh encryption of a message x under a fresh key for f_k , the resulting output is computationally indistinguishable to $f_k(x)$.

Definition 1 (Correctness of rFE). A rFE scheme $\Pi = (\text{Setup}, \text{KGen}, \text{Enc}, \text{Dec})$ for a randomized functionality $f : \mathcal{K} \times \mathcal{X} \times \mathcal{R} \rightarrow \mathcal{Y}$ is correct if the following distributions are computationally indistinguishable:

$$\{\text{Dec}(\text{sk}_{k_j}, c_i)\}_{k_j \in \mathcal{K}, x_i \in \mathcal{X}} \quad \{f_{k_j}(x_i; r_{i,j})\}_{k_j \in \mathcal{K}, x_i \in \mathcal{X}}$$

where $(\text{mpk}, \text{msk}) \leftarrow \text{Setup}(1^\lambda)$, $\text{sk}_{k_j} \leftarrow \text{KGen}(\text{msk}, k_j)$ for $k_j \in \mathcal{K}$, $c_i \leftarrow \text{Enc}(\text{mpk}, x_i)$ for $x_i \in \mathcal{X}$, and $r_{i,j} \leftarrow \mathcal{R}$.

³ Often, and equivalently, FE schemes are parameterized by a function ensemble $\mathcal{F} = \{f_k : \mathcal{X} \times \mathcal{R} \rightarrow \mathcal{Y}\}_{k \in \mathcal{K}}$.

As for security, the setting of rFE tackles malicious encryptors. However, for our purpose, it will be sufficient to consider a weaker security guarantee that only holds for honest encryptors. In this spirit, the definition below is adapted from [1, Definition 3.3] for the special case of honest encryptors.

Definition 2 ((q_1, q_c, q_2)-NA-SIM-security of rFE). A rFE scheme $\Pi = (\text{Setup}, \text{KGen}, \text{Enc}, \text{Dec})$ for a randomized functionality $f : \mathcal{K} \times \mathcal{X} \times \mathcal{R} \rightarrow \mathcal{Y}$ is (q_1, q_c, q_2)-NA-SIM-secure if there exists an efficient (stateful) simulator $S = (S_1, S_2, S_3, S_4)$ such that for all PPT adversaries $A = (A_1, A_2)$ where A_1 makes at most q_1 key generation queries and A_2 makes at most q_2 key generation query, the output of the following two experiments are computationally indistinguishable:

REAL $_{\Pi, A}(\lambda)$	IDEAL $_{\Pi, A}(\lambda)$
$(\text{mpk}, \text{msk}) \leftarrow \text{Setup}(1^\lambda)$	$(\text{mpk}, \alpha') \leftarrow S_1(1^\lambda)$
$(x^*, \alpha) \leftarrow A_1^{\text{O}_1(\text{msk}, \cdot)}(1^\lambda, \text{mpk})$ where $x^* = (x_0, \dots, x_{q_c})$	$(x^*, \alpha) \leftarrow A_1^{\text{O}'_1(\alpha', \cdot)}(1^\lambda, \text{mpk})$ where $x^* = (x_0, \dots, x_{q_c})$
$c_i \leftarrow \text{Enc}(\text{mpk}, x_i)$ for $i \in [q_c]$	Let $\{k_1, \dots, k_{q_1}\} = \mathcal{Q}_{\text{O}'_1}$
$\text{out} \leftarrow A_2^{\text{O}_2(\text{msk}, \cdot)}(1^\lambda, \{c_i\}, \alpha)$	For $i \in [q_c], j \in [q_1]$
return $(x, \{k\}, \text{out})$	$y_{i,j} = f_{k_j}(x_i; r_{i,j})$, where $r_{i,j} \leftarrow \mathcal{R}$
	$(\{c_i\}, \alpha') \leftarrow S_3(\alpha', \{y_{i,j}\})$
	$\text{out} \leftarrow A_2^{\text{O}'_2(\alpha', \cdot)}(1^\lambda, \{c_i\}, \alpha)$
	return $(x, \{k'\}, \text{out})$

where the key generation oracles are defined in the following way:

$\text{O}_1(\text{msk}, \cdot)$ and $\text{O}_2(\text{msk}, \cdot)$: Are implemented with the algorithm $\text{KGen}(\text{msk}, \cdot)$.
The ordered set $\{k\}$ is composed of the queries made to oracles O_1 and O_2 .
 $\text{O}'_1(\text{st}', \cdot)$ and $\text{O}'_2(\text{st}', \cdot)$: Are implemented with two simulators $S_2(\alpha', \cdot), S_4(\alpha', \cdot)$.
The simulator S_4 is given oracle access to $\text{KeyIdeal}(x^*, \cdot)$, which, on input k , outputs $f_k(x_i; r)$, where $r \leftarrow \mathcal{R}$ for every $x_i \in x^*$. The ordered set $\{k'\}$ is composed of the queries made to oracles O'_1 and the queries made by S_4 to KeyIdeal .

Functional Encryption for Deterministic Functionalities Functional encryption (FE) for deterministic functionalities $f : \mathcal{K} \times \mathcal{X} \rightarrow \mathcal{Y}$ can be cast as a special case of rFE. Since f is a deterministic functionality, correctness now simply says that whenever the decryption algorithm is invoked on a fresh encryption of a message x under a fresh key for f , the resulting output equals $f_k(x)$. The definition of security is also a simple adaptation of Def. 2, with the twist that the ideal functionality in the ideal experiment is deterministic. We refer the reader to the full version [5] for the details.

2.4 Bilinear Diffie-Hellman Assumption

Our practical implementation of IB-ME is provably secure under the BDH assumption, which we recall below.

Definition 3 (BDH assumption). Let \mathbb{G} and \mathbb{G}_T be two groups of prime order q . Let $e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$ be an admissible bilinear map, and let P be a generator of \mathbb{G} . The BDH problem is hard in $(\mathbb{G}, \mathbb{G}_T, e)$ if for every PPT adversary A :

$$\mathbb{P}[A(q, \mathbb{G}, \mathbb{G}_T, e, P, P^a, P^b, P^c) = e(P, P)^{abc}] \leq \text{negl}(\lambda),$$

where $P \leftarrow_s \mathbb{G}^*$, and $a, b, c \leftarrow_s \mathbb{Z}_q^*$.

2.5 Non-Interactive Zero Knowledge

Let R be a relation, corresponding to an NP language L . A non-interactive zero-knowledge (NIZK) proof system for R is a tuple of polynomial-time algorithms $\Pi = (\mathsf{I}, \mathsf{P}, \mathsf{V})$ specified as follows. (i) The randomized algorithm I takes as input the security parameter and outputs a common reference string ω ; (ii) The randomized algorithm $\mathsf{P}(\omega, (y, x))$, given $(y, x) \in R$ outputs a proof π ; (iii) The deterministic algorithm $\mathsf{V}(\omega, (y, \pi))$, given an instance y and a proof π outputs either 0 (for “reject”) or 1 (for “accept”). We say that a NIZK for relation R is *correct* if for all $\lambda \in \mathbb{N}$, every ω output by $\mathsf{I}(1^\lambda)$, and any $(y, x) \in R$, we have that $\mathsf{V}(\omega, (y, \mathsf{P}(\omega, (y, x)))) = 1$.

We define two properties of a NIZK proof system. The first property, called adaptive multi-theorem zero knowledge, says that honest proofs do not reveal anything beyond the fact that $y \in L$. The second property, called knowledge soundness, requires that every adversary creating a valid proof for some statement, must know the corresponding witness. We defer the formal definitions to the full version [5].

3 Matchmaking Encryption

As explained in the introduction, an ME allows both the sender and the receiver, characterized by their attributes, to choose fined-grained access policies that together describe the access rights both parties must satisfy in order for the decryption of a given ciphertext to be successful.

We present two flavors of ME. In the first, which is the standard one, the receiver’s attributes and policy are independent of each other (i.e., a receiver with some given attributes can choose different policies). In the second flavor, dubbed A-ME, the receiver’s attributes and policy are tighten together. For space reasons, we defer the formal definitions for A-ME to the full version [5].

3.1 Security Model

Formally, an ME is composed of the following polynomial-time algorithms:

Setup(1^λ): Upon input the security parameter 1^λ the randomized setup algorithm outputs the master public key mpk , the master policy key kppl , and the master secret key msk . We implicitly assume that all other algorithms take mpk as input.

- SKGen**(msk, σ): The randomized sender-key generator takes as input the master secret key msk , and attributes $\sigma \in \{0, 1\}^*$. The algorithm outputs a secret encryption key ek_σ for attributes σ .
- RKGen**(msk, ρ): The randomized receiver-key generator takes as input the master secret key msk , and attributes $\rho \in \{0, 1\}^*$. The algorithm outputs a secret decryption key dk_ρ for attributes ρ .
- PolGen**(kpol, \mathbb{S}): The randomized receiver policy generator takes as input the master policy key kpol , and a policy $\mathbb{S} : \{0, 1\}^* \rightarrow \{0, 1\}$ represented as a circuit. The algorithm outputs a secret decryption key $\text{dk}_\mathbb{S}$ for the circuit \mathbb{S} .
- Enc**($\text{ek}_\sigma, \mathbb{R}, m$): The randomized encryption algorithm takes as input a secret encryption key ek_σ for attributes $\sigma \in \{0, 1\}^*$, a policy $\mathbb{R} : \{0, 1\}^* \rightarrow \{0, 1\}$ represented as a circuit, and a message $m \in \mathcal{M}$. The algorithm produces a ciphertext c linked to both σ and \mathbb{R} .
- Dec**($\text{dk}_\rho, \text{dk}_\mathbb{S}, c$): The deterministic decryption algorithm takes as input a secret decryption key dk_ρ for attributes $\rho \in \{0, 1\}^*$, a secret decryption key $\text{dk}_\mathbb{S}$ for a circuit $\mathbb{S} : \{0, 1\}^* \rightarrow \{0, 1\}$, and a ciphertext c . The algorithm outputs either a message m or \perp (denoting an error).

Note that the decryption keys dk_ρ and $\text{dk}_\mathbb{S}$ are independent, thus allowing a receiver with attributes ρ to obtain decryption keys for different policies \mathbb{S} . We also remark that the master policy key kpol could be considered as part of the master secret key msk , but we preferred to use distinct keys for clarity.

Correctness. The intuition for correctness is that the output of the decryption algorithm using decryption keys for receiver's attributes ρ and access policy \mathbb{S} , when decrypting an honestly generated ciphertext which encrypts a message m using sender's attributes σ and policy \mathbb{R} , should equal m if and only if the receiver's attributes ρ match the policy \mathbb{R} specified by the sender, and at the same time the sender's attributes σ match the policy \mathbb{S} specified by the receiver. On the other hand, in case of mismatch, the decryption algorithm returns \perp . More formally:

Definition 4 (Correctness of ME). *An ME with message space \mathcal{M} is correct if $\forall \lambda \in \mathbb{N}, \forall (\text{mpk}, \text{kpol}, \text{msk})$ output by $\text{Setup}(1^\lambda), \forall m \in \mathcal{M}, \forall \sigma, \rho \in \{0, 1\}^*, \forall \mathbb{R}, \mathbb{S} : \{0, 1\}^* \rightarrow \{0, 1\}$:*

$$\mathbb{P}[\text{Dec}(\text{dk}_\rho, \text{dk}_\mathbb{S}, \text{Enc}(\text{ek}_\sigma, \mathbb{R}, m)) = m] \geq 1 - \text{negl}(\lambda),$$

whenever $\mathbb{S}(\sigma) = 1$ and $\mathbb{R}(\rho) = 1$, and otherwise

$$\mathbb{P}[\text{Dec}(\text{dk}_\rho, \text{dk}_\mathbb{S}, \text{Enc}(\text{ek}_\sigma, \mathbb{R}, m)) = \perp] \geq 1 - \text{negl}(\lambda),$$

where $\text{ek}_\sigma \leftarrow_s \text{SKGen}(\text{msk}, \sigma)$, $\text{dk}_\rho \leftarrow_s \text{RKGen}(\text{msk}, \rho)$, $\text{dk}_\mathbb{S} \leftarrow_s \text{PolGen}(\text{kpol}, \mathbb{S})$.

Security. We now turn to defining security of an ME via two properties, that we dub *privacy* and *authenticity*. Intuitively, privacy aims at capturing secrecy of the sender's inputs (i.e., the attributes σ , the policy for the receiver \mathbb{R} , and the

plaintext m), in two different conditions: In case of a match between the sender's and receiver's attributes/policy, and in case of mismatch. This is formalized by requiring that the distributions $\text{Enc}(\text{ek}_{\sigma_0}, \mathbb{R}_0, m_0)$ and $\text{Enc}(\text{ek}_{\sigma_1}, \mathbb{R}_1, m_1)$ be computationally indistinguishable to the eyes of an attacker with oracle access to $\text{SKGen}, \text{RKGen}, \text{PolGen}$, where the values $(m_0, m_1, \mathbb{R}_0, \mathbb{R}_1, \sigma_0, \sigma_1)$ are all chosen by the adversary. The actual definition requires some care, as the adversary could, e.g., obtain a decryption key for attributes ρ and policy \mathbb{S} such that $\mathbb{R}_0(\rho) = 0 \vee \mathbb{S}(\sigma_0) = 0$ but $\mathbb{R}_1(\rho) = 1 \wedge \mathbb{S}(\sigma_1) = 1$, which clearly allows him to distinguish by evaluating the decryption algorithm. In order to exclude such "trivial attacks", we quantify privacy over all *valid adversaries*, as explained below:

- In case of a mismatch, i.e., when the adversary cannot decrypt the challenge ciphertext, it must be the case that for each attribute ρ and policy \mathbb{S} for which the adversary knows a valid decryption key: (i) Either ρ does not satisfy policies \mathbb{R}_0 and \mathbb{R}_1 ; (ii) or σ_0 and σ_1 do not satisfy policy \mathbb{S} ; (iii) or ρ does not satisfy \mathbb{R}_0 and σ_1 does not satisfy \mathbb{S} ; (iv) or ρ does not satisfy \mathbb{R}_1 and σ_0 does not satisfy \mathbb{S} .
- In case of match, i.e., when the adversary can decrypt the challenge ciphertext, it must be the case that $m_0 = m_1$, and additionally, for each attribute ρ and policy \mathbb{S} for which the adversary knows a valid decryption key, it holds that both: (i) \mathbb{R}_0 and \mathbb{R}_1 have the same evaluation on attributes ρ (i.e., $\mathbb{R}_0(\rho) = \mathbb{R}_1(\rho)$); and (ii) \mathbb{S} has the same evaluation on attributes σ_0 and σ_1 (i.e., $\mathbb{S}(\sigma_0) = \mathbb{S}(\sigma_1)$).

$\mathbf{G}_{II,A}^{\text{priv}}(\lambda)$	$\mathbf{G}_{II,A}^{\text{auth}}(\lambda)$
$(\text{mpk}, \text{kpol}, \text{msk}) \leftarrow_{\mathbb{S}} \text{Setup}(1^\lambda)$ $(m_0, m_1, \mathbb{R}_0, \mathbb{R}_1, \sigma_0, \sigma_1, \alpha) \leftarrow_{\mathbb{S}} \mathbf{A}_1^{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3}(1^\lambda, \text{mpk})$ $b \leftarrow_{\mathbb{S}} \{0, 1\}$ $\text{ek}_{\sigma_b} \leftarrow_{\mathbb{S}} \text{SKGen}(\text{msk}, \sigma_b)$ $c \leftarrow_{\mathbb{S}} \text{Enc}(\text{ek}_{\sigma_b}, \mathbb{R}_b, m_b)$ $b' \leftarrow_{\mathbb{S}} \mathbf{A}_2^{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3}(1^\lambda, c, \alpha)$ If $(b' = b)$ return 1 Else return 0	$(\text{mpk}, \text{kpol}, \text{msk}) \leftarrow_{\mathbb{S}} \text{Setup}(1^\lambda)$ $(c, \rho, \mathbb{S}) \leftarrow_{\mathbb{S}} \mathbf{A}^{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3}(1^\lambda, \text{mpk})$ $\text{dk}_\rho \leftarrow_{\mathbb{S}} \text{RKGen}(\text{msk}, \rho)$ $\text{dk}_{\mathbb{S}} \leftarrow_{\mathbb{S}} \text{PolGen}(\text{kpol}, \mathbb{S})$ $m = \text{Dec}(\text{dk}_\rho, \text{dk}_{\mathbb{S}}, c)$ If $\forall \sigma \in \mathcal{Q}_{\mathcal{O}_1} : (\mathbb{S}(\sigma) = 0) \wedge (m \neq \perp)$ return 1 Else return 0

Fig. 1. Games defining privacy and authenticity of ME. Oracles $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$ are implemented by $\text{SKGen}(\text{msk}, \cdot), \text{RKGen}(\text{msk}, \cdot), \text{PolGen}(\text{kpol}, \cdot)$.

Definition 5 (Privacy of ME). *We say that an ME II satisfies privacy if for all valid PPT adversaries A :*

$$\left| \mathbb{P} \left[\mathbf{G}_{II,A}^{\text{priv}}(\lambda) = 1 \right] - \frac{1}{2} \right| \leq \text{negl}(\lambda),$$

where game $\mathbf{G}_{II,A}^{\text{priv}}(\lambda)$ is depicted in Fig.1. Adversary A is called valid if $\forall \rho \in \mathcal{Q}_{O_2}, \forall \mathbb{S} \in \mathcal{Q}_{O_3}$ it satisfies the following invariant:

– **(Mismatch condition)**. Either

$$\begin{aligned} & (\mathbb{R}_0(\rho) = \mathbb{R}_1(\rho) = 0) \vee (\mathbb{S}(\sigma_0) = \mathbb{S}(\sigma_1) = 0) \\ & \vee (\mathbb{R}_0(\rho) = \mathbb{S}(\sigma_1) = 0) \vee (\mathbb{R}_1(\rho) = \mathbb{S}(\sigma_0) = 0); \end{aligned} \quad (1)$$

– **(Match condition)**. Or (if $\exists \hat{\rho} \in \mathcal{Q}_{O_2}, \hat{\mathbb{S}} \in \mathcal{Q}_{O_3}$ s.t. Eq. (1) does not hold)

$$(m_0 = m_1) \wedge (\mathbb{R}_0(\rho) = \mathbb{R}_1(\rho)) \wedge (\mathbb{S}(\sigma_0) = \mathbb{S}(\sigma_1)).$$

Note that in the above definition the challenge ciphertext is honestly computed. This is because privacy captures security against malicious receivers. Authenticity, on the other hand, demands that the only way to produce a valid ciphertext under attributes σ is to obtain an encryption key ek_σ from the authority, thus guaranteeing that if a ciphertext decrypts correctly, then it has been created by a sender with the proper encryption key. This captures security against malicious senders.

The latter is modeled by a game in which the attacker has oracle access to SKGen , RKGen , and PolGen . The attacker’s goal is to output a tuple (ρ, \mathbb{S}, c) such that $\text{Dec}(\text{dk}_\rho, \text{dk}_\mathbb{S}, c) \neq \perp$, and none of the encryption keys ek_σ for attributes σ (obtained by the adversary via oracle queries) satisfies the policy \mathbb{S} . Observe that the adversary is not given access to an encryption oracle. The reason for this is that we only consider security in the presence of chosen-plaintext attacks, and thus ciphertexts might be malleable,⁴ which makes it possible to forge in the authenticity game.

Definition 6 (Authenticity of ME). We say that an ME Π satisfies authenticity if for all PPT adversaries A :

$$\mathbb{P}[\mathbf{G}_{II,A}^{\text{auth}}(\lambda) = 1] \leq \text{negl}(\lambda),$$

where game $\mathbf{G}_{II,A}^{\text{auth}}(\lambda)$ is depicted in Fig.1.

Finally, a secure ME is an ME satisfying all the properties.

Definition 7 (Secure ME). We say that an ME Π is secure, if Π satisfies privacy (Def. 5) and authenticity (Def. 6).

Sometimes, we will also consider a weaker definition where there is an a priori upper bound on the number of queries an attacker can make to oracles RKGen and PolGen . We refer to this variant as security against bounded collusions. In particular, we say that an ME is (q_1, q'_1, q_2, q'_2) -secure if it has (q_1, q'_1, q_2, q'_2) -privacy and authenticity, where q_1, q'_1 (resp. q_2, q'_2) denote the number of queries to RKGen and PolGen allowed by A_1 (resp. A_2) in the privacy game.

⁴ Note that malleability (and thus the authenticity property considered in our paper) might be a desirable feature in some scenarios, as it implies a form of deniability. It could also be useful in future extensions of ME (e.g., in the spirit of proxy re-encryption).

Relation to ABE. An ME for arbitrary policies can be used as a CP-ABE with the same expressiveness. The idea is to ignore the attributes of the sender and the policy of the receiver. It is sufficient to set the ABE master public key to $(\text{mpk}, \text{ek}_\sigma)$ and an ABE receiver's decryption key to $(\text{dk}_\rho, \text{dk}_\phi)$, where ek_σ is the encryption key generated for attributes $\sigma = 0^\lambda$, dk_ϕ is the policy key for a tautology ϕ (i.e., a circuit whose output is always 1 regardless of the input), and dk_ρ is the decryption key for attributes ρ . The encryption of a message m under a policy \mathbb{R} works by running the ME encryption algorithm $\text{Enc}(\text{ek}_\sigma, \mathbb{R}, m)$. The receiver will decrypt the ciphertext by using the keys $(\text{dk}_\rho, \text{dk}_\phi)$. Since ϕ is a tautology, it does not matter under which attributes the message has been encrypted. Thus, the scheme will work as a normal CP-ABE.

Following a similar reasoning, ME implies KP-ABE. This is achieved by setting $\text{ek}_\sigma = \sigma$, and by using the same approach described above (i.e., set the sender's policy circuit \mathbb{R} to a tautology ϕ which ignores the receiver's attributes). Note that for this implication authenticity is not required, which is reminiscent of the fact that in ABE the attributes are not explicitly certified by an authority.

4 Black-Box Construction

We explore black-box constructions of ME and A-ME from several types of FE schemes. In particular, in §4.1 we give a construction of ME based on rFE and FE. As discussed in the introduction, such a construction allows us to obtain ME from weaker assumptions, at the price of achieving only security against bounded collusions. In the full version of the paper [5], we describe and analyze two additional schemes: (i) A construction of ME that is secure against unbounded collusions, based on 2FE (and thus on stronger assumptions); (ii) A construction of A-ME based on FE. All schemes additionally rely on digital signatures and on NIZK proofs.

4.1 ME from rFE

Our construction is based on the following two functionalities f^{FE} and f^{rFE} :

$$f_{\mathbb{S}}^{\text{FE}}(\sigma, m) = \begin{cases} m, & \text{if } \sigma \neq \perp \wedge \mathbb{S}(\sigma) = 1 \\ \perp, & \text{otherwise} \end{cases}$$

and

$$f_{(\rho, \text{mpk}_{\text{rFE}})}^{\text{rFE}}(\mathbb{R}, \sigma, m; r) = \begin{cases} \text{Enc}(\text{mpk}_{\text{rFE}}, (\sigma, m); r), & \text{if } \mathbb{R}(\rho) = 1 \\ \text{Enc}(\text{mpk}_{\text{rFE}}, (\perp, \perp); r), & \text{otherwise.} \end{cases}$$

Construction 1 (ME for arbitrary policies) *Let FE, rFE, SS, NIZK be respectively an FE scheme for the deterministic functionality f^{FE} , a rFE scheme for the randomized functionality f^{rFE} , a signature scheme, and a NIZK proof system for the NP relation:*

$$R_1 \stackrel{\text{def}}{=} \left\{ ((c, \text{pk}, \text{mpk}_{\text{rFE}}), (\sigma, s)) : \begin{array}{l} \exists r, m, \mathbb{R} \text{ s.t.} \\ c = \text{Enc}_{\text{rFE}}(\text{mpk}_{\text{rFE}}, (\mathbb{R}, \sigma, m); r) \wedge \\ \text{Ver}(\text{pk}, s, \sigma) = 1 \end{array} \right\}.$$

We construct an ME scheme in the following way:

- Setup**(1^λ): On input the security parameter 1^λ , the setup algorithm computes $(\text{mpk}_{\text{FE}}, \text{msk}_{\text{FE}}) \leftarrow_{\$} \text{Setup}_{\text{FE}}(1^\lambda)$, $(\text{sk}, \text{pk}) \leftarrow_{\$} \text{KGen}_{\text{SS}}(1^\lambda)$, $(\text{mpk}_{\text{rFE}}, \text{msk}_{\text{rFE}}) \leftarrow_{\$} \text{Setup}_{\text{rFE}}(1^\lambda)$, and $\omega \leftarrow_{\$} \mathcal{I}(1^\lambda)$. Finally, it outputs the master secret key $\text{msk} = (\text{msk}_{\text{rFE}}, \text{sk})$, the master policy key $\text{kpol} = \text{msk}_{\text{FE}}$, and the master public key $\text{mpk} = (\text{pk}, \omega, \text{mpk}_{\text{FE}}, \text{mpk}_{\text{rFE}})$. Recall that all other algorithms are implicitly given mpk as input.
- SKGen**(msk, σ): On input the master secret key $\text{msk} = (\text{msk}_{\text{rFE}}, \text{sk})$, and attributes $\sigma \in \{0, 1\}^*$, the algorithm returns the encryption key $\text{ek}_\sigma = (\sigma, s)$ where $s \leftarrow_{\$} \text{Sign}(\text{sk}, \sigma)$ (i.e., s is a signature on attributes $\sigma \in \{0, 1\}^*$).
- RKGen**(msk, ρ): On input the master secret key $\text{msk} = (\text{msk}_{\text{rFE}}, \text{sk})$, and attributes $\rho \in \{0, 1\}^*$, the algorithm computes the decryption key $\text{sk}_{(\rho, \text{mpk}_{\text{FE}})} \leftarrow_{\$} \text{KGen}_{\text{rFE}}(\text{msk}_{\text{rFE}}, (\rho, \text{mpk}_{\text{FE}}))$. Then, it outputs the decryption key $\text{dk}_\rho = \text{sk}_{(\rho, \text{mpk}_{\text{FE}})}$.
- PolGen**(kpol, \mathbb{S}): On input the master policy key $\text{kpol} = \text{msk}_{\text{FE}}$, and policy \mathbb{S} represented as a circuit, the algorithm computes the function key $\text{sk}_{\mathbb{S}}$ by running $\text{KGen}_{\text{FE}}(\text{msk}_{\text{FE}}, \mathbb{S})$. Then, it outputs the decryption key $\text{dk}_{\mathbb{S}} = \text{sk}_{\mathbb{S}}$.
- Enc**($\text{ek}_\sigma, \mathbb{R}, m$): On input an encryption key $\text{ek}_\sigma = (\sigma, s)$, a policy \mathbb{R} represented as a circuit, and a message m , the algorithm encrypt the message by computing $c \leftarrow_{\$} \text{Enc}_{\text{rFE}}(\text{mpk}_{\text{rFE}}, (\mathbb{R}, \sigma, m))$. Finally, it returns the ciphertext $\hat{c} = (c, \pi)$ where $\pi \leftarrow_{\$} \text{P}(\omega, (\text{pk}, c, \text{mpk}_{\text{rFE}}), (\sigma, s))$.
- Dec**($\text{dk}_\rho, \text{dk}_{\mathbb{S}}, c$): On input two keys $\text{dk}_\rho = \text{sk}_{(\rho, \text{mpk}_{\text{FE}})}$, $\text{dk}_{\mathbb{S}} = \text{sk}_{\mathbb{S}}$, and a ciphertext $\hat{c} = (c, \pi)$, the algorithm first checks whether $\text{V}(\omega, (\text{pk}, c, \text{mpk}_{\text{rFE}}), \pi) = 1$. If that is not the case, it returns \perp , and else it returns $\text{Dec}_{\text{FE}}(\text{sk}_{\mathbb{S}}, \text{Dec}_{\text{rFE}}(\text{sk}_{(\rho, \text{mpk}_{\text{FE}})}, c))$.

Correctness of the scheme follows directly by the correctness of the underlying primitives. As for security, we establish the following result, whose proof appears in the full version [5].

Theorem 1. *Let rFE, FE, SS, NIZK be as above. If rFE is $(q_1, 1, q_2)$ -NA-SIM-secure (Def.2), FE is (q'_1, q_1, q'_2) -SIM-secure, SS is EUF-CMA, and NIZK satisfied adaptive multi-theorem zero knowledge and knowledge soundness, then the ME scheme Π from Construction 1 is (q_1, q'_1, q_2, q'_2) -secure.*

5 Identity-Based Matchmaking Encryption

In this section, we present a practical ME for the identity-based setting (i.e., equality policies). As in ME, attributes are encoded by bit strings, but now each attribute $x \in \{0, 1\}^*$ satisfies only the access policy $\mathbb{A} = x$, which means that both the sender and the receiver specify a single identity instead of general policies (represented as a circuit). We will denote by snd and rcv , respectively, the target identities (i.e., the access policies) specified by the receiver and by the sender.

While any ME as defined in §3 perfectly works for this restricted setting, the problem is that in order to select the identity snd of the source, a receiver must

ask to the administrator the corresponding key dk_{snd} such that $\mathbb{S} = \text{snd}$. (Recall that the sender, instead, can already specify the target identity $\mathbb{R} = \text{rcv}$ on the fly, during encryption.) In particular, if the receiver is interested in decrypting ciphertexts from several distinct sources, it must ask for several decryption keys dk_{snd} , which is impractical.⁵

We resolve this issue by removing algorithm `PolGen` from the syntax of an IB-ME, so that the decryption algorithm takes directly as input the description of the target identity snd (i.e., $\text{Dec}(\text{dk}_\rho, \text{snd}, c)$). This way, the receiver can specify the target identity the source must satisfy on the fly, without talking to the authority.

5.1 Security of IB-ME

The choice of removing the `PolGen` algorithm has an impact on the security properties for IB-ME. Below, we revisit each security guarantee in the identity-based setting and explain how (and why) the security definition has to be adapted. We refer the reader to Fig. 2 for the formal definitions.

Privacy of IB-ME. We cannot require that the sender’s identity remains hidden in case of a decryption failure due to a mismatch condition. In particular, a malicious receiver can always change the sender’s target identity in order to infer under which identity a ciphertext has been encrypted.

More formally, consider the adversary that chooses a tuple $(m, m, \text{rcv}, \text{rcv}, \sigma_0, \sigma_1)$, and receives a ciphertext c such that $c \leftarrow^s \text{Enc}(\text{ek}_{\sigma_b}, \text{rcv}, m)$, where the encryption key ek_{σ_b} corresponds to identity σ_b ; the attacker can simply pick a target identity snd' such that, say, $\sigma_0 = \text{snd}'$ (whereas $\sigma_1 \neq \text{snd}'$), and thus distinguish σ_0 from σ_1 by decrypting c with dk_ρ and target identity snd' .⁶ On the other hand, privacy might still hold in case of mismatch, as long as the keys dk_ρ held by the receiver correspond to identities ρ that do not match the receiver’s target identity. Thus, in the security game, an attacker is now valid if for every decryption key dk_ρ obtained from the oracle, it holds that $\rho \neq \text{rcv}_0$ and $\rho \neq \text{rcv}_1$, where the target identities $\text{rcv}_0, \text{rcv}_1$ are chosen by the adversary. Lastly, note that in case of a match, if a receiver has identity ρ and specifies a policy snd , it can automatically infer that $\sigma = \text{snd}$ and $\text{rcv} = \rho$. For this reason, the privacy game does not consider any match condition.

This relaxed form of privacy is enough and desirable in many scenarios. Intuitively, it guarantees that nothing is leaked to an unintended receiver who doesn’t match the sender’s policy; on the other hand, an intended receiver can choose which ciphertexts to decrypt by trying different policies. This feature is essential in our bulletin board application (Section §6) because it allows parties, e.g., journalists and political activists, to select which type of messages to read.

⁵ This is *not* an issue for an ME that supports arbitrary policies, as in that case, a single policy encodes a large number of attributes.

⁶ This attack can be generalized to show that privacy does not hold if the `PolGen` algorithm (and thus the policy key `kpol`) is made public.

IB-ME works well in this scenario since it provides enough flexibility to the intended receivers while protecting senders from possible attackers.

Finally, we note that the above security definition does not guarantee that the message m remains secret with respect to an *honest receiver* that chooses the “wrong” target identity snd . The latter is, however, a desirable feature that our practical scheme will satisfy (cf. Remark 1).

Authenticity of IB-ME. Turning to unforgeability in the identity-based setting, the forgery (c, ρ, snd) is considered valid if for all encryption keys ek_σ obtained by the adversary it holds that $\sigma \neq \text{snd}$, and moreover the identity ρ is not held by the adversary (i.e., the adversary cannot “forge to itself”).

$\mathbf{G}_{\Pi, \mathbf{A}}^{\text{ib-priv}}(\lambda)$	$\mathbf{G}_{\Pi, \mathbf{A}}^{\text{ib-auth}}(\lambda)$
$(\text{mpk}, \text{msk}) \leftarrow \text{Setup}(1^\lambda)$ $(m_0, m_1, \text{rcv}_0, \text{rcv}_1, \sigma_0, \sigma_1, \alpha) \leftarrow \mathbf{A}_1^{\text{O}_1, \text{O}_2}(1^\lambda, \text{mpk})$ $b \leftarrow \{0, 1\}$ $\text{ek}_{\sigma_b} \leftarrow \text{SKGen}(\text{msk}, \sigma_b)$ $c \leftarrow \text{Enc}(\text{ek}_{\sigma_b}, \text{rcv}_b, m_b)$ $b' \leftarrow \mathbf{A}_2^{\text{O}_1, \text{O}_2}(1^\lambda, c, \alpha)$ If $(b' = b)$ return 1 Else return 0	$(\text{mpk}, \text{msk}) \leftarrow \text{Setup}(1^\lambda)$ $(c, \rho, \text{snd}) \leftarrow \mathbf{A}^{\text{O}_1, \text{O}_2}(1^\lambda, \text{mpk})$ $\text{dk}_\rho \leftarrow \text{RKGen}(\text{msk}, \rho)$ $m = \text{Dec}(\text{dk}_\rho, \text{snd}, c)$ If $\forall \sigma \in \mathcal{Q}_{\text{O}_1} : (\sigma \neq \text{snd}) \wedge (\rho \notin \mathcal{Q}_{\text{O}_2}) \wedge$ $(m \neq \perp)$ return 1 Else return 0

Fig. 2. Games defining privacy and authenticity security of IB-ME. Oracles O_1, O_2 are implemented by $\text{SKGen}(\text{msk}, \cdot), \text{RKGen}(\text{msk}, \cdot)$.

Security definitions. The definitions below capture the very same correctness and security requirements of an ME, but translated to the identity-based case.

Definition 8 (Correctness of IB-ME). *An IB-ME $\Pi = (\text{Setup}, \text{SKGen}, \text{RKGen}, \text{Enc}, \text{Dec})$ is correct if $\forall \lambda \in \mathbb{N}, \forall (\text{mpk}, \text{msk})$ output by $\text{Setup}(1^\lambda), \forall m \in \mathcal{M}, \forall \sigma, \rho, \text{rcv}, \text{snd} \in \{0, 1\}^*$:*

$$\mathbb{P}[\text{Dec}(\text{dk}_\rho, \text{snd}, \text{Enc}(\text{ek}_\sigma, \text{rcv}, m)) = m] \geq 1 - \text{negl}(\lambda),$$

whenever $\sigma = \text{snd}$ and $\rho = \text{rcv}$, and otherwise

$$\mathbb{P}[\text{Dec}(\text{dk}_\rho, \text{snd}, \text{Enc}(\text{ek}_\sigma, \text{rcv}, m)) = \perp] \geq 1 - \text{negl}(\lambda),$$

where $\text{ek}_\sigma, \text{dk}_\rho$ are generated by $\text{SKGen}(\text{msk}, \sigma)$, and $\text{RKGen}(\text{msk}, \rho)$.

Definition 9 (Privacy of IB-ME). *We say that an IB-ME Π satisfies privacy if for all valid PPT adversaries \mathbf{A} :*

$$\left| \mathbb{P} \left[\mathbf{G}_{\Pi, \mathbf{A}}^{\text{ib-priv}}(\lambda) = 1 \right] - \frac{1}{2} \right| \leq \text{negl}(\lambda),$$

where game $\mathbf{G}_{\Pi, \mathbf{A}}^{\text{ib-priv}}(\lambda)$ is depicted in Fig 2. Adversary \mathbf{A} is called valid if $\forall \rho \in \mathcal{Q}_{\mathcal{O}_2}$ it satisfies the following invariant:

- (**Mismatch condition**). $\rho \neq \text{rcv}_0 \wedge \rho \neq \text{rcv}_1$

Definition 10 (Authenticity of IB-ME). We say that an IB-ME Π satisfies authenticity if for all PPT adversaries \mathbf{A} :

$$\mathbb{P}[\mathbf{G}_{\Pi, \mathbf{A}}^{\text{ib-auth}}(\lambda) = 1] \leq \text{negl}(\lambda),$$

where game $\mathbf{G}_{\Pi, \mathbf{A}}^{\text{ib-auth}}(\lambda)$ is depicted in Fig.2.

Definition 11 (Secure IB-ME). We say that an IB-ME Π is secure if it satisfies privacy (Def. 9) and authenticity (Def. 10).

5.2 The Scheme

We are now ready to present our practical IB-ME scheme.

Construction 2 (IB-ME) The construction works as follows.

Setup(1^λ): Let $e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$ be a symmetric pairing, and P a generator of \mathbb{G} , with \mathbb{G} , and \mathbb{G}_T of an order q that depends on λ . We also have three hash functions $H : \{0, 1\}^* \rightarrow \mathbb{G}$, $H' : \{0, 1\}^* \rightarrow \mathbb{G}$, $\hat{H} : \mathbb{G}_T \rightarrow \{0, 1\}^\ell$, modeled as random oracles, and a polynomial-time computable padding function $\Phi : \{0, 1\}^n \rightarrow \{0, 1\}^\ell$. We require that for all $m \in \{0, 1\}^n$ one can verify in polynomial time if m has been padded correctly, and moreover that $\Phi(m)$ is efficiently invertible. On input the security parameter 1^λ , the setup algorithm samples two random $r, s \in \mathbb{Z}_q$, and sets $P_0 = P^r$. Finally, it outputs the master public key $\text{mpk} = (e, \mathbb{G}, \mathbb{G}_T, q, P, P_0, H, H', \hat{H}, \Phi)$ and the master secret key is $\text{msk} = (r, s)$. Recall that all other algorithms are implicitly given mpk as input.

SKGen(msk, σ): On input the master secret key msk , and identity σ , the algorithm outputs $\text{ek}_\sigma = H'(\sigma)^s$.

RKGen($\text{mpk}, \text{msk}, \rho$): On input the master secret key msk , and identity ρ , the algorithm outputs $\text{dk}_\rho = (\text{dk}_\rho^1, \text{dk}_\rho^2, \text{dk}_\rho^3) = (H(\rho)^r, H(\rho)^s, H(\rho))$.

Enc($\text{mpk}, \text{ek}_\sigma, \text{rcv}, m$): On input an encryption key ek_σ , a target identity $\text{rcv} = \rho$, and a message $m \in \{0, 1\}^n$, the algorithm proceeds as follows:

1. Sample random $u, t \in \mathbb{Z}_q$.
2. Compute $T = P^t$ and $U = P^u$.
3. Compute $k_R = e(H(\rho), P_0^u)$ and $k_S = e(H(\rho), T \cdot \text{ek}_\sigma)$.
4. Compute $V = \Phi(m) \oplus \hat{H}(k_R) \oplus \hat{H}(k_S)$.
5. Output ciphertext $C = (T, U, V)$.

Dec($\text{mpk}, \text{dk}_\rho, \text{snd}, c$): On input the master public key mpk , a decryption key dk_ρ , a target identity $\text{snd} = \sigma$, and a message m , the algorithm proceeds as follows:

1. Parse c as (T, U, V) .
2. Compute $k_R = e(\text{dk}_\rho^1, U)$ and $k_S = e(\text{dk}_\rho^2, H'(\sigma)) \cdot e(\text{dk}_\rho^3, T)$.
3. Compute $\Phi(m) = V \oplus \hat{H}(k_R) \oplus \hat{H}(k_S)$
4. If the padding is valid, return m . Otherwise, return \perp .

Correctness The correctness of the scheme only depends on the computation of k_R and k_S as evaluated by the decryption algorithm. Here, we require that the padding function Φ satisfies the property that a random string in $\{0, 1\}^\ell$ has only a negligible probability to form a valid padding w.r.t. the function Φ .⁷ Let k_R, k_S be the keys computed during encryption, and k'_R, k'_S the ones computed during decryption. The scheme is correct since $\forall \sigma, \rho, \text{rcv}, \text{snd} \in \{0, 1\}^*$, $\text{ek}_\sigma \leftarrow \text{SKGen}(\text{msk}, \sigma)$, $\text{dk}_\rho \leftarrow \text{RKGen}(\text{msk}, \rho)$:

1. If $\sigma = \text{snd}$ and $\rho = \text{rcv}$:

$$\begin{aligned} k_R &= e(H(\rho), P_0^u) = e(H(\rho)^r, P^u) = \\ &= e(\text{dk}_\rho^1, U) = k'_R, \text{ and} \\ k_S &= e(H(\rho), T \cdot \text{ek}_\sigma) = e(H(\rho), T \cdot H'(\sigma)^s) = \\ &= e(H(\rho), T) \cdot e(H(\rho)^s, H'(\sigma)) = \\ &= e(\text{dk}_\rho^3, T) \cdot e(\text{dk}_\rho^2, H'(\sigma)) = k'_S \end{aligned}$$

2. Otherwise, if $\rho \neq \text{rcv} = \rho'$ or $\sigma \neq \text{snd} = \sigma$

$$\begin{aligned} k_R &= e(H(\rho'), P_0^u) \neq e(H(\rho)^r, P^u) = \\ &= e(\text{dk}_\rho^1, U) = k'_R, \text{ or} \\ k_S &= e(H(\rho), T \cdot \text{ek}_\sigma) = e(H(\rho), T \cdot H'(\sigma)^s) = \\ &= e(\text{dk}_\rho^3, T) \cdot e(\text{dk}_\rho^2, H'(\sigma)) \neq \\ &= e(\text{dk}_\rho^3, T) \cdot e(\text{dk}_\rho^2, H'(\sigma')) = k'_S. \end{aligned}$$

Since k'_R (resp. k'_S) is hashed by the random oracle \hat{H} , then $\hat{H}(k'_R)$ (resp. $\hat{H}(k'_S)$) is statistically close to a random string of length ℓ . Hence, with overwhelming probability, $V \oplus \hat{H}(k_R) \oplus \hat{H}(k'_S)$, where either $k_R \neq k'_R$ or $k_S \neq k'_S$, will produce an invalid padding, and the decryption algorithm returns \perp .

Remark 1 (Plaintext secrecy w.r.t. unauthorized-but-honest receivers). We note that the plaintext is information-theoretically hidden from the point of view of an honest receiver which specifies a target identity that does not match the sender's identity. Moreover, the latter holds even given the internal state of the receiver at the end of the decryption procedure. In fact, since $\hat{H}(k_S)$ is statistically close to uniform, and $|\hat{H}(k_S)| = |\Phi(m)| = \ell$, the decryption algorithm will compute a symmetric key k_S different to the one generated during encryption.⁸

⁷ This can be achieved, e.g., by setting $\ell = n + \lambda + 1$, and by appending to each message the string $1||0^\lambda$.

⁸ It is important to recall that a similar guarantee does not hold in the identity-based setting, when the receiver is semi-honest (cf. §5.1).

Security. As for security, we establish the following result, whose proof appears in the full version [5].

Theorem 2. *Let \mathbb{G}, \mathbb{G}_T be two groups of prime order q , and let $e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$ be an admissible bilinear map. If the BDH problem is hard in $(\mathbb{G}, \mathbb{G}_T, e)$ (Def. 3), then the IB-ME scheme Π from Construction 2 is secure (Def. 11) in the random oracle model.*

6 IB-ME Performance Evaluation and Application to Tor

In this section, we demonstrate that our IB-ME is practical and we use it to implement a novel system for anonymous but authentic communication. We first show in §6.1 the performance evaluation of our IB-ME implementation. We then describe in §6.2 an application for IB-ME built on top of our implementation. The proposed application is a *bulletin board hidden service* that allows parties to collect or exchange anonymous messages that have an expected format and come from authentic sources. It allows users to exchange IB-ME messages over the Tor network, specifically, using the Tor Hidden Services feature (cf. §6.2). Our bulletin board prototype can be used for covert communication by journalists or activists under authoritarian governments. It improves upon systems such as SecurePost ([35]) for verified group anonymity by providing much stronger privacy guarantees since ciphertexts can be vetted *before* decryption.

6.1 Implementation and Evaluation of the IB-ME cryptosystem

We provide an experimental evaluation of the IB-ME cryptosystem. To this end, we implemented a proof of concept in Python 3.6.5 using Charm 0.50 [2], a framework for prototyping pairing-based cryptosystems (among others). Since our IB-ME is defined using symmetric pairings (also called Type-I pairings), we instantiate it with a supersingular curve with a 512-bit base field (curve **SS512** in Charm), which gives approximately 80 bits of security [39]. The execution environment is an Intel NUC7i7BNH with an Intel Core i7-7567U CPU @ 3.50GHz and 16 GB of RAM, running Ubuntu 18.04 LTS.

Table 2. Performance of high- and low-level cryptographic operations of IB-ME

Operation	Minimum (ms)	Average (ms)
Setup	2.197	2.213
RKGen	2.200	2.225
SKGen	3.400	3.429
Encryption	6.942	7.012
Decryption	4.344	4.385

Table 2 shows the cost in milliseconds associated to the main high- and low-level cryptographic operations of IB-ME. We executed these experiments in 50

different runs of 10 times each and both the minimum and average timing was taken for each operation; we use the Python module `timeit` for these measurements. It can be seen that the average timings for the main high-level operations of IB-ME, namely Encryption and Decryption, are 7.012 ms and 4.385 ms, respectively. These results show that the scheme is highly practical.

It is worth mentioning that there is room for improvement in the implementation if we use optimizations such as pre-computation of some pairing operations when one of the arguments is fixed (which occurs in the two pairings during decryption since one argument is a decryption key) or is reused (the two pairings in the encryption function have $H(\rho)$ as an argument), which can lead to speeds-up around 30%, as reported in [18]. Another potential optimization is the use of multipairings in the decryption operation. A promising direction would be to redefine the scheme in a Type-III pairing setting, which allows for more performant curves [22].

Finally, Table 3 shows a summary of the space costs associated to different elements of our IB-ME. We analyze both the theoretical cost and the actual values with the parameters of the experiment. In addition to the use of Charm’s curve `SS512` (which implies that the size of $|\mathbb{G}| = 512$ bits and $|\mathbb{G}_T| = 1024$), we use for the size of identity bitstrings $|\mathbb{G}|$, for the size of messages $n = |\mathbb{G}_T|$, and for the padding output size $\ell = n + \lambda + 1 = 1105$.

Table 3. Space costs of IB-ME elements.

Element	Theoretical cost	Size (in bits)
Encryption key	$ \mathbb{G} $	512
Decryption key	$3 \mathbb{G} $	1536
Message	n	1024
Ciphertext	$2 \mathbb{G} + \ell$	2129
Ciphertext expansion	$\frac{\ell}{n} + \frac{2 \mathbb{G} }{n}$	≈ 2

6.2 An Anonymous Bulletin Board

Here, we describe the implementation of a bulletin board hidden service that is powered by our IB-ME scheme (cf. §5). In a nutshell, our application allows senders to post encrypted messages to an anonymous bulletin board, hosted by a Tor hidden service [45]. To this end, senders specify a target identity string that acts as the receiver’s access policy, as well as the encryption key corresponding to their own identity. Conversely, receivers can fetch encrypted messages from the bulletin board, and try to decrypt them with their own decryption keys (associated with their identity) and the expected identity of the sender. Only those encrypted messages where there is a match between sender and receiver can be decrypted correctly.

Our system protects every party’s privacy in several aspects. First of all, thanks to the nature of Tor hidden services, the IP addresses of each party and

the connection between the client and the server remain hidden. Secondly, if decryption fails nothing is revealed to the parties.

Next, we will give a brief overview of Tor Hidden Services.

Tor and Hidden Services Tor [43] is the most prominent P2P anonymous system, totaling more than 2 million users and 6,000 relays. It allows clients to access the Internet anonymously by hiding the final destination of their connections. It achieves this by creating random circuits between the client and the destination (e.g., web server), where every relay is aware only of its incoming and outgoing links.

Various services can be set up so that they are accessible only within the Tor network. These Tor *Hidden Services* [45], or HS, are run without revealing their IP addresses and can be reached with no prior information. In order to deploy an HS, the owner needs to initialize the service by choosing some relays that will act as introduction points (IPs). The service will keep an open Tor circuit to each IP that will be used as the entry points to access the HS. The IPs' identities are communicated to Tor by creating a service descriptor entry. This entry contains all the information needed to access the service (e.g., description ID, list of IPs, etc.). Then, the entry is uploaded to the hidden service directory (HSDir) which stores the description entries of all available HSs. A node that wants to connect to an HS will (1) retrieve from HSDirs the correct description entry, (2) establish a Tor circuit to a random relay known as the *rendezvous point*, RP in short, and (3) reveal to one of the hidden service's IP (contained in the description entry) the address of the RP. The HS can now open a Tor circuit to the RP, so that the node and the HS can communicate without revealing their respective IP addresses.

Our Anonymous Bulletin Board Our application is composed of two parts: a web server implemented as a Tor hidden service, and a command line client that is used to upload and download data from the server.

A user that wants to post a message to the bulletin board can use the client to encrypt it (using their IB-ME encryption key ek_σ and an identity string policy rcv for the intended receiver), and upload the ciphertext to the web server through the Tor network. These ciphertexts are publicly available.

A receiver can now use the client to download all the ciphertexts and try to decrypt each of them, using the receiver's decryption key dk_ρ and the sender's identity policy snd (given as input to the client). The client will report to the user the outcome of the decryption phase, showing all the successfully decrypted messages. The role of the web server is to store encrypted messages and to offer a simple REST API that allows clients to post and read these messages. In our prototype, we do not include any additional security measure, but in a real-world deployment, specific countermeasures should be taken in order to protect against potential denial of service attacks from clients (e.g., by requiring a proof-of-work along with the request) and/or include some authentication mechanisms. We refer the reader to Fig. 3 for an overview of the system.

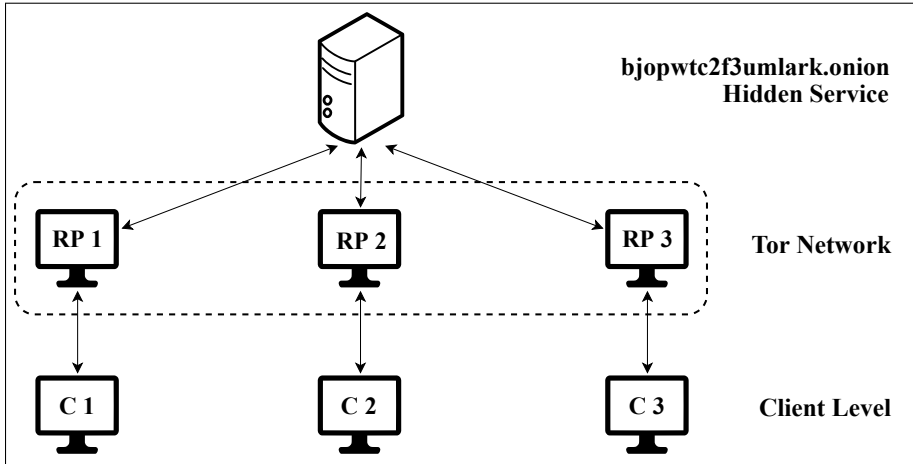


Fig. 3. Example of interaction between three clients C1, C2, C3 and the anonymous bulletin board (<http://bjopwtc2f3umlark.onion>) using Tor. The relays RP1, RP2, and RP3 are the rendezvous points shared between the service and the respective clients. Each party communicates with the respective RP using a Tor circuit.

As in any identity-based cryptosystem, key management requires a key generation service that generates and distributes encryption and decryption keys. This service could be implemented as another Tor hidden service, or even integrated with an existing HSDir (already assumed to be trusted because downloaded from legitimate servers), that automatically converts email addresses or phone numbers into keys. Another possibility is to assume the existence of an off-line authority so that users of the application obtain their keys through an out-of-band channel. In our prototype, we assume the latter option for simplicity.

Finally, note that the performance cost of our Tor application is dominated by the network latency of the Tor relays. Since the main focus of the paper is the new cryptographic primitive, we report only the performance evaluation of our IB-ME scheme (cf. §6.1).

7 Conclusions

We have proposed a new form of encryption, dubbed matchmaking encryption (ME), where both the sender and the receiver, described by their own attributes, can specify fine-grained access policies to encrypted data. ME enables several applications, e.g., communication between spies, social matchmaking, and more.

On the theoretical side, we put forward formal security definitions for ME and established the feasibility of ME supporting arbitrary policies by leveraging FE for randomized functionalities in conjunction with other more standard cryptographic tools. On the practical side, we constructed and implemented practical

ME for the identity-based setting, with provable security in the random oracle model under the BDH assumption. We also showcased the utility of IB-ME to realize an anonymous bulletin board using the Tor network.

Our work leaves open several important questions. First, it would be interesting to construct ME from simpler assumptions. Second, it is conceivable that our black-box construction could be instantiated based on better assumptions since we only need secure rFE w.r.t. honest encryptors; unfortunately, the only definition that is specifically tailored for this setting [3] has some circularity problems [26,1]. Third, a natural direction is to come up with efficient ME schemes for the identity-based setting without relying on random oracles, or to extend our scheme to the case of fuzzy matching [6]. Further extensions include the setting of chosen-ciphertext security, ME with multiple authorities, mitigating key escrow [17,23], and creating an efficient infrastructure for key management and revocation.

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